



Interaction Analysis and Decoupling of Axial-Torsional Vibrations in Rotary Drilling Systems

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ABSTRACT

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This research aims to investigate the interaction analysis and decoupling of axial-torsional vibrations in rotary drilling systems. The primary focus lies in proposing effective compensators for the decoupling process, allowing the extension of the proposed methodology from single-input single-output (SISO) systems to multi-input multi-output (MIMO) systems. Specifically, the objective is to address the strongly interactive terms between the inputs and outputs of the rotary drilling systems. By achieving this, the interconnected multi-loop components can be treated as a series of SISO subsystems, reflecting the comprehensive dynamics of the original system. The study carefully examines two distinct multivariable systems: a theoretical two-input two-output (TITO) system and an actual rotary drilling system. The results demonstrate the efficiency of the proposed approach, emphasizing its superiority over conventional methods. Notably, the proposed methodology effectively reduces dynamic error, settling time, and rise time, highlighting its potential for enhancing the overall performance and robustness of rotary drilling systems.

1. INTRODUCTION

Typically, multivariable systems, in contrast to their single-variable counterparts, often confront numerous challenges within industrial contexts, primarily stemming from the significant coupling between their various inputs and outputs. As a result, researchers have extensively focused on analyzing these interaction dynamics to streamline system complexities [1]. Notably, the relative gain array (RGA) stands out as a widely adopted technique employed to optimize input-output pairings for improved system performance [2]. In addition to these interaction analysis techniques, the growing significance of multivariable control systems has prompted the development of various controllers, with the PID controller emerging as the most prevalent and widely recognized in industrial applications [3]. However, the crucial task of parameter tuning remains a key consideration in the effective design and implementation of such controllers.

Rotary drilling systems play a pivotal role in the extraction of natural resources, serving as a critical

mechanism for accessing subterranean reservoirs of oil, gas, and other valuable minerals. The efficiency and reliability of these systems directly impact the productivity and cost-effectiveness of drilling operations in the energy sector [4], [5]. However, the complex nature of the drilling process, characterized by intricate axial-torsional vibrations, presents a significant challenge in maintaining stable and precise drilling operations. The interplay between axial and torsional dynamics within the system often results in undesirable interaction effects, leading to reduced drilling accuracy, increased wear and tear on equipment, and potential safety concerns [6]. Efforts to address the intricacies of axial-torsional vibrations in rotary drilling systems have spurred extensive research aimed at understanding the underlying dynamics and developing effective control strategies. In this context, the analysis of system interaction and the implementation of decoupling techniques emerge as critical focal points for enhancing drilling system performance. By deciphering the complex interrelationships between the various input and output parameters, researchers strive to isolate and mitigate the adverse effects of coupling, thereby fostering a more stable and responsive drilling environment [7], [8].

This study delves into the comprehensive exploration of interaction analysis and decoupling methodologies tailored to the intricate dynamics of axial-torsional vibrations in rotary drilling systems. By leveraging advanced concepts from the field of control theory and system engineering, the research endeavors to uncover nuanced insights into the underlying mechanics of the coupled system. The proposed compensators and decoupling strategies are poised to transform the conventional understanding of system dynamics, paving the way for enhanced control mechanisms and heightened operational efficiency in rotary drilling applications. This study seeks to establish a robust framework for addressing the challenges posed by axial-torsional vibrations in rotary drilling systems.

2. ROTARY DRILLING SYSTEMS

2.1 System description

One of the prominent systems within the petroleum industry is the rotary drilling system, primarily designed for the excavation of boreholes from the surface to targeted subsurface reservoirs [9]. This system achieves its function by inducing rotational motion to the drilling tool, a process meticulously governed by the top drive mechanism [10]. The linkage between the Top drive and the drill bit is facilitated through a composite assembly of drill pipes commonly referred to as the drill string. The successful operation of this equipment hinges on three fundamental functions [11], [12].

The rotary drilling system can be dissected into two integral components: the drilling rig, situated prominently on the terrestrial surface, and the drill string, which operates beneath the earth's surface [13]. The drilling rig encompasses a towering structure known as a derrick or mast, housing essential lifting and floor equipment, essential for the manipulation, fastening, and unfastening of rods, as well as the substitution of drilling tools. In contrast, the drill string encompasses the subterranean segment of the drilling apparatus. It comprises the upper component, constituted by a series of rods, and the lower portion identified as the Bottom Hole Assembly (BHA). The BHA encompasses crucial elements such as the drilling tool, the drill collar, and stabilizers [14-16]. During drilling operations, three distinct types of vibrations may manifest either concurrently or individually, each capable of causing significant damage and failures within the system's equipment. These vibrations can be categorized based on their propagation directions into three primary types: axial vibrations, lateral vibrations, and torsional vibrations [17]. It is worth noting that multiple types of vibrations can coexist, resulting in coupled modes, such as axial-torsional, torsional-lateral, or lateral-axial-torsional. This study places particular emphasis on the coupled axial-torsional vibration mode, primarily due to its prevalence and its potential deleterious effects on rotary drilling systems [18].

2.2 Coupling Modes of Mechanical Vibrations

2.2.1 Torsional-Lateral Vibrations

In specific drilling scenarios, the response of the drill

string to torsional vibrations may be influenced by the lateral coupling effect, leading to the emergence of critical angular velocities not apparent in the response of uncoupled torsional vibrations. Therefore, a comprehensive consideration of both coupling effects on the angular velocity of the drill string and the interaction between the drill bit and the geological formation becomes imperative in such instances [19]. Although such coupling occurrences are relatively infrequent, their investigation remains significant [22]. Regrettably, the current research study under scrutiny has not specifically addressed this particular type of coupling. However, other types of coupled vibrations have been discussed in the next subsections.

2.2.2 Lateral-Axial Vibrations

Within the domain of rotary drilling systems, lateral vibrations can instigate the whirling phenomenon, which constitutes a critical state capable of inflicting damage upon the drill string and associated equipment [20]. Recent scientific inquiries have elucidated that random whirling, in particular, exerts a more pronounced influence on axial vibrations compared to forward and backward whirling. Specifically, the occurrence of random whirling induces alterations in the drill string's curvature, consequently engendering axial vibration waves. It is worth highlighting that the emergence of whirling and bit-bounce typically transpires simultaneously, often following prior torsional vibrations. In light of the salience of torsional vibrations and their intricate coupling with other vibration modes, the present study will center its focus on coupled vibrations encompassing torsional vibrations [19]. It is essential to emphasize that the coupling effects observed in drilling systems are intricate and contingent upon multiple factors, including drilling conditions, formation properties, and drill string configuration [21]. Consequently, this present study primarily concentrates on coupled axial-torsional vibrations, which represent one of the most prevalent forms of coupled vibrations encountered in drilling operations. The primary objective herein entails the proposition of a PID controller capable of effectively mitigating the detrimental impacts of coupled axial-torsional vibrations on the drilling system. Through the enhancement of controller robustness, the proposed approach seeks to improve system performances.

2.2.3 Axial-Torsional Vibrations

The coupling phenomenon between axial and torsional vibrations, closely linked to the transformation of WOB transmitted to the drill bit through the drill-string, represents a prevalent occurrence during drilling operations. This coupling mode may engender the stick-slip phenomenon, eventually leading to whirling, wherein axial vibration waves are generated and propagate towards the surface [22]. Moreover, the arrival of a reflected axial vibration wave from the surface or a contact point within the drill string can trigger the release of the drill bit from the stick phase, precipitating the slip phase and culminating in a repetitive cycle. Given the insightful

analysis of these interactions, the present study dedicates its focus to the mode of coupled axial-torsional vibrations, owing to its frequent manifestation and pronounced impact on the rate of penetration (ROP) and equipment integrity. This endeavor aims to undertake a comprehensive investigation of the coupled phenomena involving the bit-bounce and the stick-slip [23].

3. MATHEMATICAL MODEL

The mathematical model of coupled torsional-axial vibrations in a drilling system with nonlinear torque on the bit can be described by the following set of equations [17].

3.1 Torsional dynamics

The mathematical equation that describes the torsional dynamic is given by equation (1).

$$I\ddot{\theta} + c\dot{\theta} + k\theta = T(\theta, z) \quad (1)$$

where, I = moment of inertia of the drill string θ = angular displacement of the drill string c = damping coefficient k = torsional stiffness of the drill string $T(\theta, z)$ = nonlinear torque on the bit, which is a function of the angular displacement θ and axial displacement z of the bit [15].

3.2 Axial dynamics

The mathematical equation that describes the torsional dynamic is given by equation (2).

$$m\ddot{z} + b\dot{z} + kz = F(\theta, z) \quad (2)$$

where, m = mass of the drill string z = axial displacement of the drill string b = damping coefficient k = axial stiffness of the drill string $F(\theta, z)$ = axial force on the bit, which is a function of the angular displacement θ and axial displacement z of the bit [21]. The nonlinear equation of T and F englobed the interaction between the axial and the torsional vibration. The equation are given as follows:

$$T = k_1\theta + k_2z^3 \quad (3)$$

$$F = k_2\theta^3 + k_1z \quad (4)$$

3.3 Transfer functions of the TITO system

The For the given coupled torsional-axial vibration system with interactions, let us denote the transfer function matrix as:

$$G(s) = \begin{bmatrix} G_{\theta \rightarrow T} & G_{\theta \rightarrow F} \\ G_{z \rightarrow T} & G_{z \rightarrow F} \end{bmatrix} \quad (5)$$

Where:

$G_{\theta \rightarrow T}$: represents the transfer function from the input torsional motion to the output, torsional motion.

$G_{\theta \rightarrow F}$: represent the transfer function from the input torsional motion to the output axial force.

$G_{z \rightarrow T}$: represent the transfer function from the input torsional motion to the output torsional motion.

$G_{z \rightarrow F}$: represent the transfer function from the input axial motion to the output axial force.

Considering the interaction terms between axial and torsional motions, the transfer function matrix elements can be expressed as:

$$G_{\theta \rightarrow T}(s) = \frac{1}{Is^2 + cs + k - \frac{dT}{d\theta}} \quad (6)$$

$$G_{\theta \rightarrow F}(s) = \frac{dT}{dz} \frac{1}{Is^2 + cs + k - \frac{dT}{d\theta}} \quad (7)$$

$$G_{z \rightarrow T}(s) = \frac{dF}{d\theta} \frac{1}{ms^2 + bs + k - \frac{dF}{dz}} \quad (8)$$

$$G_{z \rightarrow F}(s) = \frac{1}{ms^2 + bs + k - \frac{dF}{dz}} \quad (9)$$

Thus, the obtained transfer functions are given as follows:

$$G_{\theta \rightarrow T}(s) = \frac{1}{Is^2 + (c + k_{at})s + (k - k_{ta})} \quad (10)$$

$$G_{\theta \rightarrow F}(s) = \frac{1}{Is^2 + (c + k_{at})s + (k - k_{ta})} \quad (11)$$

$$G_{z \rightarrow T}(s) = \frac{1}{ms^2 + (b + k_{ta})s + (k - k_{at})} \quad (12)$$

$$G_{z \rightarrow F}(s) = \frac{1}{ms^2 + (b + k_{ta})s + (k - k_{at})} \quad (13)$$

Then, the transfer functions written in the matrix form as follows:

$$G(s) = \begin{bmatrix} \frac{1}{Is^2 + cs + k - k_1} & \frac{3k_2z^2}{Is^2 + cs + k - k_1} \\ \frac{3k_2\theta^2}{ms^2 + bs + k - k_1} & \frac{1}{ms^2 + bs + k - k_1} \end{bmatrix} \quad (14)$$

Here k_{ta} and k_{at} represent the coupling stiffness terms for axial and torsional motions, respectively. These transfer functions account for the coupled dynamics between the axial and torsional vibrations, providing comprehensive representation of the MIMO system with interactive terms

4. INTERACTION ANALYSIS OF TITO DRILLING SYSTEM

Owing to the significant impact of interaction terms within MIMO systems, numerous approaches have been proposed and extensively examined in existing literature to comprehensively assess these effects, as evidenced in studies such as [2], [21], and [22]. To facilitate the evaluation process and identify the optimal input-output pairing with minimal interaction, this study leverages the utilization of the RGA method. The assessment of the method's efficacy is quantitatively conducted through an analysis of the system's steady-state gain, serving as a robust metric for gauging performance outcomes. The RGA method operates on the steady-state gain matrix of the multivariable systems in their open-loop configuration, a fundamental principle articulated by equation (15).

$$\Lambda = \lambda_{ij} * \left[\lambda_{ij}^{-1} \right]^T \quad (15)$$

Here, the symbol $*$ signifies the Hadamard product, representing the element-wise multiplication, while λ_{ij} represents the eigenvalue of the element in the (i, j) position, as derived from equation (16).

$$\lambda_{ij} = \lim_{s \rightarrow 0} G(s) \tag{16}$$

Empirical evidence suggests that the aggregate of the RGA matrix components invariably equates to 1. Consequently, when the elements of the matrix are in close proximity to 1, the interactions are characterized as weak. Conversely, if the elements deviate significantly from 1, the interactions are deemed strong, categorizing the system as highly interactive. The interaction analysis of the system is obtained as:

$$\Lambda = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} \tag{17}$$

Based on equation (17), it is clear that the system is strongly coupled and necessitates decoupling. In the next session we will discuss the explicit decoupling that will be applied to rotary drilling system under axial-torsional vibrations.

5. DECOUPLING PRINCIPE

In this study, the presented approach revolves around the dissection of the intricately interdependent multivariable system via the application of an explicit decoupling methodology. The subsequent step involves the development of a PID controller, drawing upon the established equivalence between the traditional feedback mechanism and the Internal Model Control (IMC) framework [12]. For the sake of simplicity, the focus in this section is directed toward a two-input two-output (TITO) system, delineated by its transfer function $G(s)$ in (6); however, it is worth noting that the methodology can readily extend to systems of higher order complexity.

$$\mathbf{G}(s) = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \tag{18}$$

The initial introduction of the explicit decoupling concept aims to nullify the impact of interconnected loops within the complex multivariable system. This strategy involves the inclusion of a suitable compensator preceding the system process. The current study sheds light on the explicit decoupling principle as applied to a broad-spectrum TITO system, accentuated in Figure. 1.

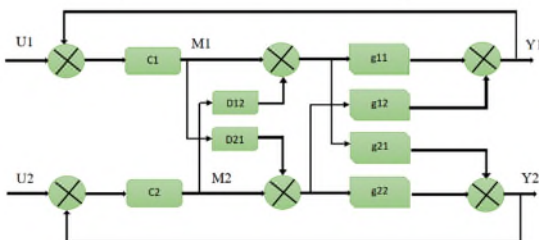


Figure. 1 Explicit decoupling principle with decentralized controller for a general TITO system.

Where $\mathbf{D}(s) = \begin{pmatrix} 1 & D_{12} \\ D_{21} & 1 \end{pmatrix}$ and $\mathbf{C}(s) = \begin{pmatrix} C_1 & 0 \\ 0 & C_2 \end{pmatrix}$ are the decoupler and decentralized controller respectively.

In order to eliminate the interaction effect of the first loop on the second for the studied TITO system, equation (19) should be satisfied [13].

$$g_{22} * D_{21} * M1 + g_{21} * M1 = 0 \tag{19}$$

However, for the elimination of the interaction effect of the second loop on the first, equation (20) should be satisfied.

$$g_{11} * D_{12} * M2 + g_{12} * M2 = 0 \tag{20}$$

From (19) and (20), the decouplers' terms have deduced as given by (9) and (10) respectively [2].

$$D_{12} = -\frac{g_{12}}{g_{21}} \tag{21}$$

$$D_{21} = -\frac{g_{21}}{g_{22}} \tag{22}$$

To design decouplers for the given transfer function, we can use the concept of decoupling control, where we aim to eliminate or minimize the interaction terms between different inputs and outputs. To decouple the system, we need to find decoupling matrices $D1(s)$ and $D2(s)$ such that:

$$D_{12}(s)G(s) = \begin{bmatrix} G_{\theta \rightarrow T}^{decoupled}(s) & 0 \\ 0 & G_{z \rightarrow F}^{decoupled}(s) \end{bmatrix} \tag{23}$$

$$D_{21}(s)G(s) = \begin{bmatrix} G_{\theta \rightarrow F}^{decoupled}(s) & 0 \\ 0 & G_{z \rightarrow T}^{decoupled}(s) \end{bmatrix} \tag{24}$$

The decoupling matrices $D1(s)$ and $D2(s)$ can be designed using various methods, such as the inverse of the static gain matrix or through state feedback. However, the specific design methodology may depend on the system's characteristic, the desired control objectives, and the available control inputs and measurements. We can defined the decoupling matrices $D1(s)$ and $D2(s)$ as the inverse of the static gain matrix.

$$D_{12}(s) = \begin{bmatrix} Is^2 + cs + k - k_1 & 0 \\ 0 & ms^2 + bs + k - k_1 \end{bmatrix}^{-1} \tag{25}$$

$$D_{21}(s) = \begin{bmatrix} Is^2 + cs + k - k_1 & 0 \\ 0 & ms^2 + bs + k - k_1 \end{bmatrix}^{-1} \tag{26}$$

To decouple the system while considering the presence of coupling and cross-coupling terms, we need to employ more sophisticated techniques such as modal analysis, state-space methods, or specific decoupling algorithms. Decoupling a system with non-zero coupling terms often requires a thorough understanding of the system dynamics and interrelationships between the different variables. Additionally, the design of decoupling controllers may involve using state feedback or dynamic compensation techniques to create control laws that effectively eliminate or minimize the undesired coupling effects. The specific decoupling approach and methodology depend on the system's characteristics, the

complexity of the coupling terms, and the control objectives. For a comprehensive decoupling design that accounts for non-zero cross-coupling terms, it is advisable to perform a detailed analysis of the system dynamics and to explore advanced control strategies tailored to the specific requirements and constraints of the system.

6. RESULTS AND DISCUSSION

6.1 Coupled Axial-torsional Model

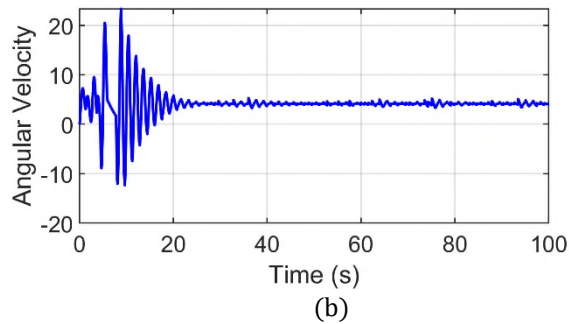
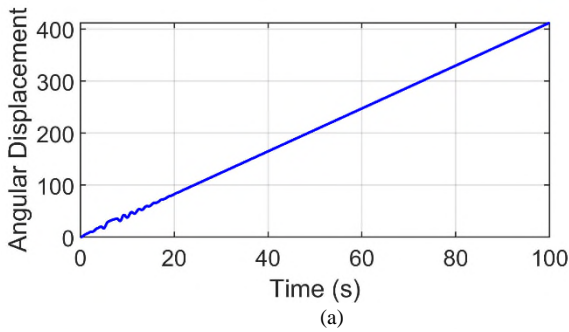


Figure.2 Coupled model responses: a) Angular displacement, b) Angular velocity.

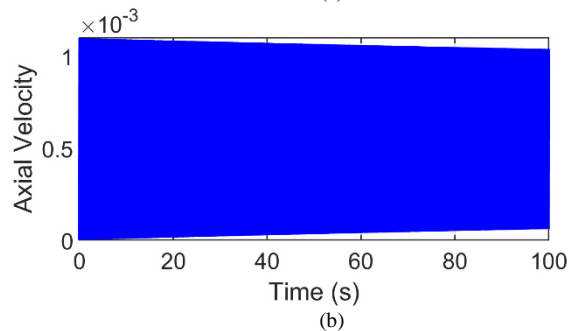
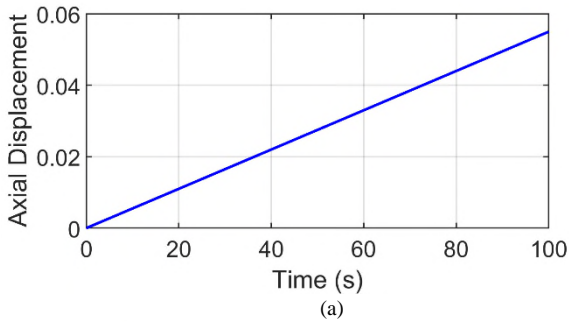


Figure.3 Coupled model responses: a) Axial displacement, b) Axial velocity.

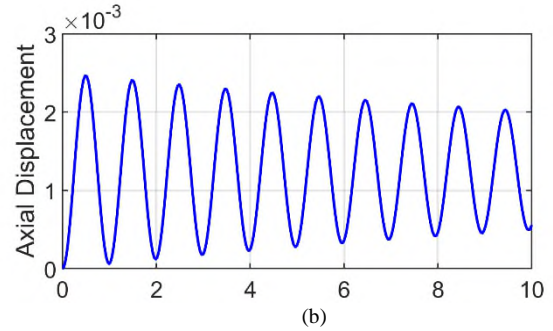
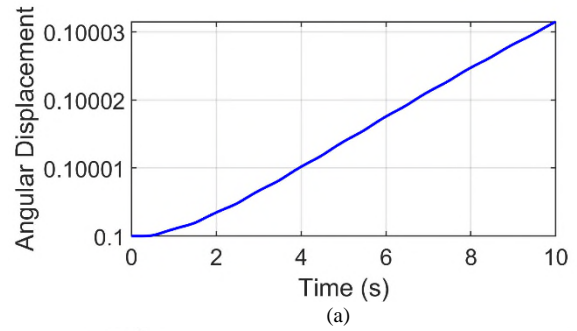


Figure. 4 Coupled model second responses: a) Angular displacement, b) Angular velocity.

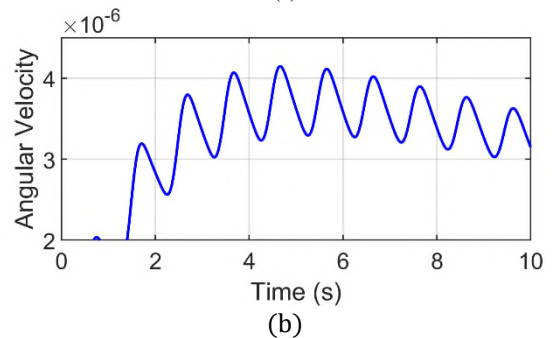
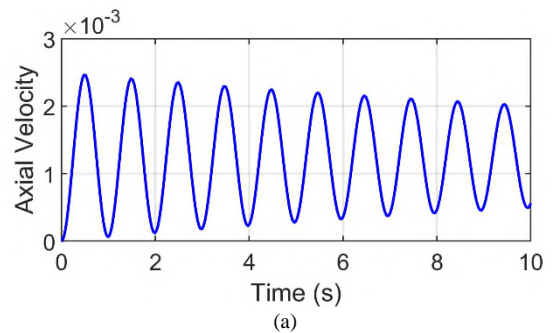
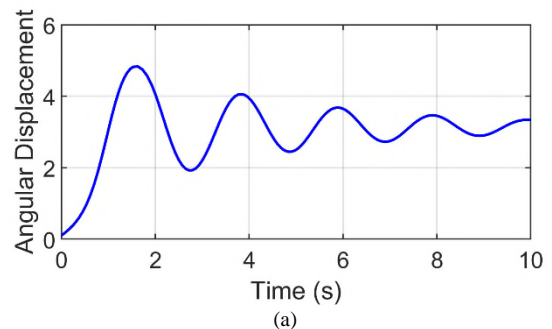


Figure. 5 Coupled model second responses: a) Axial displacement, b) Axial velocity.

6.2 Decoupled Torsional Model



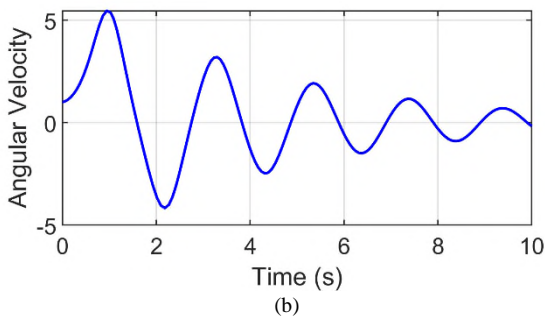


Figure.6 Decoupled torsional model responses: a) Angular displacement, b) Angular velocity.

6.3 Decoupled Axial Model

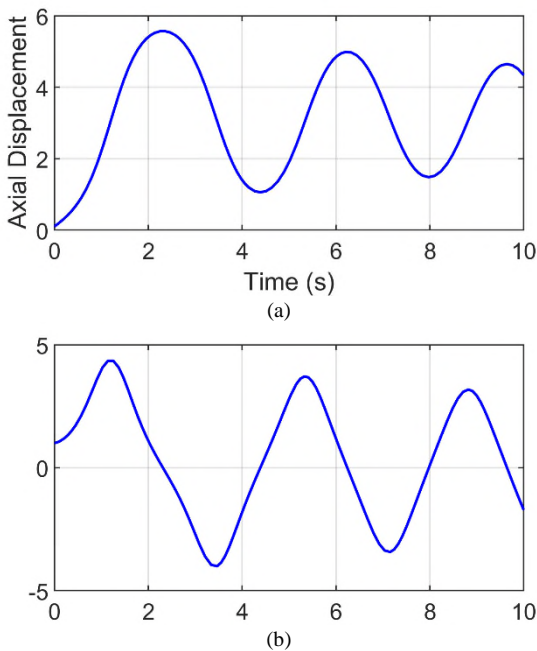


Figure.7 Decoupled axial model responses: a) Axial displacement, b) Axial velocity.

7. CONCLUSIONS

In conclusion, this study delved into the intricate dynamics of axial-torsional vibrations within rotary drilling systems, emphasizing the critical importance of interaction analysis and subsequent decoupling processes. The proposed compensators effectively tackled the challenges posed by the strong interactivity between the system's inputs and outputs, enabling the transformation of interconnected multi-loop components into a comprehensive set of SISO subsystems. This approach not only showcased remarkable adaptability across various multivariable systems but also demonstrated its notable efficiency in significantly reducing dynamic error, settling time, and rise time. The results underscore the pivotal role of this methodology in augmenting the overall performance and robustness of rotary drilling systems, thus paving the way for more refined and effective drilling practices in the future.

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