Monopoly pricing in an M/M/1 queue with breakdowns and repairs

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Introduction

The literature on the strategic behavior of customers in queuing systems is very rich. This literature has given many useful ideas in the conception of queueing systems with rational customers, whose analysis combines two different and important disciplines namely: queueing and game theory.

The study of the rational queueing systems was initiated by Naor [6], who studied an M/M/1 queue with a reward-cost structure, where the system state is observable. His work was completed by Edelson and Hildebrand [4] in the unobservable case. Since then, there has been an increasing number of papers that deal with the strategic behavior of customers in different queuing systems.

In some of these works, authors seek to determine the optimal price of service to be offered by the system according to the strategic behavior of the customers. Chen and Frank [1] have adapted the classic Naor's model [6], where the server (firm) can adjust his service price according to the current state of the queue. Then, Chen and Frank [2] completed this work in the unobservable case.

The works cited so far deal with reliable queueing systems. However, in many real-world situations, the server is subject to random failures that affect the system characteristics. Among the works that have dealt with the analysis of the strategic behavior of the customers in queueing models with unreliable server, we can cite that of Economou and Kanta [3] which extended the model of Naor [6], including the failures and the repairs periods of the server while considering the totally observable case and the almost observable case. More later, Li et al [4] completed this work by dealing with two other types of information: the almost unobservable case and the totally unobservable case.

Recently, another type of modeling has emerged in the queuing systems analysis with reward-cost structure. The latter is represented by dynamic game between the different agents (customers, server, social optimizer, ...) intervening in the system. Ziani et al. [7] studied an M/M/1 queue with three agents (the social optimizer, the service provider, and the customers), where they analyzed the interactions that occur between them and modeled the problem as a three-stage dynamic game.

As we mentioned above, there are many works that have considered the server breakdown problem and others that have considered the service pricing problem in the study of the Markovian queueing systems with strategic customers. Our work combines these both problematic. We study the pricing of the service provided in an M/M/1 queueing system with an unreliable server and strategic customers where the system state is totally unobservable. We use the game theory tools to model the strategic interactions between the server and the customers and we analyze the impact of the service price selected by the server on customers' decisions and the consequences of the customer's decision on the utility of the server. To do that, we propose a two-stage game model between the server and the customers.

Key Words: Strategic customers, Pricing, M/M/1 Queue with server breakdowns and repairs, Two-stage dynamic game, Nash Equilibrium.

Model Description

Fig. 1 Schema of model.

Modeling as a game

We assume that the server acts first and chooses a service price P that it will apply to all customers. Customers arrive sequentially to the system and they are informed of the price displayed by the server. They also have knowledge about the system parameters, but they are not informed on the system state. Each customer arriving to the system should choose between: join the system or choose an outside opportunity.

Therefore, each customer arriving to the system is at the same level of information as the customers who arrived before him and those who will arrive after him. Thus, only the service price P and the system parameters will be used for a customer's decision making. Given the homogeneity of customers, we can assume that a proportion of a q customers decides to join the system and another proportion (1–q) customers chooses the outside opportunity.

Thus, the server interaction with the customers and the sequencing in the decision process bring back to a two-stage dynamic game. At the first stage of the game, the server imposes a service price P. At the second stage, even if the customers arrive to the system sequentially and each arriving customer must make his decision to enter or choose an outside opportunity on the basis of one and the same information for all, which is the price that the service displays, the appropriate game model would be a sequential game with imperfect information with an unknown number of players (customers). We can then consider that customers make their decisions simultaneously.

For a service price P and a proportion q of customers deciding to join, the server utility would be:

$$U_1(P, \alpha = (q, 1-q)) = \lambda q P$$

and the customer utility would be:

$$U_2(P, \alpha = (q, 1-q)) = q(R - P - C.\overline{T}_s) + (1-q)v$$

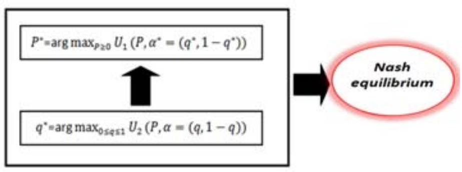

Fig. 2 Schema of resolution.

Numerical example

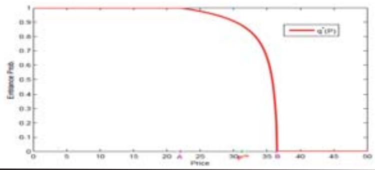

Fig. 3 Entance probabilities vs. Price for R=40, v=3, C=1, μ = 4, θ = 1 and λ = 2.5

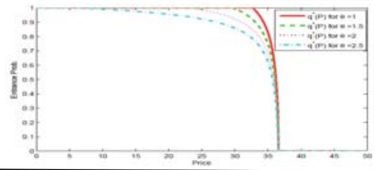

Fig. 4 Entance probabilities vs. Price for R=40, v=3, C=1, μ = 4, r=6 and λ = 2.5

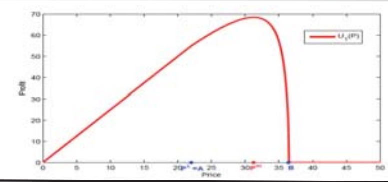

Fig. 5 $U_1(P)$ vs. Price for R=40, v=3, C=1, μ = 4, r=3, θ = 1 and λ = 2.5

Conclusion

Inspired from Li et al. (2014) [5] and Chen and Frank (2004) [2], we studied the interactions between the strategic behaviors of the server and the customers in an M/M/1 queuing system with unreliable server where the state of the system is totally unobservable. This situation was modeled by a two-stage dynamic game (leader-follower).

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