Analysis of an (s,Q) Inventory model with continuous Review and partially backordered demands throw Petri nets

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Doctoriales of Operations Research

12th and13th December 2018

Abstract :

In this work, we analyze an inventory system with (s,Q) control policy with continuous review and batch demands arriving according to a Poisson process and serving instantaneously. If the system is out of stock, an arriving primary demand join a virtual limited orbit and repeat its request after a random time with a constant policy. The model is representing by a new tool called "Batch Deterministic and stochastic Petri nets"(BDSPNs). The most important performance measures for the continuous Markov chain represented by the stock level and the number of demands in the orbit are derived at the steady state.

Key-words: (s,Q) control policy, BDSPNs, Inventory systems, Markov chains, Performance measures.

I. Introduction :

During last few years, many researches are focused on the problem of how companies could operate their stocks, according to different control policies existing on the literature , to prevent the situation of shortages. A lot of suggestions are proposed in several papers, which take into account the different parameters of the policy including (the review, the re-order level, the commodity size, etc) . In the other hand, the more realistic inventory systems are those described by the presence of randomly events (parameters). These type of inventory systems is called the stochastic inventory systems which are the most difficult to analyze. Many researchers modeled these inventory systems by several methods .The most well-known in the literature are : queuing theory [5] and Markov chains [1,2,3,6].

For example in [1,2] the author model an (R,s,S) inventory system with periodic review through discrete Markov chain. In [3], Artalejo used the same tool and introduced the retrial notion in the inventory systems with shortages and infinite source of demands. Since then, several researchers have taken over the work of Artalejo while considering a finite source of requests (see the contribution of Periyasamy [6]).

Another important tool is Petri nets created in 1962 by Carl Adam Petri. Its powerful is due to its graphical and mathematical representation, which offer us more simplifications in the analysis of the considered model. Even so, we see just some applications of this tool in the inventory systems. As an example, we can seat the contribution of Labadi [4], who introduced a new model called "Batch Deterministic and Stochastic Petri Nets". He proved its applicability, by modeling a continuous review (s,S) control policy with lost sales demands and batch demand.

That is what we used in our work, in order to model an (s,Q) inventory system with continuous review, batch retrials demands and infinite source of demands. The general assumptions are:

1. A batch demands arrives according to a Poisson process and is served instantaneously from the on hand inventory. we suppose that the source of demands is infinite so in the shortages situation, the company should take in to account a limited number of demands whose enters in the orbit and reject the rest, in order to well-run this situation. Hence the notion of partially backlogged requests
2. If the system is out of stock, an arriving demand enters in a virtual limited orbit, and repeat its request after a random time (under the constant repeated calls policy) until the stock is regenerated. If a demand arrives in the system and finds a no available stock and the orbit is full, then it's immediately rejected.

II. Description of the model and assumptions :

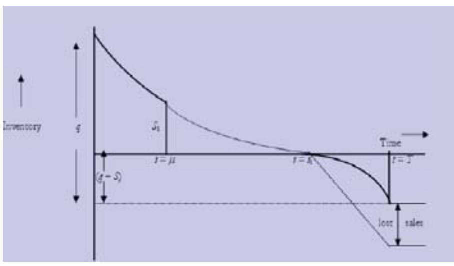

Fig1. An inventory system according to a partial backlog demand assumption.

Assumptions:

1. Primary batch demands with a fixed size n, arrive according to a Poisson process with rate λ and are independent from their size, the service is instantaneous.
2. The lead times are independent, identically distributed following an exponential low with rate µ.
3. An arriving primary demand finding the system out of stock joins a virtual limited orbit. But it makes successive attempts according to an exponential low with rate Θ, after it can be satisfied.
4. If a primary demand finding the system out of stock and the orbit is full, is rejecting from the system immediately.
5. The maximum stock level S>2s. where S and s multiple of n.
6. The size of the orbit is K=k*n.
7. The policy of retrial is a constant policy.
8. The re-order quantity is Q=S-s.

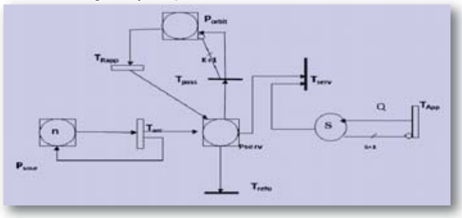

Fig2 . Petri nets representing the (s,Q) inventory system with retrials batch demand.

1. Quantitative analysis of the model:

A. Transit regime and stationary distribution: We get from the different reduced M-marking (Fig3), which is an isomorphs to a continuous Markov chain, the transition matrix given in (Fig4).

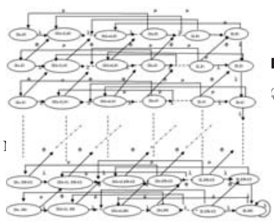

Fig3. The reachability graph

$$Q]_{nl,nm} = \begin{pmatrix} A_0 & C_0 & 0 & \dots & \dots & \dots & 0 \\ B_1 & A_1 & C_1 & \dots & \dots & \dots & 0 \\ 0 & B_2 & A_2 & C_2 & 0 & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots \\ \vdots & 0 & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \vdots & \ddots & 0 & B_{k-1} & A_{k-1} & C_{k-1} \\ 0 & 0 & \dots & 0 & 0 & B_k & A_k \end{pmatrix}$$

Fig4. The transition matrix of CMTC.

We solved the balance equations system below :

$$\begin{cases} A_0\Pi^0 + B_1\Pi^1 = 0 \\ C_{j-1}\Pi^{j-1} + A_j\Pi^j + B_{j+1}\Pi^j = 0, \forall j \in \{1,.., K-1\} \\ C_{K-1}\Pi^{K-1} + A_K\Pi^K = 0 \end{cases}$$

Where :

$$\Pi^j = (\pi_0^j, \pi_1^j, .., \pi_S^j), \forall j \in \{0,.., K\}$$

from the following algorithm :

Début

Etape0: Compute for $j \in \{1,2,..., K\}, T_{i,j}$

Etape1: Compute for $j = K, M_{i,j}$

- then compute

$$r_0^{K-1} = \frac{\theta\sum_{i=0}^{S-1} M_{i,K}}{\lambda - \theta\sum_{i=0}^{S-1} T_{i,K}}$$

and $r_{i+1}^K = M_{i,K} + T_{i,j} r_0^{K-1}, \forall i \in \{0,.., S-1\}$

Etape2:

- Put $j = j-1$
- Compute $M_{i,j}$

$$r_0^{j-1} = \frac{\theta\sum_{i=0}^{S-1} M_{i,j}}{\lambda - \theta\sum_{i=0}^{S-1} T_{i,j}}$$

and

$r_{i+1}^j = M_{i,j} + T_{i,j} r_0^{j-1}, \forall i \in \{0,.., S-1\}$

Etape3: put j=0 , compute: $r_{i+1,0} = M_{i,0}, \forall i \in \{0,.., S-1\}$

Replace

$$\pi_i^j = r_i^j \left(\sum_{j=0}^{K}\sum_{i=0}^{S} r_i^j\right)^{-1}, \forall (i,j) \in \{0,.., S\} \times \{0,.., K\}$$

Fin

B. Performance measures

:

a. *The mean inventory level:*

$$\overline{N_s} = \sum_{j=0}^{K}\sum_{i=1}^{S} i\pi_i^j$$

b. *The expected number of demands in the orbit :*

$$\overline{N_w} = \sum_{i=0}^{S}\sum_{j=1}^{K} j\pi_i^j$$

c. *The mean frequency of the lost demands :*

$$\overline{N_p} = \lambda\left(1 - \left(\sum_{j=0}^{K-1}\pi_0^j + \sum_{i=1}^{S}\sum_{j=0}^{K}\pi_i^j\right)\right)$$

d. *The mean frequency of orders:*

$$\overline{N_c} = \mu\sum_{i=0}^{s}\sum_{j=0}^{K}\pi_i^j$$

e. *Function cost according to the model:*

$$C_T(s,Q) = c_s\overline{N_s} + c_c\overline{N_c} + c_w\overline{N_w} + c_p\overline{N_p}$$

Where c_s, c_c, c_w, c_p are respectively the holding cost, the re-order cost, waiting cost and rejecting cost.

III. Conclusion et perspectives :

In this contribution, we model an inventory system with batch demand according to the (s,Q) policy with continuous review through a new model called "Deterministic and stochastic Petri Nets", where we take in to account the shortages of stock. In this case, a demand join a virtual limited orbit according to FIFO policy, and makes successive attempts after it can be satisfied from the on hand inventory. In the other side, we take different random parameters (lead times, retrial and the arriving process of demands). These parameters are supposed independent, and they give to our model a realistic view application in real inventory systems. The reachability graph for the different M-marking allowed us the possibility of studying the transit regime of the corresponding Markov chain. We elaborated an algorithm from witch we obtain the stationary distribution, in order to find some important performances measures.

Perspectives:

1. Compute other performances measures of our model.
2. Do a simulation of the model, to compare with analytical results.
3. Compare with other model in the literature.

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