Séminaire Mathématique de Béjaïa (LaMOS)
Volume 13, 2014, pp. 59 - 64

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A Taylor Series Approximation for the Performance Measures of the (R, s, S) Inventory Model

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Résumé Real life inventory problems are often very complicated and they are resolved only through approximations. Therefore, it is very important to justify this approximation and to estimate the resultant error. This paper presents a functional approximation of the inventory model with $(R; s; S)$ policy, built on a Taylor series approximation. Using the underlying Markov chain with respect to the perturbation of the demand distribution, we obtain quantitative estimates with an exact computation of constants. Numerical examples are carried out to illustrate the performance of this approach.

Mots-Clés : Inventory Systems, Periodic Policy, Modeling, Markov Chains, Taylor series approximation.

9.1 Introduction

The stock can be viewed as the accumulation of products that can be used to meet future demand. Inventory investment helps ensure business continuity and allows the company to produce a steady pace. The presence of intermediate storage in a production line reduces the risk of production stop in case of failure of one of the machines. On the other hand, a low level of inventories increases the risk of rupture and can cause downtime. Unmet demand of the client may have negative sequence cons (loss of customer confidence) in addition to loss of income. Essentially, inventory management considers two questions :

- How can we maintain the stock has a high enough level ?
- What exactly does "enough ?

The problems of inventory management are among the most studied by specialists in operations research. They meet frequently and when they can be controlled by quantitative techniques, the profits are substantial. Stochastic models of inventory management are more realistic because they take into account the uncertain behavior of some parameters. However, they are more difficult to analyze. They can contain a large number of parameters. In general, these models are very complicated and can only be solved by approximation. In this work, we propose to apply for the first time, the Taylor series expansions method for the stochastic inventory management model (R, s, S) [3]. The disturbance concerns the demand law. The results used are those for Markov chains with finite state space. Our results were compared with those established by B.Rabta [1, 2].

9.2 The Model

Consider the model (R, s, S) with a demand under the compound Poisson law. Customers arrive according to a Poisson distribution. The number of customers arriving during a period T is then Poisson variable $N(T)$. The client requests an amount nth random amount D_n. Suppose that D_n are independent and identically distributed with common law :

$$D(k) = P(D_n = k); k = 1; 2; \dots.$$

The hypothesis of delivery is zero makes the position of a stock process P identical to the level stock process N. For such problems of inventory management, there is evidence that the policy (R, s, S) is optimal. Following this policy, the state X_n of the stock is inspected at dates $t_n = nR$. If the level of stock X_n is equal or less than s, the manager places an order so has bring the stock level S. The size of the order is equal to $X_n = Z_n - S$. If by against the stock level is above the threshold s, we do not place an order and wait for the next time for review expects.

9.2.1 The corresponding Markov chain

The stock state X_{n+1} at the end of the period $(n+1)$ is given by :

$$X_{n+1} = \begin{cases} (X_n - D_{n+1})^+ & \text{If } X_n > s; \\ (S - D_{n+1})^+ & \text{Otherwise.} \end{cases}$$

9.2.2 The Transition Matrix

Suppose that at time n, $X_n = k$, then : $X_{n+1} = (k - D_{n+1})^+$. The correspending transition matrix is given as follow :
It is easy to observe that X_n is an irreductible, homogeneous and aperiodic Markov Chain with finite state space $E = \{0; 1, \dots S\}$.

9.3 The Taylor Series Expansion Method

The main idea of this method is adopted precisely the same already known fundamental mathematical analysis. The application of the method of the Taylor series expansion for the

P=

	0	1		s	$s+1$		S
0	$\sum_{S}^{\infty} a_k$	a_{S-1}	$\cdots$	a_{S-s}	a_{S-s-1}	$\cdots$	a_0
1	$\sum_{S}^{\infty} a_k$	a_{S-1}	$\cdots$	a_{S-s}	a_{S-s-1}	$\cdots$	a_0
	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
s	$\sum_{S}^{\infty} a_k$	a_{S-1}	$\cdots$	a_{S-s}	a_{S-s-1}	$\cdots$	a_0
$s+1$	$\sum_{s+1}^{\infty} a_k$	a_s	$\cdots$	a_1	a_0	0	0
	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	0
S	$\sum_{S}^{\infty} a_k$	a_{S-1}	$\cdots$	a_{S-s}	a_{S-s-1}	$\cdots$	a_0

Figure 9.1. The Transition Matrix

stationary distribution is to write the stationary distribution of the perturbed system as a power series which depends on the stationary distribution of the ideal system, transition matrices and deviation matrix from the ideal system. It is in this context that we try to apply this method to the model (R, s, S) inventory management, where the function in question is none other than the vector of stationary distribution, after giving explicitly the conditions of its application. The major usefulness of the approach of the Taylor series expansion is based on two factors : fast convergence of the series and the ability to compute the remainder of the Taylor series efficiently.

9.3.1 The Taylor Series Expansion numerical results

Using a special recursive algorithm, we calculated the stationary distribution for the disturbed model. We applied the TSEM to test the sensitivity of the model when disrupts the arrival request rate λ to $\lambda' = \lambda + \Delta$. The considered parameters are : $R = 1; 6 = S; s = 3; = 5$, where $\Delta = 0.1$. The transition matrix of the system is constructed performed via a computer program.

$$P = \begin{pmatrix} 0,384039 & 0,175467 & 0,175467 & 0,140374 & 0,084224 & 0,033690 & 0,006738 \\ 0,384039 & 0,175467 & 0,175467 & 0,140374 & 0,084224 & 0,033690 & 0,006738 \\ 0,384039 & 0,175467 & 0,175467 & 0,140374 & 0,084224 & 0,033690 & 0,006738 \\ 0,384039 & 0,175467 & 0,175464 & 0,140374 & 0,084224 & 0,033690 & 0,006738 \\ 0,734974 & 0,170374 & 0,084224 & 0,033690 & 0,006738 & 0,000000 & 0,000000 \\ 0,559507 & 0,175467 & 0,140374 & 0,084224 & 0,033690 & 0,006730 & 0,000000 \\ 0,384039 & 0,175467 & 0,175467 & 0,140374 & 0,084224 & 0,033690 & 0,006738 \end{pmatrix}$$

Figure 9.2. The Corresponding transition Matrix

9.3.2 The Stationary Distributions

The stationary distributions vectors are computed and given as bellow :

$$\pi = (0.4162; 0.1727; 0.1674; 0.1304; 0.0767; 0.0302; 0.0060), \pi_Q = (0.4542; 0.1809; 0.1721; 0.1317; 0.0760; 0.0294; 0.0057).$$

Then, the projector of the stationary transition matrix and the deviation Matrix are :

$$\Pi_P = \begin{pmatrix} 0,416287 & 0,172774 & 0,167402 & 0,130486 & 0,076747 & 0,030288 & 0,006017 \\ 0,416287 & 0,172774 & 0,167402 & 0,130486 & 0,076747 & 0,030288 & 0,006017 \\ 0,416287 & 0,172774 & 0,167402 & 0,130486 & 0,076747 & 0,030288 & 0,006017 \\ 0,416287 & 0,172774 & 0,167402 & 0,130486 & 0,076747 & 0,030288 & 0,006017 \\ 0,416287 & 0,172774 & 0,167402 & 0,130486 & 0,076747 & 0,030288 & 0,006017 \\ 0,416287 & 0,172774 & 0,167402 & 0,130486 & 0,076747 & 0,030288 & 0,006017 \\ 0,416287 & 0,172774 & 0,167402 & 0,130486 & 0,076747 & 0,030288 & 0,006017 \end{pmatrix}$$

Figure 9.3. The Projector Matrix

$$D = \begin{pmatrix} 0,5514 & -0,1701 & -0,1593 & -0,1206 & -0,0692 & -0,0268 & -0,0053 \\ -0,4486 & 0,8299 & -0,1593 & -0,1206 & -0,0692 & -0,0268 & -0,0053 \\ -0,4486 & -0,1701 & 0,8407 & -0,1206 & -0,0692 & -0,0268 & -0,0053 \\ -0,4486 & -0,1701 & -0,1593 & 0,8794 & -0,0692 & -0,0268 & -0,0053 \\ -0,1278 & -0,2028 & -0,2431 & -0,2180 & 0,8603 & -0,0573 & -0,0113 \\ -0,2936 & -0,1685 & -0,1894 & -0,1704 & -0,1149 & 0,9484 & -0,0115 \\ -0,4486 & -0,1701 & -0,1593 & -0,1206 & -0,0692 & -0,0268 & 0,9947 \end{pmatrix}$$

Figure 9.4. The Deviation Matrix

9.4 Conclusion

In this work, we described the inventory system (R, s, S) by a discrete and homogeneos Markov chain, with finite state space, which is ergodic admitting a unique invariant distribution. We also applied the Taylor series expansion in order to predict its caracteristics in case of deviation of one of its parameters. The main difficulty of the TSEM is the stopping criterion for the reminder term approximation. Based on the results of K. Abbas ans

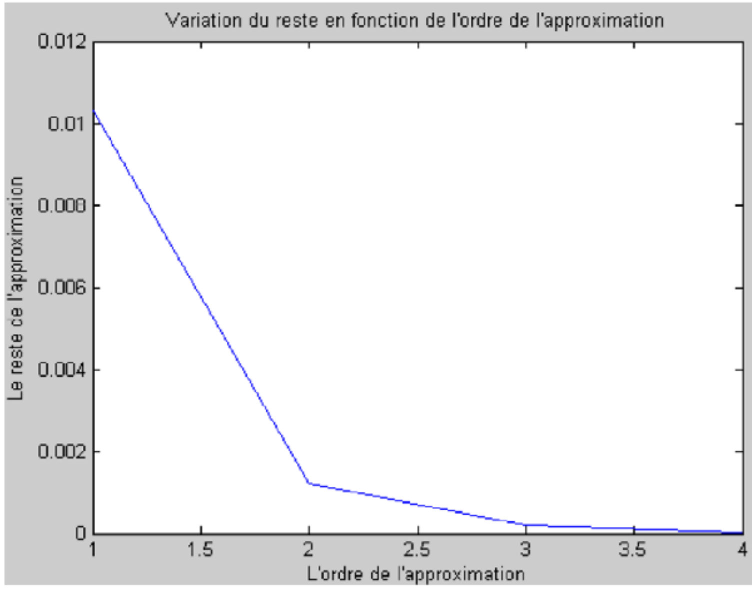

Figure 9.5. Variation of the Reminder Term

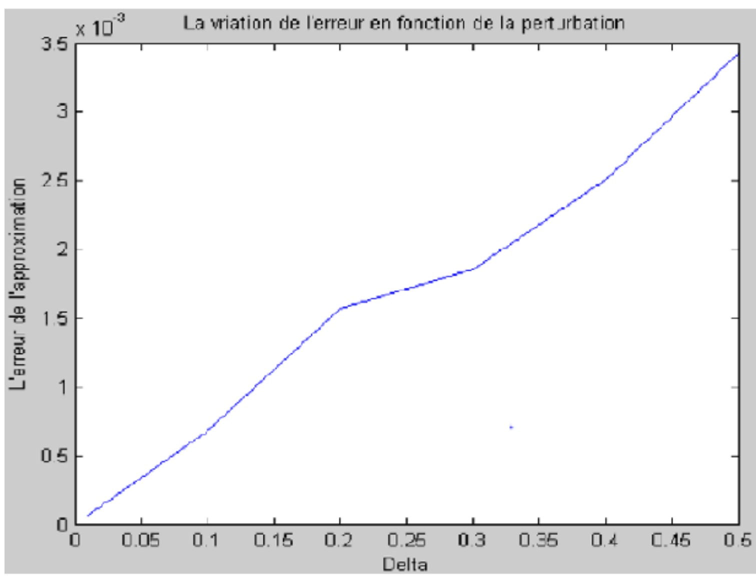

Figure 9.6. Variation of the approximation error

B.Heidergott [3], we were able to get good results from which we obtained the stopping order for the characteristic polynomial and an estimation of the error.

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