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Modeling by Hybrid Petri Nets

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Résumé A hybrid Petri net can be obtained if one part is discrete and another part is continuous. This paper is a survey of the author's works on hybrid Petry nets. The author presented this Petri nets formalism with real application examples. Finally, he introduced the notion of continuous Petri nets (autonomous and timed), which differentiates them from discrete Petri nets, their usefulness and some extensions proposed in the literature concerning the definition of instantaneous firing rates.

Key words : Petri net Hybrid Petri nets, continuous petri nets, formalism, application examples.

Many systems are naturally hybrid, i.e., their modeling needs at least one continuous state variable and at least one discrete state variable. In some cases, a discrete system, or part of a system, can be approximated by a continuous model.

Petri nets (PNs) are widely used to model discrete event dynamic systems (computer systems, manufacturing systems, communication systems, etc.). Continuous Petri nets (in which the markings are real numbers and the transition firings are continuous) were defined more recently ; such a PN may model a continuous system or approximate a discrete system. A hybrid Petri net can be obtained if one part is discrete and another part is continuous. The talk is basically a survey of the work of the author's team on hybrid PNs.

In a discrete PN, the marking of a place may correspond either to the Boolean state of a device (for example a resource is available or not), or to an integer (for example the number of parts in a buffer). A general analysis method is to compute the set of reachable states and deduce the different properties of the system. But when a PN contains a large number of tokens, the number of reachable states explodes and this is a practical limitation of the use of Petri nets. To illustrate this point, consider a manufacturing line composed of three machines M_1, M_2 and M_3 in order, and two intermediate buffers B_1 and B_2 with respective finite capacities, C_1 and C_2 (Figure 1). The parts move on the machines, and wait in the intermediate buffers if required. We assume that there are always unworked parts upstream M_1 and available space downstream M_3. The number of reachable states of this system is $N = 2^3(C_1 + 1)(C_2 + 1)$; then $N = 1352$ for $C_1 = C_2 = 12$. For a set composed of 10 machines and 9 buffers each with capacity 12, $N = 2^{10} \times 13^9$ which is greater than 10^{13} states! This observation led us to define continuous PNs and hybrid PNs. In a continuous

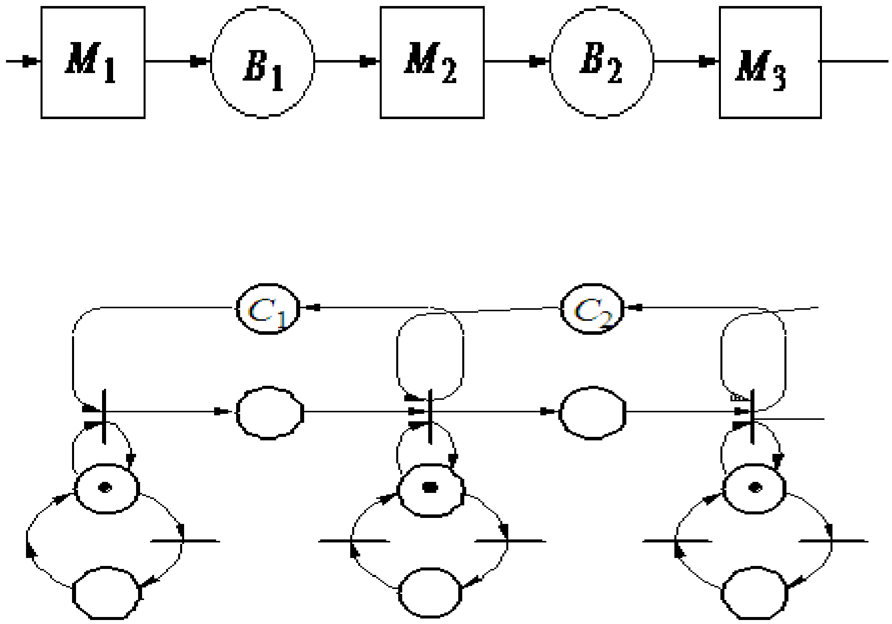

Figure 1.1. A production line.

PN, the markings of places are real numbers and the firing of transitions is a continuous process. For the example considered, the flow of parts on the machine may be approximated by a continuous flow and the numbers of parts in the buffers may be approximated by real numbers. However, the state of each machine (operational or not) is necessarily discrete. Hence, a hybrid model can be used for this system.

Continuous Petri nets were introduced in (David and Alla, 1987). The concept of hybrid Petri net was introduced in the same paper, then developed in (Le Bail et al., 1991).

A continuous PN may be either autonomous (no time is involved) or with firing speeds associated with transitions. A timed model may be used for the performance evaluation of systems. Various timed continuous PN models have been defined which differ by the calculation of the instantaneous firing speeds of the transitions. They provide good approximations for performance evaluation when a PN contains a large number of tokens. All the models work on the same basic rule. The only difference is the way in which the instantaneous firing speeds are defined; it follows that other definitions of this firing speed can be chosen.

Some authors have explicitly added new concepts and results to the initial definitions of continuous and hybrid Petri nets (references can be found in the survey references at the end of this abstract) : timings to place in continuous Petri nets, special places and transitions added in order to model systems processing batches of parts.

The survey is organized as follows :
After the original motivation of this research, discrete Petri nets, continuous Petri nets, and hybrid Petri nets are briefly presented. It is particularly shown the passage from a discrete state space to a continuous one, while keeping the positive property of the variables.

Timing in discrete Petri nets and continuous Petri nets are evocated and the choices made for hybrid Petri nets are presented. Several definitions for the maximal speeds associated with transitions are given. The most important is in the constant continuous PN (CCPN) where the maximal firing speed is constant. In that case, the dynamic behavior of the model remains event driven, and the number of reachable states is low what may be the size of the markings. This constitutes a fundamental advantage of the CCPN model. It authorizes very fast simulations.

Autonomous Petri nets (i.e., not involving time) and their modeling power are explained. Simple dynamic behaviors are presented, such as the influence of the discrete part on the continuous one and vice-versa, and the transformation of discrete markings in continuous ones.

Various examples of applications show the powerful aspect of the this modeling tool : a performance evaluation of a production System, a water Supply System, a transfer Line and a controlled system via Communication Networks.

Continuous systems have been studied for a long time. Modeling, analysis and control of discrete event systems have undergone major developments in recent decades. In recent years, a need has emerged to consider systems which are partially continuous and partially discrete.
Continuous PNs and Hybrid PNs can be used for modeling and analyzing these systems.

Références

1. R. DAVID and H. ALLA, "Continuous Petri Nets", 8th European Workshop on Application and Theory of Petri Nets. Zaragoza (SP), June 1987.
2. J. LE BAIL, H. ALLA, and R. DAVID, "Hybrid Petri Nets", European Control Conference. Grenoble (F), June 1991, pp. 1472-1477.
3. R. DAVID and H. ALLA, Discrete, Continuous, and Hybrid Petri Nets, Springer, Heidelberg, 2010.
4. H. ALLA and R. DAVID, "Continuous and Hybrid Petri Nets", Journal of Circuits, Systems and Computers, Vol. 8, N° 1, 1998.