Séminaire Mathématique de Béjaïa (LaMOS)
Volume 11, 2012, pp. 39 - 41

9

Embedding balanced binary trees in hypercube

Kamal KABYL

Laboratory of Modeling and Optimization of Systems
University of Bejaia, 06000 Bejaia, Algeria.

Résumé The hypercube is a structure whose topology is used in different fields such as computer science, combinatorics, code theory, etc. Thus, the study of spanning of graphs in the hypercube has received much interest these later years. The problem consist of giving the smallest dimension of a hypercube in which a given graph is embeddable. We talk then about optimal hypercube and cubical dimension of the graph. Arfati et al. [1] have proved that the problem of deciding if a graph is embeddable in a hypercube of a given dimension is NP-complete, Corneil and Wagner [2] have shown that this problem is NP-complete even in the case where G is a tree.. In this paper, we introduce two new classes of balanced binary trees for which the cubical dimension is given.

Mots clés : Hypercubes,Trees, Embedding, Isomorphism.

9.1 Introduction

An embedding of the graph G in the graph H is an application on V(G) into V(H) which preserves the adjacency. In the case where V(G) = V(H) we say that the embedding is total. In a general way, the study of an embedding of G in H turns to see if G is isomorphic to a subgraph of H.
An important class to study is the one embedding trees in the hypercube. This study results from the large using of trees in many domains : computer science, social science, operations research, combinatorial optimization. . . .
We define in this paper two new classes of balanced binary trees for which the cubical dimension is determined.
A tree T is said Cn-valuated if we can mark every edge by an integer from $\{1, 2, ..., n\}$ such that for any path P of T, there exists an integer $k \in \{1, 2, ..., n\}$, for which an odd number of edges are marked by k. Havel and Moravek [Havel (1984)] have proved that a graph G is embeddable in Q_n if and only if there exists a Cn-valuation of G. We introduce two new classes of balanced binary trees having 2^n vertices for which the cubical dimension is given. Furthermore, these classes of graphs satisfied the conjecture of Havel [3].

9.2 two new classes of balanced binary trees

9.2.1 The Class AD_n

For $n \geq k \geq 1$, AD_n^k is obtained from the tree $\widehat{\widehat{D}}_n$ while inserting a new vertex on the edge at distance k from the root in the copy of D_n and another new vertex on the edge at distance k from the root in the other copy of D_n. AD_n^1, AD_n^2 and AD_n^n are given by the figure below

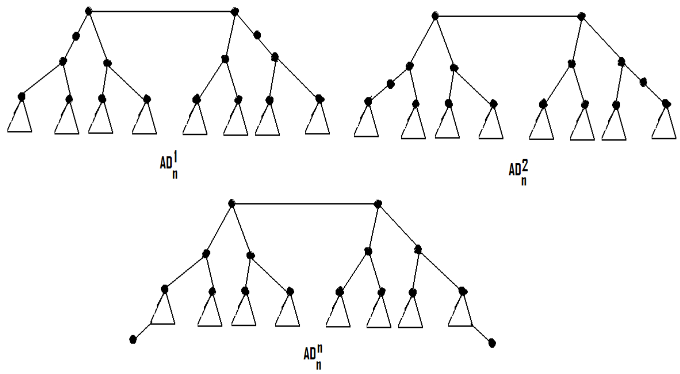

FIGURE 9.1.

Theorem 1. *For any $n \geq k \geq 1$, AD_n^k is a subgraph of its optimal hypercube Q_{n+2}.*

9.2.2 The Class $A\widehat{D}_n^k$

For $n \geq k \geq 0$, $A\widehat{D}_n^k$ is obtained from the binary tree $\widehat{\widehat{D}}_n$ while inserting two new vertices on the edge at distance k from the root of the D_n in the binary tree $\widehat{\widehat{D}}_n$. $A\hat{D}_n^1$, $A\hat{D}^2{}_n$ and $A\hat{D}_n^n$ given by the figure below

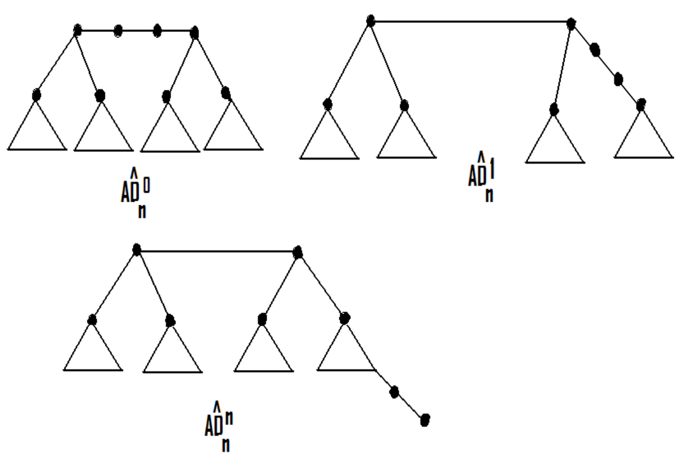

FIGURE 9.2.

Theorem 2. *For any $n \geq k \geq 0$, $A\widehat{D}_n^k$ is a subgraph of its optimal hypercube Q_{n+2}.*

9.3 Conclusion

In this work, we are interested in embedding of trees in the hypercube. Many researchers have studied this problem, their works have characterized some classes of tree. All trees are embeddable in the hypercube. The problem is to find the smallest dimension of the hypercube

in which a given tree is embeddable.
we have frequently used the term of Cn-valuation to determine the dimensions of certain classes of trees. We have also introduced two new classes of of balanced binary trees obtained from the complete binary tree, which the Cubical dimension is determined

Références

1. J. Arfati, C.H. Papadimitriou, P. Papageorgiou. The complexity of cubical graphs. *Proceedings of 11th international Kolloquium on automata,languages and programming*, 51–57, 1984.
2. D.G. Corneil, A. Wagner. Embeding trees in a hypercube is NP-complete. *Siam J. Comput*, 19 :570–590, 1990.
3. I. Havel. On hamiltonian circuit and spanning trees of hypercubes. *Časopis p̌ro pestovani matematiky*, 109 : 135–152, 1984.
4. M. Havel, I. *On hamiltonian circuits and spanning trees of hypercubes*. Cas prest. Mat **109**, 135-152, (1984).
5. M. I. Havel, P. Liebl. *Embedding the polytomic tree into the n-cube*. Cas prest. Mat **98**, 307-314, (1973).
6. M. Havel, I. and Moravek, J. *B-valuation of graphs*. Czech- Math .jour ., **22**, 338-351,(1972).
7. M. Choudum, S.A. and Raman, I. *Embedding height balanced trees and Fibonacci trees in hypercubes*. J Appl Math Comput., **30**, 39-52,(2009).