Banking Efficiency and the Economic Transition Process

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Résumé

Ce papier s'interroge sur le rôle du système bancaire dans le processus de transition économique. Cela est considérée dans le contexte d'un modèle à générations imbriquées avec croissance endogène. Il y a deux technologies de production, l'une pour la production d'un bien final et l'autre pour la production du bien d'investissement. Le rendement du capital investit dans la production du bien d'investissement est stochastique. Les banques collectent l'épargne des ménages et financent la production du bien d'investissement en respectant des règles prudentielles.

On montre que l'accumulation du capital est constituée de plusieurs phases et que le processus de transition est conditionnée par la perfection du marché du crédit. On montre aussi, que des banques efficientes permettent à l'économie de mieux résister aux mauvaises performances du secteur de production du bien d'investissement. Cependant, dans le cas de banques inefficaces, la situation peut dégénérer en crise de confiance dans le système bancaire retardant le processus de transition de plusieurs années. L'impact est d'autant plus important que le marché du crédit est imparfait et que l'économie est au début de son processus de transition.

Mots Clés : Efficacité bancaire, crise de confiance, processus de transition

Abstract

This paper investigates the role of the banking system in the economic transition process. This is considered in the context of an overlapping generation model with endogenous growth. The are two production technologies, one for the production of a final good and the other for the production of an investment good. The return of the capital invested in the investment good technology is stochastic. Banks collect the saving of households and finance the production of the investment good while respecting some prudential rules.

We show that the capital accumulation is constituted of several phases and that the economic transition process is conditioned by the credit market perfection. We also show that efficient banks make the economy more resistant to bad performances of the investment good sector. However, in case of inefficient banks, the situation can degenerate in a confidence crisis in the banking system delaying the transition process by several years. This negative impact is more severe when the economy is at the beginning of its transitional process and when its credit market is more imperfect.

Keys Words : Banking efficiency, confidence crisis, transition process.

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1 Introduction

The modern economic literature on growth starts with Robert Solow's work in the mid 1950s. It focused on the role of capital and labor resources and the use of technology as the sources of growth. The contribution of the financial structure to growth was first analyzed at the end of the sixteenth (Goldsmith (1969) and McKinnon (1973)). However, it is only during the nineteenth that the relationship between the financial sector development and the economic growth became a generally accepted and robust observation.

As Paul Wachtel notes "the interest in the economic role of banks is in part due to two recent development in the world economy: the emergence of transition economy banks and the frequent occurrence of banking crises in both developed and less developed countries". Indeed, the Eastern Europe and the Former Soviet Union had planned economies and did not have modern banking sectors. One of the first steps of their transition process was the formation of commercial banks. However, the credit evaluation and risk management were irrelevant in the newly established banking system. In countries such as Czech Republic, Romania, Bulgaria and Poland, the banks distress was frequent and the governments was obliged to recapitalize its bank more than once within a decade. The consequence was an inhibition of the transition process. Although, the financial system restructuring has been completed in many of the cited countries, macro-economic shocks may lead to a breakdown of the financial system, which affects the average growth rate over a decade or more and inhibiting the transition process.

Nowadays, in the transition economies as well as in less developed countries, depositors' lack of confidence in the financial system remains an acute problem which inhibits the collection of savings and causes low economic growth.

In the present paper we develop a theoretical model that analyzes the relationship between the banking efficiency and the dynamic of the transition process. In the first section we consider an efficient banking system and establish important results for the economic transition. We show that the process of capital accumulation is constituted of several phases and that the speed of the economic transition is conditioned by the credit market perfection. Because banks are efficient, the economy resist to bad performances of the investment good sector .

In the second section, we consider an inefficient banking system. The bad performances of the investment good sector may affect the bank ability to honor the deposit contract causing a depositors' loss of confidence in the banking system. In this case, the transition process is inhibited and the convergence to the long term trap is delayed by several years. We show that this negative impact is more severe when the economy is at the beginning of its transitional process and when its credit market is more imperfect.

2 The Model

We construct an overlapping generation model. Each generation consists of a continuum of agent living for two periods. There are two production technologies in the economy; a technology for the production of a final good and another for the investment good. The final good can be used for consumption and production of the investment good. The investment good is used as an imput in the final good production technology. Moreover, there is a bank managed by a central banker which gather the household's saving and lend for the entrepreneurs of the investment good sector.

2.1 Technology

2.1.1 The Final Good Production Technology

The final good are produced with the investment good, K_t, and labor, L_t, using the Cobb-Douglas technology

$$Y_t = A_t.K_t^{\alpha}.L_t^{1-\alpha}$$

where A_t is a productivity term that introduces a positive externality, depending on the per capita capital level

$$A_t = K_t^{1-\alpha} = \left(\frac{K_t}{L_t}\right)^{1-\alpha}$$

The output per capita is therefore

$$y_t = A_t.k_t^{\alpha}$$

The inputs price in terms of the final good are

$$\begin{aligned} \rho_t &= \alpha \\ w_t &= w(k_t) = (1-\alpha)k_t \end{aligned} \tag{1}$$

2.1.2 The Investment Good Production Technology

The investment good is produced with the final good according to the following technology; one indivisible unit of a final good invested at t gives R_{t+1} units of investment good at $t+1$.

$$R_{t+1} = \begin{cases} R+\sigma & \text{with probability} \quad \pi \\ R-\sigma & \qquad\qquad\qquad 1-\pi \end{cases}$$

Where $\alpha(R+\sigma) > 1$ and $\alpha(R-\sigma) < 1$. Let S_t a variable that take the value H if the return R_t is equal to $R+\sigma$ and L in the other case. Note that $\forall\, t > 0$

$$\begin{aligned} E_t(R_{t+1}) &= E(R) \\ &= (R + (2\pi - 1)\sigma) \end{aligned}$$

2.2 The Agents

Each agent of the $[0, 1]$ continuum of agents of the generation, $t-1$, who are born at $t-1$ supply inelastically one unit of labor at the first period of its life in the final good production sector so that the total labor supply is

$$L_t = 1$$

Each agent earn a wage w_t at t, and consume only at the end of its life i.e. at date $t+1$.

At the beginning of the second period they have to decide how to use their wages. A proportion p_t become entrepreneur and decide to produce the investement good. The remainder proportion, $1-p_t$, can deposit its saving at the bank at a deposit rate of r_{t+1} or simply store it without depreciation if there is no confidence in the banking system.

We assume that $w(k_0) < 1$ which means that initially, each entrepreneur has to borrow $1-w_t$ to finance its project. This impose the following condition on k_0

$$k_0 < k_\alpha = \frac{1}{1-\alpha}$$

When $k_t > k_\alpha$ we have $w(k_t) = \dfrac{k_t}{k_\alpha} > 1$ and all the agents will be entrepreneurs since they can self-finance their project. In this case, the bank don't have any economic role (this is not the case if we relax the hypothesis of homogenous agents).

Let denote n the integer such that

$$n \leq w(k_t) < (n+1)$$

or

$$nk_\alpha \leq k_t < (n+1)k_\alpha$$

since the investment good project is indivisible, n is also the proportion of projects that are realized at period $t+1$

$$p_t = n \quad \text{where } n \text{ is the integer } / \ nk_\alpha \leq k_t < (n+1)k_\alpha$$

To determine the proportion p_t for $k_t < k_\alpha$ we should specify the properties of the banking role.

2.3 The Interest Rate

At the beginning of period $t+1$, i.e. at date t, each entrepreneur have to borrow $1 - w_t$ to implement its project. At the same time the gross deposit return rate r_{t+1} which is also the gross interest rate on loans is determined by the following conditions:

$$\alpha E(R) \ \geq r_{t+1} \tag{2}$$

This condition insure that the bank finance profitable project with an expected return superior or equal to the cost of borrowing.

$$r_{t+1}\,(1 - w_t) \leq \lambda\alpha E(R) \tag{3}$$

The second condition is a solvency one which insure that the interest payments are inferior to the cost of default on loans as represented by the term $\lambda\alpha E(R)$. The parameter $\lambda \in \,]0,1]$ represents the degree of perfection of the credit market. When the credit market is perfect, the bank can take all the project return in case of default on loans.

$$r_{t+1} \ > 1 \tag{4}$$

This condition is necessary to collect the agents' saving otherwise they will retain their saving out of the bank until the end of their life.

Proposition 1

for $k_t \leq k_a$ condition (3) dominates (2)

for $k_t > k_a$ condition (2) dominates (3)

where $k_a = \frac{1 \ \lambda}{1 \ \alpha}$

Proof. See appendix I. ■

At earlier stage of the economic development ($k_t \leq k_a$) (3) dominates (2). Figure (1) illustrate this case

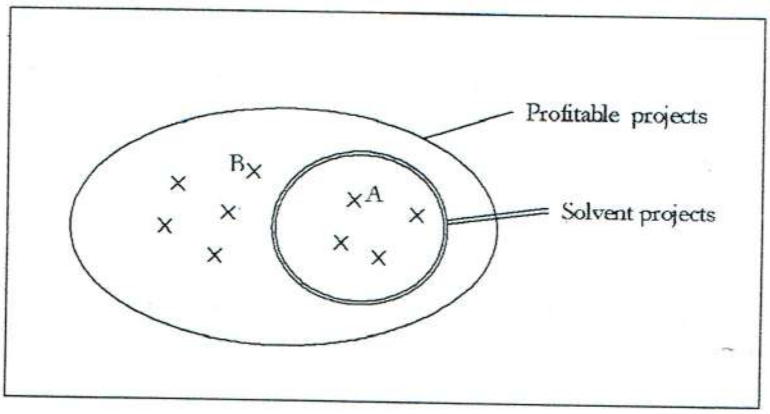

Figure 1: An illustration of "(3) dominates (2)"

Hence, (3) dominates (2) means that if (3) is satisfied then (2) is satisfied. This is the case for project A which is profitable and solvent. Whereas (2) may be satisfied and (3) is not. This is the case of project B which is profitable but not solvent. At the equilibrium, the interest rate is fixed such that the solevency constraint is satisfied. If the interest rate is fixed such that the production of the investment good is profitable, the entrepreneur of project B, for example, has an incentive to default because the cost of default is less than the interest payment.

At advanced stage of the economic development when the economy accumulate a sufficient level of capital ($k_t > k_a$) the entepreneur's share in the project financement increases sufficiently that (2) dominates (3). Figure (2) illustrate this case

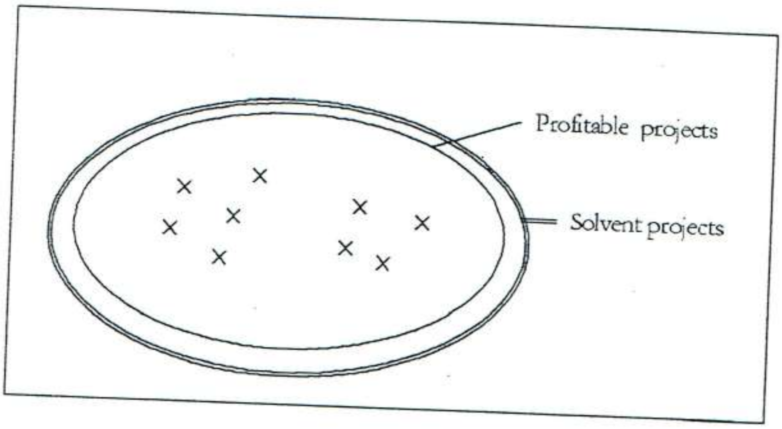

Figure 2: An illustration of "(2) dominates (3)"

Hence, (2) dominates (3) means that if (2) is satisfied then (3) is satisfied. Thus, all profitable projects become solvable. At the equilibrium, the interest rate is fixed such that the production of investment good is profitable.

At the equilibrium, the interest rate is determined by the following

$$r_{t+1} = \begin{cases} \dfrac{\lambda \alpha E(R)}{1-(1-\alpha)k_t} & \text{if } k_m \leq k_t \leq k_a \\ \alpha E(R) & \text{if } k_t > k_a \end{cases} \tag{5}$$

Where k_m is obtained by setting $\dfrac{\lambda\alpha E(R)}{1-(1-\alpha)k_t} = 1$ and verify

$$k_m = \frac{1-\lambda\alpha E(R)}{1-\alpha} \tag{6}$$

Figure (3) illustrate the equation (5)

For $k_t < k_m$, satisfying the solvency constraint means setting an interest rate $r_{t+1} < 1$. But at this interest rate level the potential depositors will prefer to hold their capital until the end of their life. Hence, in the region of lower

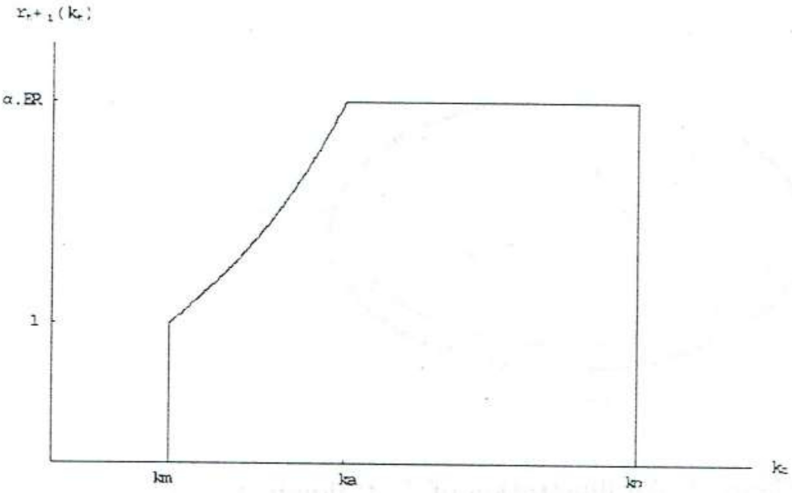

Figure 3: The Interest rate and the Economic Development

stage of development, $]0, k_m[$, the domestic saving is hold out of the banking system depriving the economy from important rsources for its growth dynamic.

To verify the condition (4) the parameters have to be choosen such that

$$\alpha(R + (2\pi - 1)\sigma) > 1$$

2.4 The Bank Reserves

Before determining the expression of the bank reserves we should determine in which cases the entrepreneurs default on their loans at the end of period $t + 1$ and the correspondant amount of default per project.

Proposition 2

for $k_t \leq k_b$ there is a systematic default of the entrepreneurs if and only if $S_{t+1} = L$

for $k_t > k_b$ there is no default whatever the state S_{t+1}

where $k_b = k_a + \frac{2\pi\lambda\sigma}{(1-\alpha)E(R)}$

Proof. See appendix II. ■

If $k_t \leq k_b$ and $S_{t+1} = L$ happens at the end of period $t+1$, the entrepreneurs have an incentive to default on their loans. This occurs because the cost of default $\lambda\alpha(R - \sigma)$ is inferior to the interest payment $r_{t+1}\,(1 - w_t)$.

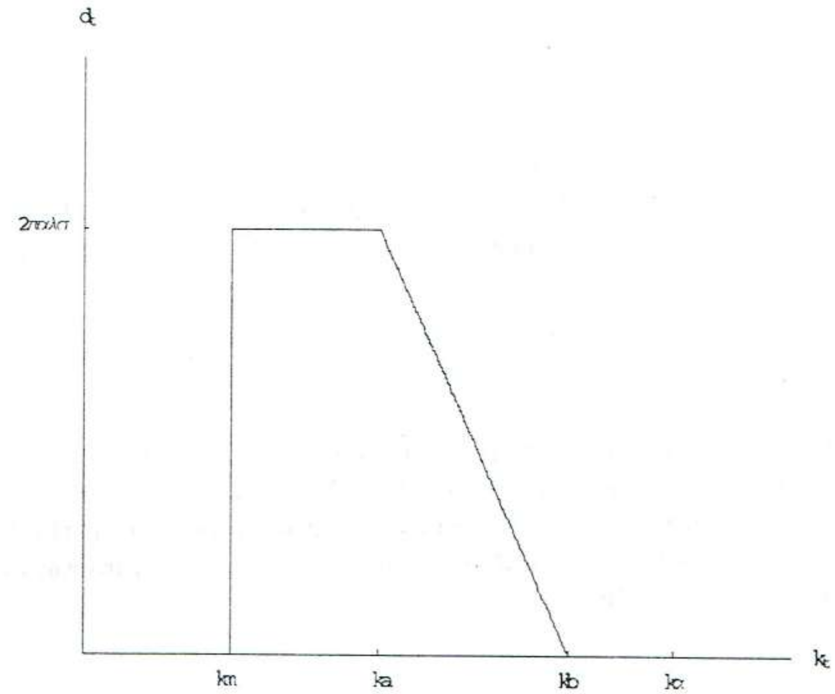

Figure 4: The possible default per project = f(Economic Develpment)

The amount of the default per project is therefore equal to the difference between the interest payment and the amount that the bank can seize

$$d_t = r_{t+1}\,(1 - w_t) - \lambda\alpha(R - \sigma)$$

using (5) we obtain

$$d_t = \begin{cases} 2\pi\alpha\lambda\sigma & \text{if } k_m \leq k_t \leq k_a \\ \alpha E(R)\left\{1 - [1-\alpha]\, k_t\right\} - \alpha\lambda(R-\sigma) & \text{if } k_a < k_t \leq k_b \\ 0 & \text{if } k_t > k_b \end{cases} \tag{7}$$

The following figure illustrate equation (7):

In regions $[0, k_m]$ and $[k_\alpha, +\infty[$ the bank don't play any role that's why the bank reserves as well as the interest rate are set to zero. The economic transition from region $[k_m, k_a]$ to $[k_a, k_b]$ is accompanied with the reduction of the amount of the possible default per project. In region $[k_m, k_a]$, the solvency constraint dominates the profitability one that's why the bank reserves are at their maximum and the interest rate is lower than that of regions $[k_a, k_b]$, $[k_b, k_\alpha]$. In region $[k_b, k_\alpha]$, there is no default and the bank reserves are zero. In this region, the number of realized projects and the growth rate are maximum.

Since p_t is the proportion of entrepreneurs and $1-p_t$ is the proportion of savers, the total amount of deposit is $(1-p_t)\,w_t$ and the total amount of loans is $p_t(1-w_t)$. Because of the possibility of default the bank have to cut reserves to pay its depositors the total amount of $r_{t+1}\,(1-p_t)\,w_t$ at the end of period $t+1$. Let B_t denote the bank reserves at the beginning of period $t+1$, which means that the bank reserves must compensate for the total default amount.

$$B_t = p_t d_t \tag{8}$$

Note that if the entrepreneurs pay their loans at the end of period t, which corresponds to $S_t = H$, then the period t bank reserves B_{t-1} will serve for the next period. To determine the necessary amount of reserves that the bank will cut at each period from the total deposit we formulate it as a fraction β_t of the salarial mass w_t then we have

$$\begin{aligned} B_t &= \beta_t w_t + 1_{\{S_t=H\}} B_{t-1} \\ &= \beta_t w_t + 1_{\{S_t=H\}} \beta_{t-1} w_{t-1} \end{aligned} \tag{9}$$

where

$$(1-p_t)\,w_t = \beta_t w_t + p_t(1-w_t) \tag{10}$$

The latter equation means that the bank deposit is decomposed in bank reserves and loans to the entrepreneurs.

2.5 The Proportion of Entrepreneurs

- In section $(I.2)$ we showed that the proportion of entrepreneurs for $k_t > k_\alpha$ is

$$\boxed{p_t = floor\left[\frac{k_t}{k_\alpha}\right]} \tag{11}$$

- The proportion of entrepreneurs for $k_t \in [k_m, k_\alpha]$, is obtained from equation (10):

$$\begin{aligned} p_t &= (1-\beta_t) w_t \\ &= (1-\beta_t)\frac{k_t}{k_\alpha} \end{aligned} \tag{12}$$

Combining (9) and (12) we obtain

$$\beta_t = \begin{cases} \dfrac{d_t}{1+d_t} & \text{if} \quad S_t = L \\ \dfrac{d_t}{1+d_t} - \dfrac{k_{t-1}}{k_t}\dfrac{d_{t-1}}{1+d_{t-1}} & \text{if} \quad S_t = H \end{cases} \tag{13}$$

and

$$\boxed{p_t = \begin{cases} \dfrac{1}{(1+d_t)}\dfrac{k_t}{k_\alpha} & \text{if} \quad S_t = L \\ \left(\dfrac{1}{1+d_t} + \dfrac{k_{t-1}}{k_t}\dfrac{d_{t-1}}{1+d_{t-1}}\right)\dfrac{k_t}{k_\alpha} & \text{if} \quad S_t = H \end{cases}} \tag{14}$$

- For $k_t \in]k_0, k_m]$, there is no bank. A fraction $1-\tau$ of agents decide to hold their wage until the end of their life and the remainder fraction τ collect their wage to produce the investment good. They sign share contracts and at the end of the period the project return is divided proportionally to the amount of invested capital. Let denote p_t^m the proportion of realized projects. The following equation determine p_t^m :

$$\int_0^{p_t^m} (1-w_t)dn = \int_0^{\tau} w_t dn - \int_0^{p_t^m} w_t dn$$

which means that the amount of capital flow needed to achieve p_t^m projects is equal to the shareholders total capital. We obtain

$$\boxed{p_t^m = \tau w_t = \tau \frac{k_t}{k_\alpha}} \tag{15}$$

Note that the entrepreneurs have no incentive to repay the shareholders' total piece in the cake. If some entrepreneurs default, this may dissuade the future generations from signing share contracts causing a slowdown of the investment and an economic recession.

The following figure illustrate how the proportion of financed projects at period $t+1$ varies with the level of accumulated capital k_t , if we suppose that $S_t = L$

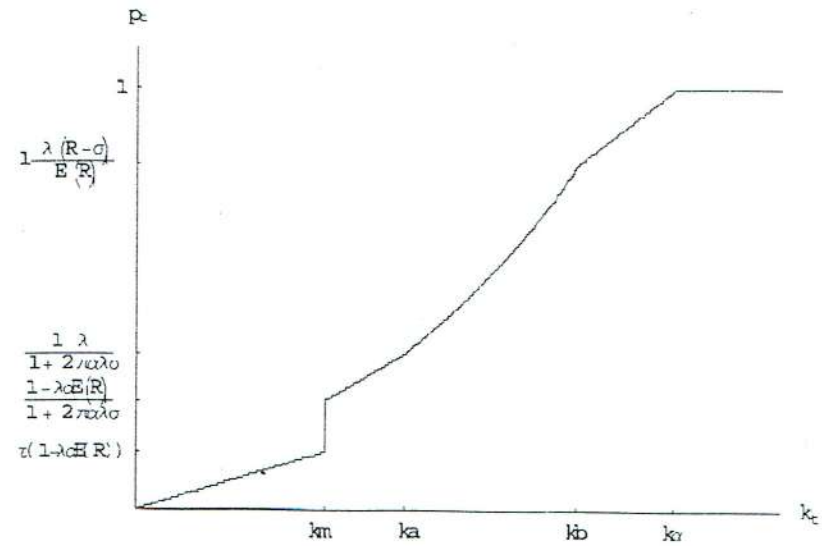

Figure 5: The proportion of projects = f (Economic Development) for St=L

2.6 The Growth Dynamic

2.6.1 Theoretical Expressions

The quantity k_{t+1} of the investement good produced at the end of period $t+1$ is equal to the proportion of realized projects multiplied by the return R_{t+1} of one project. Since R_{t+1} is stochastic we have

$$\begin{aligned} E_t\left(k_{t+1}\right) &= p_t E_t\left(R_{t+1}\right) \\ &= p_t E\left(R\right) \end{aligned}$$

using equations (11), (14) and (15) we obtain the avearage rate of growth of the economy for period $t+1$, $(t \geq 1)$

$$\boxed{E_t\left(\frac{k_{t+1}}{k_t} \setminus k_t < k_m\right) = (1-\alpha)\, E\left(R\right)} \tag{16}$$

$$\boxed{E_t\left(\frac{k_{t+1}}{k_t} \setminus k_t > k_\alpha\right) = floor\left[\frac{k_t}{k_\alpha}\right] E(R)} \tag{17}$$

and from the following equations

$$E_t\left(\frac{k_{t+1}}{k_t} \setminus S_t = L,\ k_m \le k_t < k_\alpha\right) = \frac{(1-\alpha)\,E(R)}{1+d_t}$$

$$E_t\left(\frac{k_{t+1}}{k_t} \setminus S_t = H,\ k_m \le k_t < k_\alpha\right) = (1-\alpha)\,E(R)\left(\frac{1}{1+d_t} + \frac{k_{t-1}}{k_t}\frac{d_{t-1}}{1+d_{t-1}}\right)$$

we obtain

$$E_t\left(\frac{k_{t+1}}{k_t} \setminus k_m \le k_t < k_\alpha\right) = \pi E_t\left(\frac{k_{t+1}}{k_t} \setminus S_t = H,\ k_m \le k_t < k_\alpha\right) +$$

$$(1-\pi)E_t\left(\frac{k_{t+1}}{k_t} \setminus S_t = L,\ k_m \le k_t < k_\alpha\right)$$

and thus,

$$\boxed{E_t\left(\frac{k_{t+1}}{k_t} \setminus k_m \le k_t < k_\alpha\right) = (1-\alpha)\,E(R)\left(\frac{1}{1+d_t} + (1-\pi)\frac{k_{t-1}}{k_t}\frac{d_{t-1}}{1+d_{t-1}}\right)} \quad (18)$$

where (d_t) is given by (7). Note that for $k_t \in [k_b, k_\alpha]$ (as well as for $k_t \in [k_\alpha, +\infty[$) the avearage growth rate does not depend on the state S_t and verify

$$\frac{E_t\left(k_{t+1} \setminus k_{t-1} < k_b\right)}{k_t} > \frac{E_t\left(k_{t+1} \setminus k_{t-1} > k_b\right)}{k_t} = (1-\alpha)\,E(R)$$

2.6.2 Comments

1. When the good state $S_t = H$ occurs the period t reserves will serve in period $t+1$ which decreases the amount of reserves that the bank cut from period $t+1$ deposit increasing, by the same time, the proportion of financed project at period $t+1$. This expalins why the growth average when $S_t = H$ is superior to that corresponding to $S_t = L$.

2. The growth rate average on period $t+1$ of the economy depends not only on the economy level of development as caracterized by the five following regions $[0, k_m]$, $[k_m, k_a]$, $[k_a, k_b]$, $[k_b, k_\alpha]$ and $[k_\alpha, +\infty[$ but also on the

state S_t realised at period t for the two first regions. It becomes clear that the initial level of development k_0 and the history $(S_1, S_2, ..., S_t)$ of the variable S affect the growth rate at period $t+1$ since it determines in which region of development the economy is.

3. Remembering equations (??), (6) and (??) we see that $[k_m, k_a]$, $[k_a, k_b]$ and $[k_b, k_\alpha]$ depend on the degree of perfection of the credit market as represented by λ. We will return back to this affect in section (2.5).

4. To ensure the possibility for an economy to move from the initial region $[k_m, k_a]$ (where the solvability constraint dominates the profitability constraint) to region $[k_a, k_b]$ (where the project finance is conditioned by its profitability) we have to impose a condition on our parameter choice. Suppose that $k_t \in [k_m, k_a]$ and $S_t = L$ then

$$k_{t+1} = \begin{cases} \dfrac{(1-\alpha)(R+\sigma)}{1+2\pi\alpha\lambda\sigma} k_t & \text{with probability} \quad \pi \\ \dfrac{(1-\alpha)(R-\sigma)}{1+2\pi\alpha\lambda\sigma} k_t & \qquad\qquad 1-\pi \end{cases}$$

The condition is therefore

$$\frac{(1-\alpha)(R+\sigma)}{1+2\pi\alpha\lambda\sigma} > 1 \tag{19}$$

In the other case, their will be recession even if the good return state was realized. It is easy to show that the transition from region $[k_a, k_b]$ to $[k_b, k_\alpha]$ and from $[k_b, k_\alpha]$ to $[k_\alpha, +\infty]$ is possible when the condition (19) is satisfied.

2.6.3 The Numeric Simulation

a) *Parametrization*

We choose $\alpha = 0.37$. The parameters k_0, R, σ, π, and λ should satisfy the following conditions that was presented previously

$$\begin{cases} \alpha(R+\sigma) > 1 \\ \alpha(R-\sigma) < 1 \\ \alpha(R+(2\pi-1)\sigma) > 1 \\ \dfrac{(1-\alpha)(R+\sigma)}{1+2\pi\lambda\alpha\sigma} > 1 \\ k_m \leq k_0 < \dfrac{1}{1-\alpha} \end{cases} \tag{20}$$

The following values

α	π	σ	R	λ	k_0
0.37	$[0.9, 1]$	1	2	$[0, 1]$	$]k_m, 1.58]$

satisfies this conditions but are not unique. This choice ensures that

Proposition 3 *With the above parameters choice we have*

$$\boxed{p_t \leq 1 \quad \forall\, t}$$

Proof. See appendix III. ∎

b) *The Economy Growth Path*

If we denote $G(k_t) = E_t\left(\frac{k_{t+1}}{k_t} \setminus k_t, S_t = L\right)$ than G has the folowing pace

G gives the one-period minimum growth rate avearge of the economy in regions $[0, k_m]$, $[k_m, k_a]$, $[k_a, k_b]$, $[k_b, k_\alpha]$ and $[k_\alpha, +\infty[$. Note that it is in region $[k_b, k_\alpha]$ that the economy realizes the maximum growth rate. Remember that in this region the bank reserves are set to zero because the entrepreneurs don't

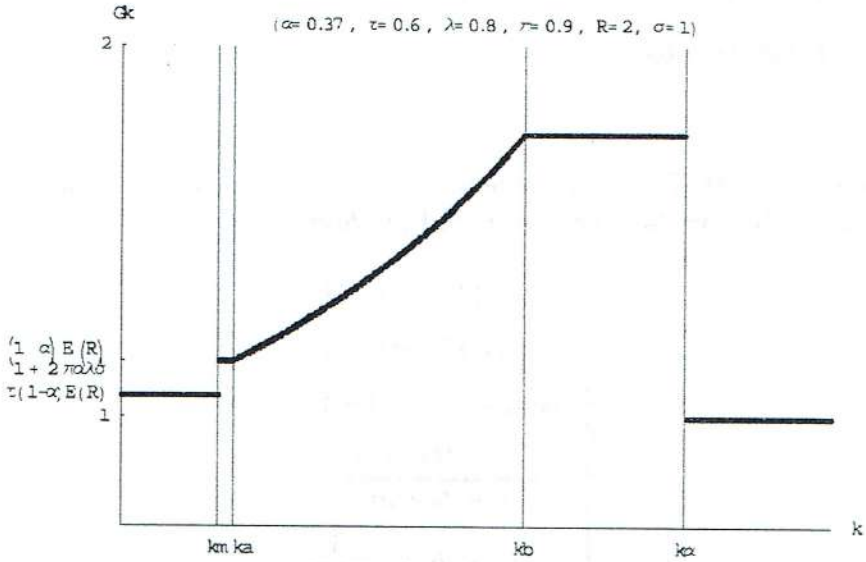

Figure 6: G = G(kt)

default on loans. This lead to an increasing proportion of financed project as shown in figure (5). The region $[k_\alpha, +\infty[$ corresponds to a steady state that's why the growth rate is equal to 1.

The dynamic of the growth rate avearge depends on the history of (S) and can therefore alternate between the defferent levels that appears in figure (6). To show this we consider in the following paragraph two senarios.

An Example of two Senarios

Assume that the initial capital level of the economy is $k_0 \in [k_m, k_a]$ and let see the economy dynamic under the following two senarios S_1 and S_2 where $S_i = (S_i(t))_{t\in[1,T]}$ and

$$S_1(t) = H \qquad \text{for} \quad t \in [1, T]$$

$$S_2(t) = \begin{cases} L & \text{for} \quad t \in \{6, 7, 8\} \\ H & \text{for} \quad t \in [1, T] \setminus \{6, 7, 8\} \end{cases}$$

As figure (7) shows, for the two senarios, during the five first periods the capital level is increasing passing from region $[k_m, k_a]$ to region $[k_b, k_\alpha]$.

For the senario (S_1), the capital level continues increasing and reaches its long term level by the end of the seventh period. At the same time, the one period growth rate average reaches its long term value of one.

For the second senario (S_2), the realization of low returns during the sixth, seventh and eighth periods retards the reaching of the long term trail but don't affect its values.

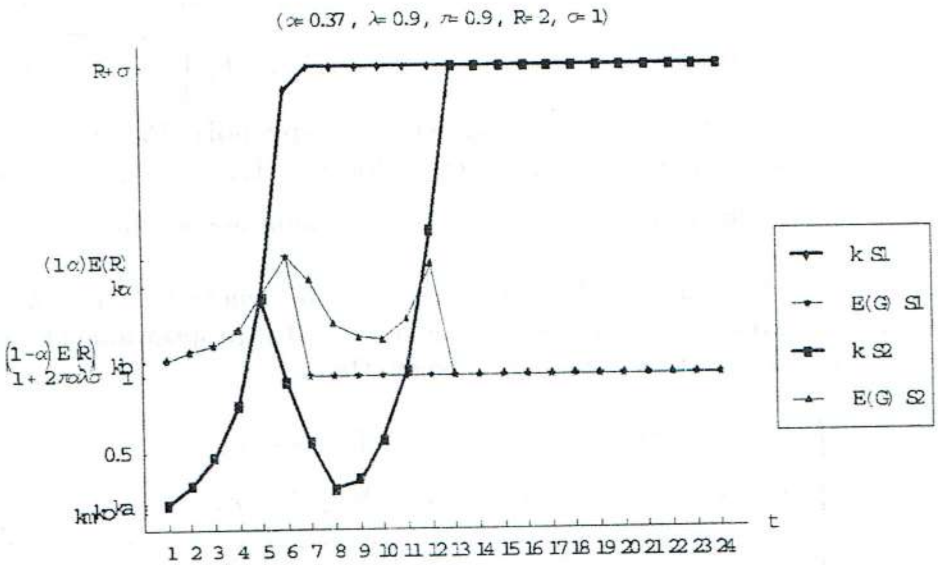

Figure 7: The Growth Path Under Two Senarios

The Growth Rate Average Under T periods

Using Equations (16), (17) and (18) and given the initial capital level k_0 we can calculate the growth rate avearage over T periods, $E_0\left(\left[\frac{k_T}{k_0}\right]^{\frac{1}{T+1}}\right)$. To do this we have developed an algorithm presented in appendix (IV). Using this algorithm we calculate the growth rate average for $T = 1, ...10$, $\lambda \in]0,1]$ and $k_0 = \frac{1-\lambda_0 \alpha E(R)}{1-\alpha}$ where $\lambda_0 = 0.2$. The results are summarized in figure (8).

If we consider k_m, k_a and k_b as functions of λ we can denote it $k_m(\lambda)$, $k_a(\lambda)$ and $k_b(\lambda)$ and its expression continue to be given by (6), propositions (1) and (2) . It is easy to note that $k_0 < k_\alpha$ and to show that

$$\begin{cases} k_0 < k_m(\lambda) & \text{for } \lambda < \lambda_0 \\ k_m(\lambda) \leq k_0 < k_a(\lambda) & \text{for } \lambda_0 \leq \lambda < \lambda_1 \\ k_a(\lambda) \leq k_0 < k_b(\lambda) & \text{for } \lambda_1 \leq \lambda < \lambda_2 \\ k_b(\lambda) \leq k_0 & \text{for } \lambda_2 \leq \lambda \end{cases} \tag{21}$$

where $\lambda_1 = \lambda_0 \alpha E(R) = 0.21$ and $\lambda_2 = \frac{\lambda_0 \alpha \left(E(R)\right)^2}{R-\sigma} = 0.58$.

Inequations (21) with figure (6) expalins the pace of the growth average for the first period.

The Effect of the Credit Market Imperfection

As mentioned in the comments of section (1.6.2) the credit market perfection affects the different regions of the economy development. Indeed, we have

$$\frac{\partial k_m}{\partial \lambda} < 0,\ \frac{\partial k_a}{\partial \lambda} < 0,\ \frac{\partial (k_b - k_m)}{\partial \lambda} > 0 \text{ and } \frac{\partial (k_\alpha - k_b)}{\partial \lambda} > 0$$

Hence, when the credit market level of perfection increase the region $[0, k_m]$ where the bank have no role is reduced . Whereas, the regions of intermediate level of development $[k_b, k_m]$ and $[k_\alpha, k_b]$ are enlarged. Therefore, the more perfect a credit market is the faster will the economy convergence to its long-term trail. In fact, we have to remember that in region $[0, k_m]$, the entrepreneurs default may evict the investment and thereby causes a recession.

Note that given an intermediate level of development $k_t \in [k_m, k_b]$, an amelioration of the credit market perfection reduces the necessary amount of bank

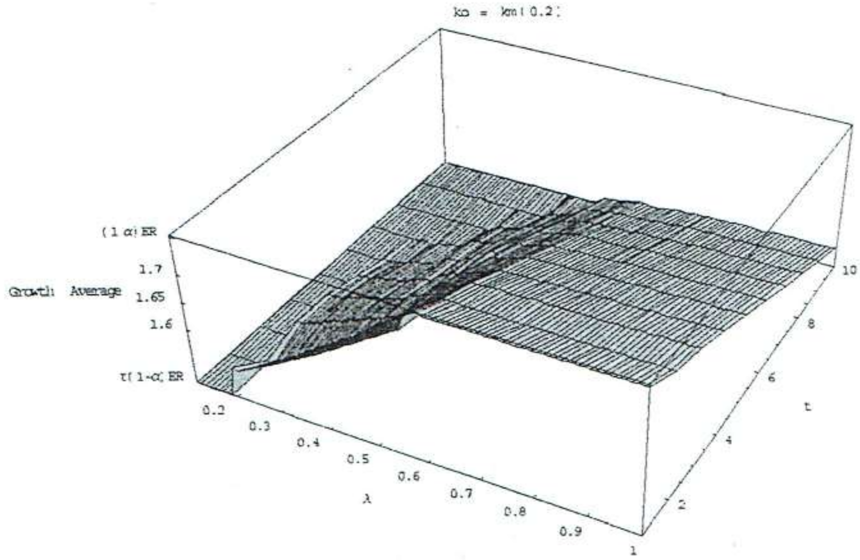

Figure 8: The Effect of the Credit Market Perfection on the Growth Rate Average for Period [0,t], t =1,...,10

reserves which increases the proportion of realized projects and fastens the convergence to the long-term trail. Note also that the interest rate increases in region $k_t \in [k_m, k_a]$ because it becomes no longer the region where the solvability constraint dominates the profitability one.

Figure (9) illustrates the above remarks. It shows the different regions, the bank reserves per project and the interest rate for the values 0.4 and 0.9 of λ.

Figure (10) shows how an improvement of the credit market perfection affect the different regions of development and the corresponding levels of one-period avearage growth. Note that the region of high development is not affected by the credit market perfection. So that the latter don't affects the long-term trail but only the growth for the intermediary zones.

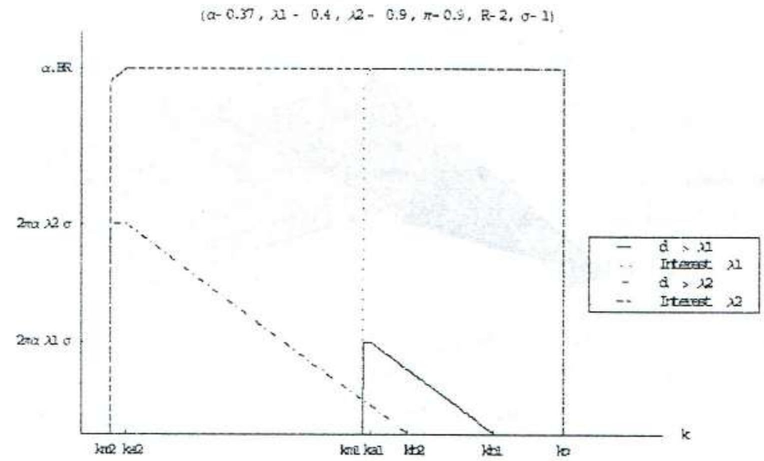

Figure 9: The Effect of an improvement of the Credit Market Perfection on the Banking Attitude

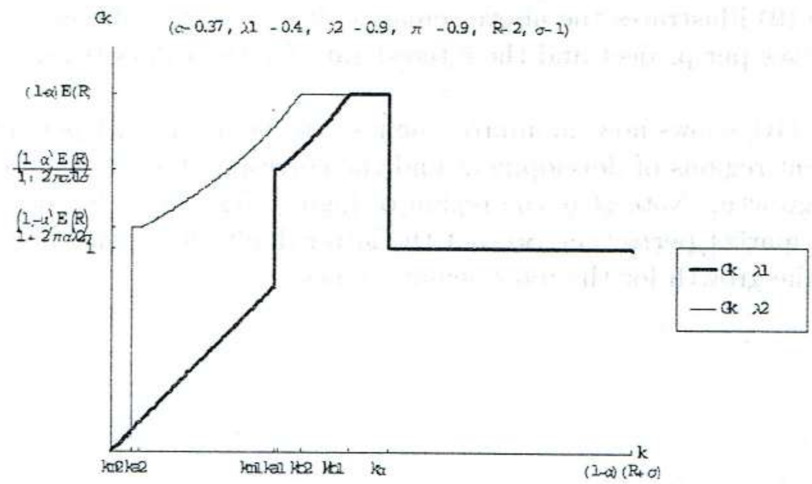

Figure 10: The Effect of an Improvement of the Credit Market Perfection on The Growth Levels

3 Case of Information Assymetry

We assume that there is an information asymmetry between the entrepreneurs and the bank about the risk of the investment good production technology. The bank beleives that one indivisible unit of a final good invested at t gives $\hat{R}_{t+1}$ units of investment good at $t+1$, where

$$\hat{R}_{t+1} = \begin{cases} R+\hat{\sigma} & \text{with probability } \pi \\ R-\hat{\sigma} & \text{with probability } 1-\pi \end{cases}$$

and $\hat{\sigma} < \sigma$. Thus we have

$$Var(\hat{R}_{t+1}) < Var(R_{t+1})$$

which means that the bank underestimate the risk of the investment good technology. Note that $\forall\ t$

$$E_t\left(\hat{R}_{t+1}\right) = \hat{E(R)} = R + (2\pi-1)\hat{\sigma} < E(R)$$

3.1 The Interest Rate

The new interest rate fixed at the equilibrium is given by replacing R_{t+1} with $\hat{R}_{t+1}$ in equation (5)

$$\hat{r}_{t+1} = \begin{cases} \dfrac{\lambda\alpha \hat{E(R)}}{1-(1-\alpha)k_t} & \text{if } k_t \leq k_a \\ \alpha \hat{E(R)} & \text{if } k_t > k_a \end{cases} \tag{22}$$

3.2 The Bank Reserves and the Proportion of Entrepreneurs

From equation (22) we can show as in section one that the bank anticipate the default of the entrepreneurs, at the end of period $t+1$, if $S_{t+1} = L$ and $k_t \leq \hat{k}_b$ where

$$\hat{k}_b = k_a + \frac{2\pi\lambda\,\hat{\sigma}}{(1-\alpha)(R+\hat{\sigma}\ (2\pi-1))} < k_b \tag{23}$$

In this case, the anticipated amount of default on each loan $\hat{d}_t$ is obtained by replacing σ by $\hat{\sigma}$, k_b by $\hat{k}_b$ and k_m by $\hat{k}_m$ in equation (7). It is easy to show

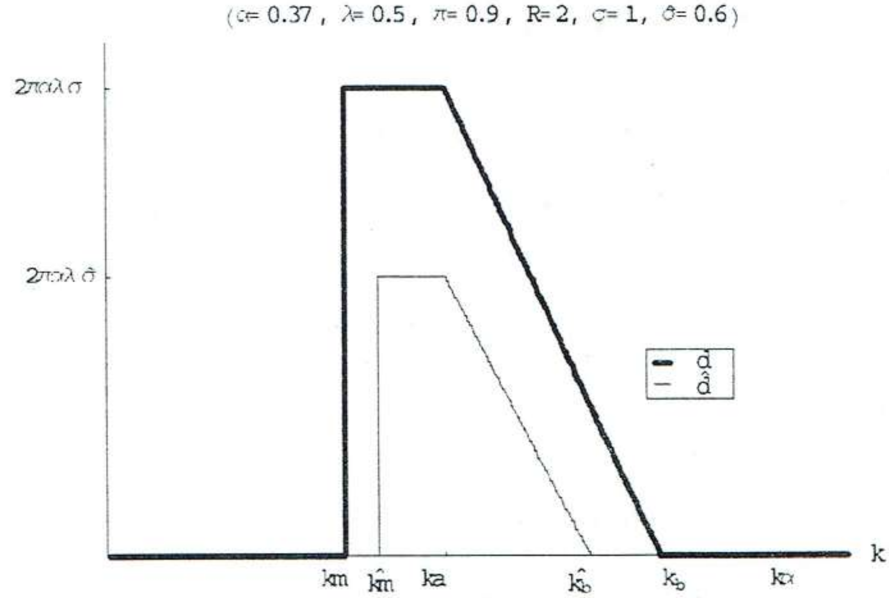

Figure 11: The Reserves Per Project

that

$$\boxed{\hat{d}_t < d_t \quad \forall\, t}$$

This result is obtained because the bank underestimates the risk of the investment good technoogy. As figure (11) shows, the amount of reserves per project that it actually detains, $\hat{d}_t$, is inferior to the necessary one, d_t for any stage of the economy development. Note also, that the information assymetry tighten the zone of possible default since

$$\begin{cases} \hat{k}_m > k_m \\ \hat{k}_b < k_b \end{cases}$$

Hence, the information assymetry is beneficial to the entrepreneurs when the capital level k_t, is in region $\left[\hat{k}_b, k_b\right]$. As figure (12) shows the proportion of financed project increase. However, when the capital level is in region $\left[k_m, \hat{k}_b\right]$, the information assymetry penalize the entrepreneurs since the bank refuse to finance their projects fearing insufficient return $\left(E\left(\hat{R}\right) < E(R)\right)$

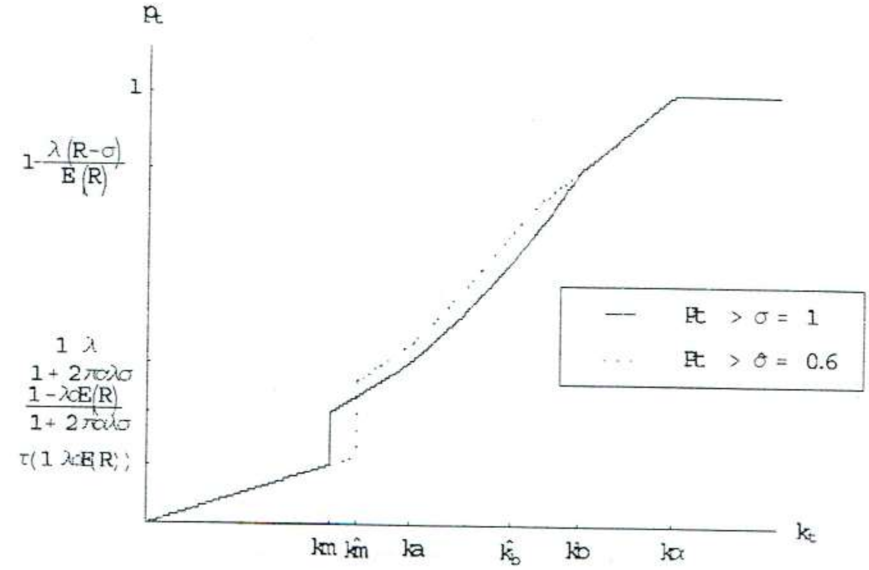

Figure 12: The Proportion of Projects

The proportion $\hat{p}_t$ of financed projects, at the beginning of period $t+1$, is obtained by a similar reasoning to that of the previous section, and its expression is obtained by replacing $\hat{d}_t$ by d_t and $\hat{d}_{t-1}$ by d_{t-1} in equation (14). Following equation (9) the bank reserves are

$$\boxed{\hat{B}_t = \hat{p}_t\ \hat{d}_t}$$

The neceesary bank reserves are obtained by multiplying the proportion of financed projects $\hat{p}_t$ by the real possible default per project d_t

$$\tilde{B}_t = \hat{p}_t\ d_t > \hat{B}_t$$

This means that in case of the entrepreneurs default the bank is unwilling to repay their depositors the interests it promise them. It can repay the maximum level of

$$\hat{B}_t + \hat{p}_t\ \lambda\alpha(R-\sigma) < \tilde{B}_t + \hat{p}_t\ \lambda\alpha(R-\tilde{\sigma}) = r_{t+1}(1-\hat{p}_t)w_t$$

3.3 The Depositors Confidence Crisis in the Bank

3.3.1 The Origin

The problem arises when the amount the depositors receive is inferior to their initial deposit, i.e.:

$$\hat{B}_t + \hat{p}_t \, \lambda\alpha(R-\sigma) < (1-\hat{p}_t)w_t \tag{24}$$

To see if this is possible we have to distinguish the following six regions

$k_t \in$	$]\hat{k}_m, k_a]$	$]\, k_a, \hat{k}_b]$	$]\hat{k}_b, k_\alpha]$
$S_t = L$	I	III	V
$S_t = H$	II	IV	VI

Remember that for $k_t > k_\alpha$ the bank don't play any economic role which justifies that we don't consider the correspondent regions. Note that in each region, the bank reserves $\hat{B}_t$ have a different expression.

Proposition 4 *Under conditions (20) the confidence crises may occur in region (I). Under (20) it may occur in region (II) , (III) and (V) if the following sufficient conditions are respectively satisfied*

in region (II) :	$\pi < \frac{1-\alpha(R-\sigma)}{2\alpha\,\hat{\sigma}}$
in region (III):	$\hat{\sigma} < \frac{R(1-\alpha(R-\sigma))}{1-\alpha(R-\sigma)(2\pi-1)}$
in region (V):	$k_t < \frac{1-\lambda\alpha(R-\sigma)}{1-\alpha}$

Proof. See appendix V. ■

For regions (IV) and (VI) we haven't found a simple sufficient condition for the inequality (24) to be verified. The simulation shows however that the crisis can occur in this region (IV) (see section 2.4)

3.3.2 The Depositors' Reaction

At the end of period $t+1$, when the bank pays the depositors of generation $(t-1)$ a quantity of final goods inferior to their initial deposit, potential depositors of generation (t) have no confidence in the bank management and decide to hold the wages they earn at $t+1$ out of the banking system. We assume that this loss of confidence lasts for one generation after the default event. This means that generation $(t+2)$ depositors put their wages in the bank if there is no further crisis. This is guaranteed if we assume that the bank is restructured after the confidence crisis so that it know exactly the risk of the investment technology i.e. $\hat{\sigma} = \sigma$.

3.3.3 The Economic Consequences

As a consequence of the confidence crisis, the bank don't play any economic role during period $t+1$. The proportion p_{t+1} of generation (t) potential entrepreneurs have no access to loans and are, therefore, unable to realize their projects. Hence, they decide to cooperate by putting their capital in common and signing share contracts involving that some entrepreneurs give their capital to others. At the end of the period the project return is divided proportionally to the amount of invested capital. Let denote p_{t+1}^c the new proportion of entrepreneurs. The following equation determine p_{t+1}^c :

$$\int_0^{p_{t+1}^c}(1-w_{t+1})dn = \int_0^{p_{t+1}} w_{t+1}\,dn - \int_0^{p_{t+1}^c} w_{t+1}\,dn$$

which means that the amount of loans needed by the proportion p_{t+1}^c of entrepreneurs is equal to the shareholders total capital. We obtain

$$p_{t+1}^c = w_{t+1}\ p_{t+1}$$

It is clear that the number of realized project decrease largely in comparaison to p_{t+1} $(w_{t+1} < 1)$. Using equation (14) we obtain since $S_{t+1} = L$ and $k_{t+1} < k_\alpha$

$$p_{t+1}^c = \frac{(1-\alpha)^2}{1+d_{t+1}} k_{t+1}^2$$

The growth rate avearage for period $t+2$ satisfies

$$\frac{E_{t-1}(k_{t+2}^c)}{k_{t+1}} = \frac{(1-\alpha)^2 E\,(R)}{1+d_{t+1}} k_{t+1}$$

$$< 1 < \frac{(1-\alpha)\,E\,(R)}{1+d_{t+1}} = \frac{E_t(k_{t+2})}{k_{t+1}}$$

where d_{t+1} is given by equation (7). The inequality is obtained because $k_{t+1} < k_\alpha$

We conclude that an inefficient bank system (one that underestimate the risk of the investment good technlogy) can generate a crisis of confidence among depositors which in turn leads to the decline of the economic activity.

3.4 The level of Development and the Crisis Amplitude

If we define the crisis magnitude $E_{t-1}(M^c_{t+2})$ as the the growth rate average loss caused by the bank's inefficience we obtain

$$E_{t+1}(M^c_{t+2}) = \frac{E_t(k_{t+2})}{k_{t+1}} - \frac{E_{t+1}(k^c_{t+2})}{k_{t+1}}$$

$$= \frac{(1-\alpha)\,E\,(R)}{1+d_{t+1}}\,(1-(1-\alpha)\,k_{t+1})$$

We show in the appendix that

$$\boxed{\frac{\partial E_{t+1}(M^c_{t-2})}{\partial k_{t+1}} > 0} \tag{25}$$

Equation (25) means that the amplitude of the confidence crisis will be more severe if it occurs at an advanced stage of development than at a lower one. However, the effect on the transition process (the long term effect) is more severe when the crises occurs at the beginning of the transition. This is shown, for $\lambda = 0.75$, in figure (13) where the state of low return $(S = L)$ occurs respectively at date $t = 3, 4$ and 5. It is clear that the delay of convergence to the long term trail is superior when the crisis occurs at lower stage of development.

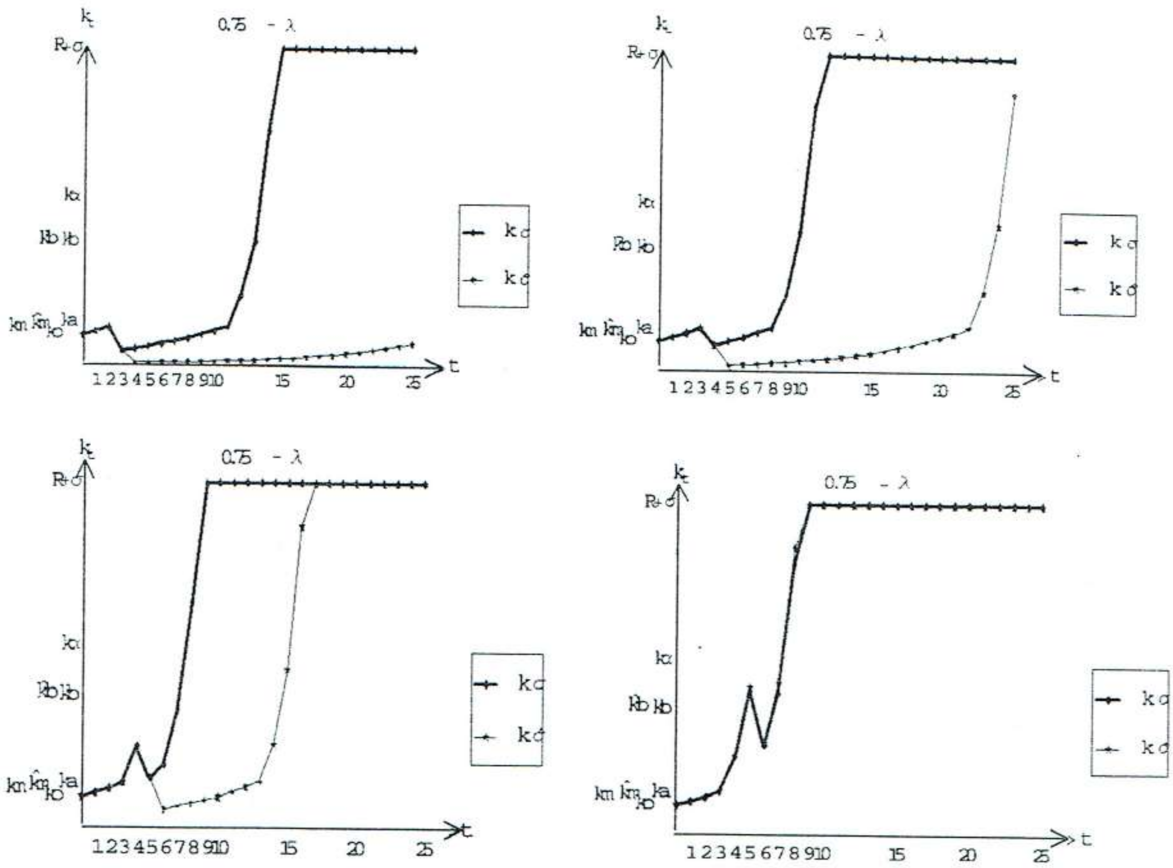

Figure 13

3.5 The Credit Market Perfection and the Crisis Amplitude

Let us assume that the transition process begin with an initial level of capital equal to k_0. Assume also that during the T first periods the return of the investment good technology is high ($S_1 = S_2 = ... = S_T = H$) and that the first low return occurs at the $T+1$ period ($S_{T+1} = L$). From our previous analysis, we know that the more perfect is the credit market the higher is the level of the capital at the end of period T (because the proportion of financed project is higher at each period). Therefore, when the low return occurs, the economy with the more perfect credit market will be the strongest and the damage of a confidence crises, if it occurs, will be the lowest. When the credit market perfection is sufficiently high there will be no confidence crises in the banking system. This last remark can be seen from inequation (24) which can be writen as follow

$$\lambda < \lambda^* \left(k_t, S_t\right)$$

where

$$\lambda^* \left(k_t, S_t\right) = \frac{1}{\alpha(R-\sigma)} \left(\left(\frac{1}{\hat{p}_t} - 1\right)(1-\alpha)k_t - \hat{d}_t\right)$$

Therefore, for a given S_T and k_T the confidence crisis don't occur at $T+1$, if the credit market perfection λ is superior to $\lambda^* \left(k_T, S_T\right)$. Figure (14) illustrates the above comments for $T = 4$ and $\lambda \in \{0.65, 0.75, 0.85\}$.

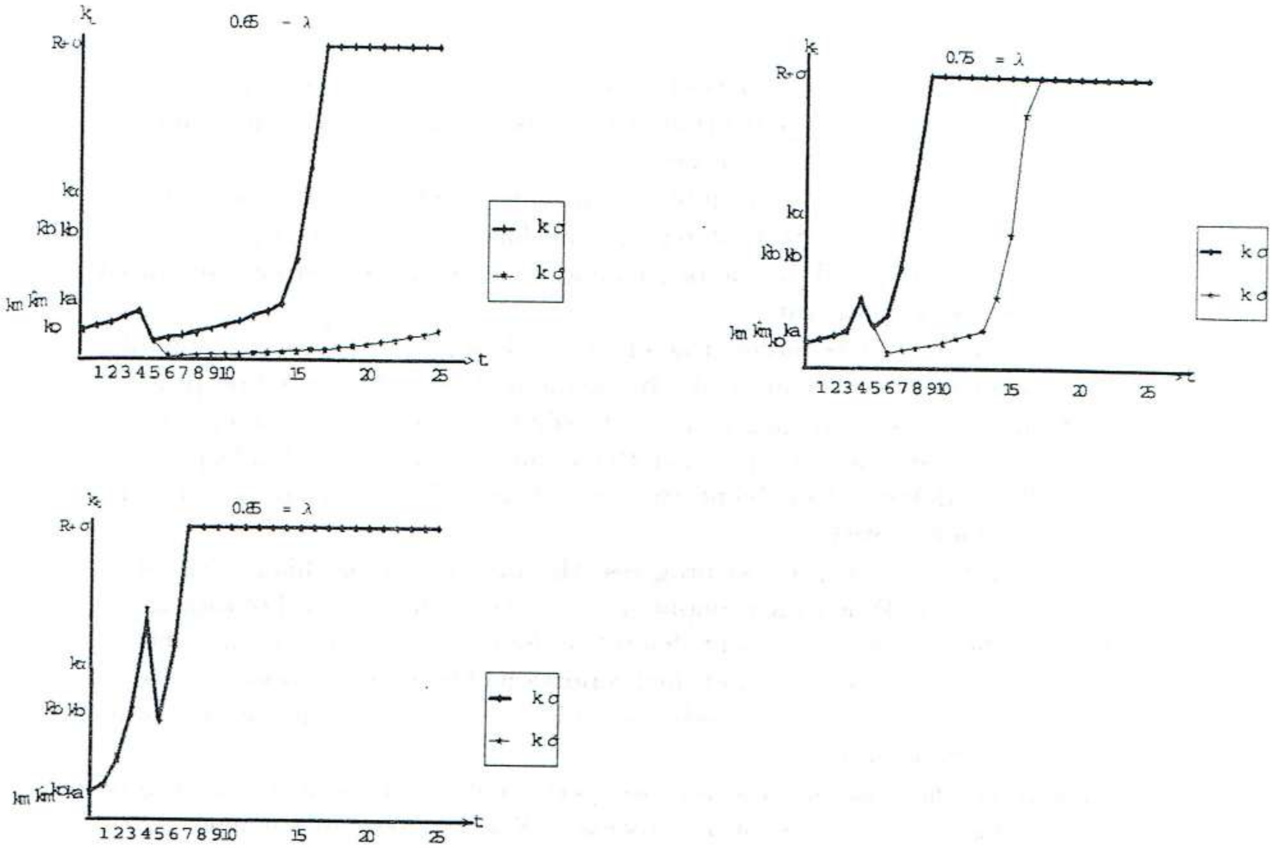

Figure 14

4 Conclusion

The paper underlies that the perfection of the banking system defined as the efficiency of banks and the perfection of the credit market, plays an important role in the economic transition process.

It proposes a model where the interest rate is the control variable of the bank. It fixes it in order to ensure that on the one hand, the financed projects are profitable and solvent and on the other hand, that depositors are remunerated a strictly positive deposit rate.

We showed that the economic role of the bank begins after the reaching of a certain economic development level. At the beginning of the transition process, the profitable projects are not necessarily solvent because the entrepreneurs' capital contribution is low compared to the amount of loans. The banks protects against a likely default of payment, in case of bad performances of projects, by the constitution of reserves.

When the transition process progress, the amount of the likely default of payment decreases. When the economy reaches a determined level of capital accumulation the entrepreneur don't default on loans. Consequently, the amount of the banks' reserve decreases and the proportion of financed projects increases. When the capital accumulation becomes sufficiently elevated, the entrepreneurs self-finance their projects.

We showed that the rare are the bad performances of the investment good sector, the faster is the transition process. When banks are efficient, a bad performance of financed projects provokes a short term recession. The consequences of this bad performance are more serious when banks are inefficient. Indeed, in this case banks underestimate the risk of projects that they finance and the reserves that it constitutes are insufficient to face a possible default of payment. In certain cases, banks will be incapable to honor the contracts signed with their depositors, which provokes a crisis of confidence in the banking system and a flight of the saving. This aggravates the recession and have long term effects on the transition process.

The negative effects of a confidence crisis are more important when it takes place in the beginning of the transition process. We showed that the more perfect is the credit market, the least will be the aftermaths, of a bad performance of the investment good sector, on the transition process. This is true even if banks are inefficient. Moreover, for a sufficiently high credit market perfection, the confidence crisis don't take place.

An extension of this work is to study the role of the banking system in the transition process of a small open economy.

References

1. Wachtel Paul and John Bonin (1999) "Financial Sector Development in Transition Economies",http://www.stern.nyu.edu/ eco/wkpapers/workingpapers00/.

2. Matsuyama Kiminori (2000) "Endogenous Inequality", *Review of Economic Studies*, 67, October 2000, 743-759.

3. Matsuyama Kiminori (2001) "Financial Market Globalization and Endogenous Inequality of Nations", ENS-Delta Seminar.

4. Bernanke Ben and Mark Gertler (1990) "Financial Fragility and Economic Performance", *The Quartly Journal of Economic*, February 1990, 87-113.

5. Aghion Philippe and Patrick Bolton (1997), "A Theory of Trickle-Down Growth and Development", *Review of Economic Studies*, 64, April 1997, 151-172.

Appendix

Appendix I

• Let $k_t \leq k_a$, assume that (3) is satisfied and let us show that (2) is satisfied.

Since (3) is satisfied we have

$$\frac{r_{t-1}\,(1-w_t)}{\lambda} \leq \alpha E(R) \tag{26}$$

but

$$\begin{aligned} k_t &\leq k_a = \frac{1-\lambda}{1-\alpha} \\ (1-\alpha)k_t &\leq 1-\lambda \\ w_t &\leq 1-\lambda \\ \frac{1-w_t}{\lambda} &\geq 1 \end{aligned}$$

Thus, with (26) we obtain

$$r_{t+1} \leq \alpha E(R)$$

which is condition (2).

• Let $k_t > k_a$, assume that (2) is satisfied and let us show that (3) is satisfied.

Using (2) we obtain

$$r_{t+1}\,(1-w_t) \leq \alpha E(R)(1-w_t) \tag{27}$$

but $k_t > k_a$ gives $1-w_t < \lambda$ so that we obtain using (27)

$$r_{t+1}\,(1-w_t) \leq \lambda\alpha E(R)$$

which is condition (3).

Appendix II

At the end of period $T+1$, the entrepreneur defaults if the interest payments $r_{t+1}\,(1-w_t)$ are superior to the cost of default $\lambda\alpha R_{t+1}$. Using equation (5) we obtain

$$r_{t+1}\,(1-w_t)-\lambda\alpha R_{t+1}=\begin{cases}\lambda\alpha\,[E(R)-R_{t+1}] & \text{if } \; k_m\leq k_t\leq k_a\\ \alpha\,[E(R)(1-w_t)-\lambda R_{t+1}] & \text{if } \; k_t>k_a\end{cases}\tag{28}$$

- <u>If $S_{t+1}=L$</u>

We have $R_{t+1}=R-\sigma<E(R)$

- <u>For $k_t\in[\,k_m,k_a]$</u> .

$$r_{t+1}\,(1-w_t)-\lambda\alpha R_{t+1}=\lambda\alpha\,[E(R)-(R-\sigma)]>0$$

- <u>For $k_a<k_t\leq k_b$</u> $=k_a+\dfrac{2\pi\lambda\sigma}{(1-\alpha)E(R)}$, in one hand, we have

$$\begin{aligned}
(1-\alpha)k_t &\leq 1-\lambda+\frac{2\pi\lambda\sigma}{E(R)}\\
w_t &\leq 1-\lambda+\frac{2\pi\lambda\sigma}{E(R)}\\
1-w_t &\geq \lambda\left(\frac{E(R)-2\pi\sigma}{E(R)}\right)\\
1-w_t &\geq \lambda\left(\frac{R+(2\pi-1)\sigma-2\pi\sigma}{E(R)}\right)\\
1-w_t &\geq \lambda\left(\frac{R-\sigma}{E(R)}\right)
\end{aligned}\tag{29}$$

in the other hand,

$$r_{t+1}\,(1-w_t)-\lambda\alpha R_{t+1}=\alpha\,[E(R)(1-w_t)-\lambda(R-\sigma)]$$

using (29) we obtain

$$r_{t+1}\,(1-w_t)-\lambda\alpha R_{t+1}\geq 0$$

- <u>For $k_t>k_b$</u>

$$1-w_t<\lambda\left(\frac{R-\sigma}{E(R)}\right)$$

and

$$\begin{aligned} r_{t-1}\,(1-w_t) - \lambda\alpha R_{t+1} &= \alpha\,[E(R)(1-w_t) - \lambda(R-\sigma)] \\ &< 0 \end{aligned}$$

Therefore, for $S_{t+1} = L$, the interest payment are superior to the default cost in the region $[k_m, k_b]$ which means that the entrepreneurs have an incentive to default on the loans.

- <u>If $S_{t+1} = H$</u>

We have $R_{t+1} = R + \sigma > E(R)$. Thus, $r_{t+1}\,(1-w_t) - \lambda\alpha R_{t+1} < 0$ for $k_t \in [\,k_m, k_a]$.

for $k_t > k_a = \dfrac{1-\lambda}{1-\alpha}$

$$\begin{aligned} r_{t-1}\,(1-w_t) - \lambda\alpha R_{t+1} &= \alpha\,[E(R)(1-w_t) - \lambda(R+\sigma)] \\ &< \alpha(R+\sigma)\,[(1-w_t) - \lambda] \end{aligned}$$

but we have also

$$\begin{aligned} (1-\alpha)k_t &> 1-\lambda \\ 1-w_t &< \lambda \end{aligned}$$

Therefore,

$$r_{t+1}\,(1-w_t) - \lambda\alpha R_{t+1} < 0$$

In this case, whatever the level of capital accumulation, the interest payments are inferior to the default cost, so that the entrepreneurs don't default on loans.

Appendix III

Let denote $t^* = \inf\{t > 0 \ / \ k_t \geq k_\alpha\}$. Thus, we have $k_{t^*-1} \in [k_b, k_\alpha[$. From equation (12) we have

$$\begin{aligned} p_{t^*-1} &= (1-\beta_{t^*-1})\frac{k_{t^*-1}}{k_\alpha} \\ &\leq \frac{k_{t^*-1}}{k_\alpha} < 1 \end{aligned}$$

Since

$$k_{t^*} = p_{t^*-1}(R+\sigma)$$

we have

$$\frac{k_{t^*}}{k_\alpha} < (1-\alpha)(R+\sigma) < 2$$

and

$$p_{t^*} = Floor\left[\frac{k_{t^*}}{k_\alpha}\right] \leq 1$$

Finally, it is easy to show by recurrence that

$$p_t \leq 1 \quad \forall\, t$$

Appendix IV

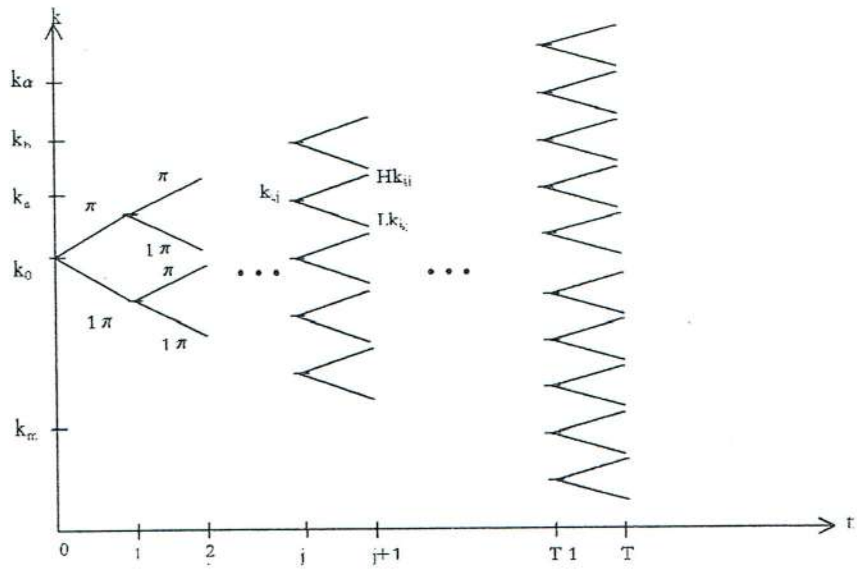

Figure 13:

The one period growth rate average over T periods is

$$E_0\left(\left[\frac{k_T}{k_0}\right]^{\frac{1}{T+1}}\right) = E_0\left(\left[\frac{k_1}{k_0}\right]^{\frac{1}{T+1}}\left[\frac{k_2}{k_1}\right]^{\frac{1}{T+1}}...\left[\frac{k_T}{k_{T-1}}\right]^{\frac{1}{T+1}}\right) \tag{30}$$

For $j = 0, ..., T-1$ there is 2^j possible values of k_j which we denote $k_{j,i}$ with $i = 1, ..., 2^j$.

Assume that at date j, the level of capital is $k_{j,i}$. The capital level at date $j+1$ will be

$$k_{j+1} = \begin{cases} k_{j+1,i} = Hk_{j,i} = p_{j,i}.(R+\sigma) & \text{if} \quad S_{j+1} = H \\ k_{j+1,i-1} = Lk_{j,i} = p_{j,i}.(R-\sigma) & \text{if} \quad S_{j+1} = L \end{cases}$$

so that

$$\left[\frac{k_{j+1}}{k_{j,i}}\right]^{\frac{1}{T+1}} = \begin{cases} \left[\frac{Hk_{j,i}}{k_{j,i}}\right]^{\frac{1}{T+1}} & \text{with probability} \quad \pi \\ \left[\frac{Lk_{j,i}}{k_{j,i}}\right]^{\frac{1}{T+1}} & \text{with probability} \quad 1-\pi \end{cases}$$

For $j = 1, ..., T-1$, we define V_j, a vector of 2^j elements, as follow

$$\begin{cases} V_1 = \left(\pi\left[\frac{Hk_{0,1}}{k_0}\right]^{\frac{1}{T+1}}, (1-\pi)\left[\frac{Lk_{0,1}}{k_0}\right]^{\frac{1}{T+1}}\right) \\ V_2 = \left(\pi V_{1,1}\left[\frac{Hk_{1,1}}{k_{1,1}}\right]^{\frac{1}{T+1}}, (1-\pi)V_{1,1}\left[\frac{Lk_{1,1}}{k_{1,1}}\right]^{\frac{1}{T+1}}, \pi V_{1,2}\left[\frac{Hk_{1,2}}{k_{1,2}}\right]^{\frac{1}{T+1}}, (1-\pi)V_{1,2}\left[\frac{Lk_{1,2}}{k_{1,2}}\right] \\ \text{Given } V_j = (V_{j,i})_{i=1,...,2^j} \quad V_{j+1} \text{ is defined by} \\ V_{j+1} = \left(\pi V_{j,i}\left[\frac{Hk_{j,i}}{k_{j,i}}\right]^{\frac{1}{T+1}}, (1-\pi)V_{j,i}\left[\frac{Lk_{j,i}}{k_{j,i}}\right]^{\frac{1}{T+1}}\right)_{i=1,...,2^j} \end{cases}$$

The one period average growth rate over T periods is the sum of the V_T elements. The inconvenience of this calculus is that the number of elements of the vector V increase exponentially with time.

Appendix V

The confidence crises occurs if and only if (24) is satisfied. But

$$\hat{B}_t + \hat{p}_t \ \lambda\alpha(R-\sigma) < (1-\hat{p}_t)w_t$$

$\iff$

$$\hat{p}_t \ \left[\hat{d}_t + \ \lambda\alpha(R-\sigma)\right] < (1-\hat{p}_t)w_t$$

$\iff$

$$\hat{p}_t \ \left[\hat{d}_t + w_t + \ \lambda\alpha(R-\sigma)\right] < w_t \ = \frac{k_t}{k_\alpha}$$

$\iff$

$$\frac{k_\alpha}{k_t}\hat{p}_t \ \left[\hat{d}_t + \frac{k_t}{k_\alpha} + \ \lambda\alpha(R-\sigma)\right] < 1 \tag{31}$$

- *In region (I)*

We have

$$\hat{d}_t = 2\pi\lambda\alpha\hat{\sigma}$$

and

$$\frac{k_\alpha}{k_t}\hat{p}_t = \frac{1}{1+2\pi\lambda\alpha\hat{\sigma}}$$

Thus,

$$\frac{k_\alpha}{k_t}\hat{p}_t \ \left[\hat{d}_t + \frac{k_t}{k_\alpha} + \ \lambda\alpha(R-\sigma)\right] = \frac{2\pi\lambda\alpha\hat{\sigma} + \frac{k_t}{k_\alpha} + \ \lambda\alpha(R-\sigma)}{1+2\pi\lambda\alpha\hat{\sigma}} \tag{32}$$

but in region (I) we have

$$\begin{aligned} k_t \ &\leq \ k_a = (1-\lambda)k_\alpha \\ \frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma) \ &\leq \ 1-\lambda+\lambda\alpha(R-\sigma) = 1+\lambda(\alpha(R-\sigma)-1) \end{aligned}$$

and according to (20) $\alpha(R-\sigma) < 1$. Hence $\frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma) < 1$.

Finally, with (32) we obtain (31) and we can conclude that uder the conditions (20) the confidence crises occurs in region (I) if $S_{t+1} = L$.

- <u>*In region (II)*</u>

We have

$$\hat{d}_t = 2\pi\lambda\alpha\hat{\sigma}$$

and

$$\frac{k_\alpha}{k_t}\hat{p}_t = \frac{1 + \dfrac{k_{t-1}}{k_t}2\pi\lambda\alpha\hat{\sigma}}{1 + 2\pi\lambda\alpha\hat{\sigma}}$$

$$\frac{k_\alpha}{k_t}\hat{p}_t \left[\hat{d}_t + \frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma)\right] = \frac{1 + \dfrac{k_{t-1}}{k_t}2\pi\lambda\alpha\hat{\sigma}}{1 + 2\pi\lambda\alpha\hat{\sigma}} \left(2\pi\lambda\alpha\hat{\sigma} + \frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma)\right) \tag{33}$$

In this region $S_{t=II}$, so that $k_{t-1} < k_t$ and (33) gives

$$\begin{aligned} \frac{k_\alpha}{k_t}\hat{p}_t \left[\hat{d}_t + \frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma)\right] &< 2\pi\lambda\alpha\hat{\sigma} + \frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma) \\ &< 2\pi\lambda\alpha\hat{\sigma} + 1 - \lambda + \lambda\alpha(R-\sigma) \end{aligned}$$

because $k_t < k_a = (1-\lambda)k_\alpha$. Hence,

$$\frac{k_\alpha}{k_t}\hat{p}_t \left[\hat{d}_t + \frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma)\right] < 1 - \lambda\left(1 - 2\pi\lambda\alpha\hat{\sigma} - \alpha(R-\sigma)\right)$$

and for

$$\pi < \frac{1-\alpha(R-\sigma)}{2\alpha\,\hat{\sigma}} \tag{34}$$

we obtain

$$\frac{k_\alpha}{k_t}\hat{p}_t \left[\hat{d}_t + \frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma)\right] < 1$$

we can conclude that uder the conditions (20) and (34) the confidence crises occurs in region (II) if $S_{t+1} = L$.

- <u>*In region (III)*</u>

In this region we have

$$\frac{k_\alpha}{k_t}\hat{p}_t = \frac{1}{1+\hat{d}_t}$$

so that

$$\frac{k_\alpha}{k_t}\hat{p}_t \left[\hat{d}_t + \frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma)\right] = \frac{\hat{d}_t + \frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma)}{1+\hat{d}_t} \tag{35}$$

In this region, $k_t \leq \hat{k}_b = k_\alpha \left(1 - \lambda + \frac{2\pi\lambda\,\hat{\sigma}}{\hat{E}(R)}\right)$. Thus,

$$\begin{aligned} \frac{k_t}{k_\alpha} &\leq 1 - \lambda + \frac{2\pi\lambda\,\hat{\sigma}}{\hat{E}(R)} \\ \frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma) &\leq 1 - \lambda\left(1 - \alpha(R-\sigma) - \frac{2\pi\,\hat{\sigma}}{\hat{E}(R)}\right) \end{aligned}$$

When the following condition is satisfied

$$\hat{\sigma} < \frac{R\,(1-\alpha(R-\sigma))}{1+\alpha(R-\sigma)(2\pi-1)} \tag{36}$$

we obtain

$$1 - \alpha(R-\sigma) - \frac{2\pi\,\hat{\sigma}}{\hat{E}(R)} = 1 - \alpha(R-\sigma) - \frac{2\pi\,\hat{\sigma}}{R+(2\pi-1)\hat{\sigma}} > 0$$

and therefore

$$\frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma) \leq 1$$

and finally using (35) we have

$$\frac{k_\alpha}{k_t}\hat{p}_t \left[\hat{d}_t + \frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma)\right] \leq 1$$

we can conclude that uder the conditions (20) and (36) the confidence crises occurs in region (III) if $S_{t+1} = L$.

- <u>*In region (V)*</u>

In this region we have

$$\frac{k_\alpha}{k_t}\hat{p}_t = \frac{1}{1+\hat{d}_t}$$

so that

$$\frac{k_\alpha}{k_t}\hat{p}_t\left[\hat{d}_t + \frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma)\right] = \frac{\hat{d}_t + \frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma)}{1+\hat{d}_t}$$

When the following condition is satisfied

$$k_t < \frac{1-\lambda\alpha(R-\sigma)}{1-\alpha} = k_\alpha\left(1-\lambda\alpha(R-\sigma)\right) \tag{37}$$

we obtain

$$\frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma) < 1$$

and therefore,

$$\frac{k_\alpha}{k_t}\hat{p}_t\left[\hat{d}_t + \frac{k_t}{k_\alpha} + \lambda\alpha(R-\sigma)\right] < 1$$

we can conclude that uder the conditions (20) and (37) the confidence crises occurs in region (V) if $S_{t+1} = L$.