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Effect of Rheological Carreau-Yasuda Parameters on Double Diffusive Convection in a square Cavity enclosure Submitted to gradient of Temperature and Concentration

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Abstract: *This paper presents a numerical study of doubly diffusive natural convection in an inclined square cavity filled with a non-Newtonian fluid subjected to a temperature and concentration gradient on the active walls, while the other walls are impermeable and adiabatic, are presented in this study. Any approximation of asymptotic parallel flow is applied to determine the onset of subcritical convection. The Boussinesq approximation and the transport equations are solved by a finite difference method. The numerical simulation presented here covers a wide range of thermal Rayleigh number* $(2000 < Ra_T < 10^4)$, *the rheological parameters of the Carreau-Yasuda fluids* ($n, a, E,$ *and* s) , *and the inclination angel,*γ,$(0° < \gamma < 90°)$. *The Nusselt and Sherwood transfer rates were improved by decreasing the power-law index n and the parameter a, or by increasing the time constant, E. Detailed results of the Nusselt and Sherwood numbers are presented and discussed.*

Keywords : Double diffusive convection, non-Newtonian, Carreau-Yasuda model, square cavity, finite difference method.

Résumé : *Cet article présente une étude numérique de la convection naturelle doublement diffusive dans une cavité carrée inclinée remplie d'un fluide non newtonien soumis à un gradient de température et de concentration sur les parois actives, tandis que les autres parois sont imperméables et adiabatiques, sont présentées dans cette étude. Une approximation d'écoulement parallèle asymptotique est appliquée pour déterminer le début de la convection sous-critique. L'approximation de Boussinesq et les équations de transport sont résolues par une méthode de différences finies. La simulation numérique présentée ici couvre une large gamme de nombre de Rayleigh thermique* $(2000 < Ra_T < 10^4)$, *de paramètres rhéologiques des fluides de Carreau-Yasuda* ($n, a, E,$ *et* s), *et de l'angle d'inclinaison,* γ, $(0° < \gamma < 90°)$. *Les taux de transfert de Nusselt et de Sherwood ont été améliorés en diminuant l'indice de power-law n et le paramètre a, ou en augmentant la constante de temps, E. Les résultats détaillés des nombres de Nusselt et de Sherwood sont présentés et discutés.*

Mots-clés : Convection doublement diffusive, non newtonienne, modèle de Carreau-Yasuda, cavité carrée, méthode de différences finies.

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Introduction :

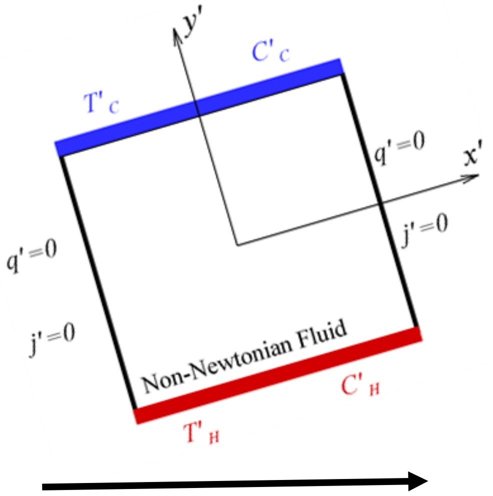
Natural convection, driven by buoyancy It is usually referred to as simultaneous temperature and concentration gradients is generally as doubly diffusive or thermosolutal convection. Researchers (Lamsaadi, Naimi, Hasnaoui, & Mamou, 2006), (Khezzar, Sigire, & Vinogradov, 2012), (Matin, Pop, & Khanchezar, 2013) have closely studied natural convection of a power law fluid in enclosures. In this case, complex flow structures often form in liquids due to the differences between thermal and solutal diffusivities (Bejan & Nield, 1999) .In reviewing the literature to date, several studies on doubly diffusive convection have revealed interesting and useful findings in recent years, which have led to a good understanding of these phenomena in different fluid systems. In square and rectangular cavities heated from below and filled with non-Newtonian fluids, we can mention the work of (Ohta, Akiyoshi, & Obata, 2002) on a numerical study of heat transfer by natural convection of pseudoplastic fluids. (Lamsaadi, Naimi, Hasnaoui, & Mamou, 2006) studied the natural convection in a vertical rectangular cavity filled with a non-Newtonian power-law fluid, and heated from the sides by constant heat fluxes. A scaling analysis was performed with a good agreement found with the numerical results; a correlation of the Nusselt number at the steady state condition was derived Both effects of the aspect ratio of a rectangular cavity and the concentration of the slurry on heat transfer are studied. Their results were correlated to predict the effects of Rayleigh number and power-law index on the Nusselt number. Natural convection of power-law fluids in a shallow cavity uniformly heated from below has been studied analytically and numerically by (Lamsaadi, Naimi, & Hasnaoui, 2005). (Khechiba, Mamou, Hachemi, Delenda, & Rebhi, 2017) studied the effect of rheological parameters on natural thermal convection in a shallow porous layer filled with a non-Newtonian. (Shahmardan & Norouzi, 2014) treated a non-Newtonian fluid flow in a channel with an enclosure. The non-Newtonian Carreau-Yasuda model was used to describe the stress dependence on the strain rate. The numerical results showed that with decreasing the power-law index, the fully development length increases.

In this paper numerically investigates doubly diffusive natural convection in an inclined square cavity filled with non-Newtonian fluid. The side-walls are subject to uniform temperature and concentration conditions concentration, while the horizontal walls are adiabatic and impermeable. The complete system of equations is solved numerically. Numerically and results are obtained for a wide range of governing parameters. The global dependence of the Nusselt and Sherwood numbers as a function of dimensionless governing parameters and boundary conditions is studied in detail.

1. Problem Definition and Mathematical Formulation:

First, the physical model considered in this work is represented on figure 1, it is a inclined square cavity saturated with a non-Newtonian fluid. The Temperature and Concentration of the left and right walls were considered to be kept uniform and constant. The vertical walls are adiabatic and impermeable.

Figure (1): The physical model and coordinate system



The effect of Soret and Dufour on heat and mass has been neglected. The fluid is incompressible. The Boussinesq (Gray & Giorgini, (1975).) approximation was adopted, the density variation, ρ, linearly with Temperature and Concentration as:

$$\rho = \rho_0 \left[1 - \beta_T (T' - T'_0) + \beta_C (C' - C'_0)\right] \tag{1}$$

Where β_T and β_C are the thermal and solutal coefficient and ρ_0 being the fluid density at reference T' and T'_0 and concentration C' and C'_0.

Considering the mentioned assumptions, the following dimensional equations of mass, liner momentum, thermal energy and concentration respectively, are expressed as follows:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \tag{2}$$

$$\rho_0\left[\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + v'\frac{\partial v'}{\partial y'}\right] = -\frac{\partial P'}{\partial x'} + \frac{\partial \tau'_{xx}}{\partial x'} + \frac{\partial \tau'_{xy}}{\partial y'} - \rho_0 g\left[\beta_T (T' - T'_0) + \beta_C (C' - C'_0)\right]\cos\gamma \tag{3}$$

$$\rho_0\left[\frac{\partial v'}{\partial t'} + u'\frac{\partial v'}{\partial x'} + v'\frac{\partial v'}{\partial y'}\right] = -\frac{\partial P'}{\partial y'} + \frac{\partial \tau'_{xy}}{\partial x'} + \frac{\partial \tau'_{yy}}{\partial y'} - \rho_0 g\left[\beta_T (T' - T'_0) + \beta_C (C' - C'_0)\right]\sin\gamma \tag{4}$$

Where :

$$\tau'_{xx} = 2\mu\frac{\partial u'}{\partial x'}, \quad \tau'_{yy} = 2\mu\frac{\partial v'}{\partial y'}$$

And

$$\tau'_{xy} = \tau'_{yx} = \mu\left(\frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'}\right)$$

$$\frac{\partial T'}{\partial t'} + u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'} = \alpha\left(\frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'^2}\right) + D_{TS}\left(\frac{\partial^2 C'}{\partial x'^2} + \frac{\partial^2 C'}{\partial y'^2}\right) \tag{5}$$

$$\frac{\partial C'}{\partial t'} + u'\frac{\partial C'}{\partial x'} + v'\frac{\partial C'}{\partial y'} = D\left(\frac{\partial^2 C'}{\partial x'^2} + \frac{\partial^2 C'}{\partial y'^2}\right) + D_{ST}\left(\frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'^2}\right) \tag{6}$$

Where u' and v' represent the velocity components, P' the pressure, t' the time, μ in the fluid viscosity, g is the gravitational acceleration, and α and D are the thermal and solutal diffusivities, respectively.

The fluid was supposed to be non-Newtonian, with the viscosity,μ, depending on the shear stress of the flow according to the Carreau-Yasuda model (Yasuda, Armstrong, & Cohen, 1981) and (Bird, Armstrong, & Hassager, 1978) :

$$\frac{\mu - \mu_\infty}{\mu_0 - \mu_\infty} = \left[1 + (E'\dot{\gamma}')^a\right]^{(n-1)/a} \tag{7}$$

Where μ_0 is the fluid viscosity at zero shear rate, and μ_∞ is the fluid viscosity at infinity shear rate, the rheological parameters such that, E_0 is the dimensional time constant, γ', is the second variant of shear rate, a, is a dimensionless parameter describing the transition region, and (less than unity for pseudo plastic fluid) is the power-law exponent characterizing the shear thinning regime (degree of shear thinning).

The dimensionless variable used are:

$$(x,y)=\frac{(x',y')}{H'},\ (u,v)=\frac{(u',v')H'}{\alpha},\ t=\frac{t'\alpha}{H'^2},\ \psi=\frac{\psi'}{\alpha},\ \Omega=\frac{\Omega' H'^2}{\alpha},\ P=\frac{P'}{P^*},$$

$$T=\frac{(T'-T_0')}{\Delta T^*},\ C=\frac{(C'-C_0')}{\Delta C^*},\quad \Delta T^*=T_H'-T_C',\quad \Delta C^*=C_H'-C_C',\quad \mu=\frac{\mu_{CY}}{\mu_0},$$

The dimensionless governing equations that inform the efficiency of the method are expressed in cost of vorticity, Ω, temperature, T, concentration, C, and stream function, ψ, are obtained as follows:

$$\frac{\partial\Omega}{\partial t}+\frac{\partial(u\Omega)}{\partial x}+\frac{\partial(v\Omega)}{\partial y}=\Pr\left[\mu\nabla^2\Omega+2\left(\frac{\partial\mu}{\partial x}\frac{\partial\Omega}{\partial x}+\frac{\partial\mu}{\partial y}\frac{\partial\Omega}{\partial y}\right)\right]+\Upsilon_\Omega \tag{8}$$

$$\frac{\partial T}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\nabla^2T+D_f\nabla^2C \tag{9}$$

$$\frac{\partial C}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}=Le^{-1}(\nabla^2C+S_r\nabla^2T) \tag{10}$$

$$\nabla^2\psi=-\Omega \tag{11}$$

The source term, Υ_Ω in Eq. (8), is given by:

$$\Upsilon_\Omega=\Pr\left[\left(\frac{\partial^2\mu}{\partial x^2}-\frac{\partial^2\mu}{\partial y^2}-2\frac{\partial^2\mu}{\partial x\partial y}\right)\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+Ra_T\left(\frac{\partial T}{\partial x}+\varphi\frac{\partial C}{\partial x}\right)\cos\gamma+\left(\frac{\partial T}{\partial y}+\varphi\frac{\partial C}{\partial y}\right)\sin\gamma\right] \tag{12}$$

The above equations are subject to the following initial conditions and dimensionless limits:

$$u=v=0,\quad \psi=0\quad \text{at}\quad x=\pm\frac{1}{2},\ \text{and}\ y=\pm\frac{1}{2} \tag{13a}$$

The thermal and solutal boundary conditions are stated as:

$$\frac{\partial T}{\partial x}=\frac{\partial C}{\partial x}=0\quad \text{at}\quad x=\pm\frac{1}{2}$$

and

$$T=C=\pm\frac{1}{2}\quad \text{at}\quad y=\pm\frac{1}{2} \tag{13b}$$

The transposition of the governing equations into dimensionless form has resulted in a dimensionless grouping such as the thermal Rayleigh number, Ra_T, and Lewis number Le:

$$Ra_T=\frac{g\beta_T\Delta T^* H'^3}{\alpha\nu},\quad Le=\frac{\alpha}{D},$$

The heat and mass transfer are expressed by the local Nusselt and Sherwood numbers (Nu) and (Sh) defined as follows, respectively:

$$Nu^{-1}=\Delta T=T_{(0,-1/2)}-T_{(0,1/2)}\quad,\quad Nu_m=\int_{-A/2}^{A/2}Nu\,dx \tag{14a}$$

$$Sh^{-1}=\Delta S=S_{(0,-1/2)}-S_{(0,1/2)}\quad,\quad Sh_m=\int_{-A/2}^{A/2}Sh\,dx \tag{14b}$$

2. Numerical Solution:

A Finite Difference Method (FDM) with uniform grid size are used to obtain numerical solution of the governing equations (8). (10), the iterative procedure was performed with the Implicit Alternate Direction method (ADI) (Douglas, Jr, & Peaceman, 1955), the stream function, ψ, was obtained by solving the equation (11), and was solved using the Over Relaxation method (SOR) (Successive Over Relaxation). The iteration process was completed on the following condition:

$$\frac{\sum_i \sum_j \left|\psi_{i,j}^{k+1} - \psi_{i,j}^{k}\right|}{\sum_i \sum_j \left|\psi_{i,j}^{k}\right|} \leq 10^{-8} \qquad (15)$$

Where Ψ_{ij} is the stream function value at the node (i, j) at the iteration.

An extensive mesh testing procedure has been conducted to ensure a mesh independent solution. Different combination of meshes have been explored for the case of $Ra_T = 10^4$, $Pr = 10$, $Le = 10$, $N = -0.5$, $n = 0.6$, $E = 0.1$, $s = 10^{-2}$, $a = 2$ and $S_r = D_f = 0$. Shows that the numerical values of Ψ_0, Nu, Sh, μ evaluated at the center of the cavity. It was confined that the grid size 100×100 ensures an independent solution as portrayed by Table 1.

Table (1): Grid independence study

$N_x \times N_y$	**50×50**	**100×100**	**200×200**
Ψ_0	13.712	13.596	14.320
Nu	3.761	3.584	3.816
Sh	9.637	8.105	9.106
μ	0.260	0.261	0.261

The comparative results are summarized in Table 2. Respectively, are obtained in a square cavity indicates the average Nusselt in different Rayleigh numbers, a good agreement is observed between the experimental and numerical codes for the predication of the Nusselt number versus Rayleigh number:

Table (2): Comparison of the averaged Nusselt number versus the Rayleigh number for a high Prandtl number.

Ra_T	**Experiment (Schneck & Veronis, 1967)**	**Present study**	**Present study vs Experiment (Schneck & Veronis, 1967)**
2000	1.13	1.19	6.0%
3000	1.60	1.53	4.4
6000	2.15	2.19	1.8

3. Results and Discusion:

The numerical simulation presented in this study concern the Double Diffusion convection in a square cavity filled with a non-Newtonian fluid. The main parameters governing the present problem are amplitude of the Carreau-Yasuda rheology parameters, namely; $n, E, a,$ and s, the Thermal Rayleigh number, Ra_T, $2000 \leq Ra_T \leq 10^4$, and the inclined angel, γ , $0° \leq \gamma \leq 90°$. In this study, all numerical results were obtained for a Prandtl number $Pr = 10$. The results obtained are presented in terms of the both Nusselt and Sherwood numbers.

Fig. 2. Illustrates the effect of power-law index, n, and Rayleigh number, Ra_T, on the Nusselt and Sherwood numbers. The Nusselt and Sherwood numbers increase with increasing Rayleigh number, Ra_T. For a given value of Rayleigh number, Ra_T, the heat and mass transfer rates increase as the power-law index, n, decreases and produce an effect on it a similar to the produced by the increase, Ra_T, for a given, n. However, it should be noted that the convection in more sensitive to any change in Ra_T with the shear thinning $(0 < n < 1)$ than to the Newtonian behavior $(n = 1)$. In n<1 shear fluids shows that for any decrease in the power law index n, the onset of subcritical convection occurs at a convection of finite amplitude, which indicated clearly the existence of subcritical convection.

Figure (2): Effect of parameter, a, and power-law index, n, on: Nusselt number, Nu, and Sherwood number, Sh, fro $Ra_T = 10^4$, $Le = 10$, $N = -0.5$, $E = 0.1$, $s = 10^{-2}$, $\gamma = 0°$ and $S_T = D_f = 0$.

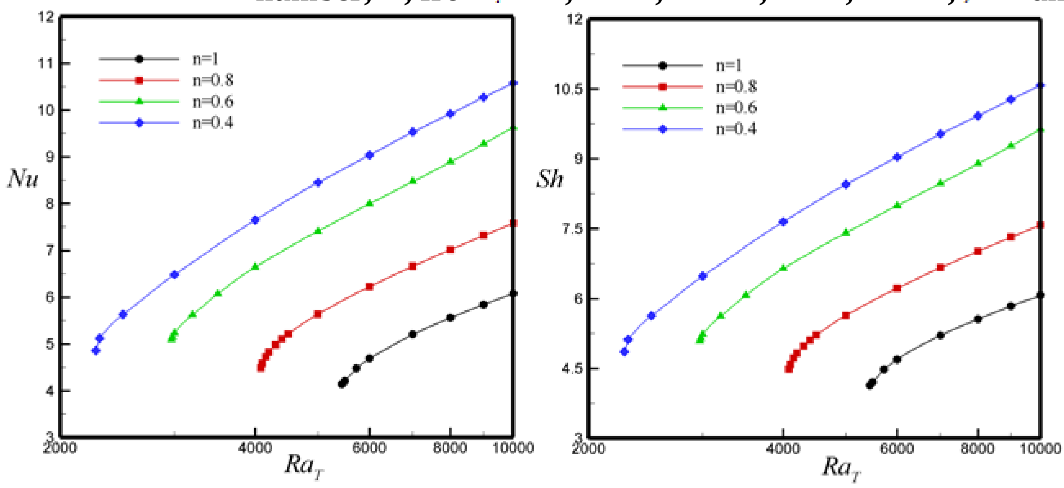


Fig. 3. Illustrates the Nusselt and Sherwood numbers for different various of parameters, E, and power-law index, n, it shows that the Nusselt and Sherwood numbers increases as the parameter, E, increases. Furthermore, the convection heat and mass transfer increases with increasing of parameter, E, and decreasing power-law index, n, from 1 to 0.4. Note that the Nusselt and Sherwood numbers are sensitive to both the power law index and the dimensionless time constant.

Figure (3): Effect of time constant parameter, E, and power-law index, n, on: Nusselt number, Nu, and Sherwood number, Sh, for $Ra_T = 10^4$, $Le = 10$, $N = -0.5$, $a = 2$, $s = 10^{-2}$, $\gamma = 0°$ and $S_T = D_f = 0$.

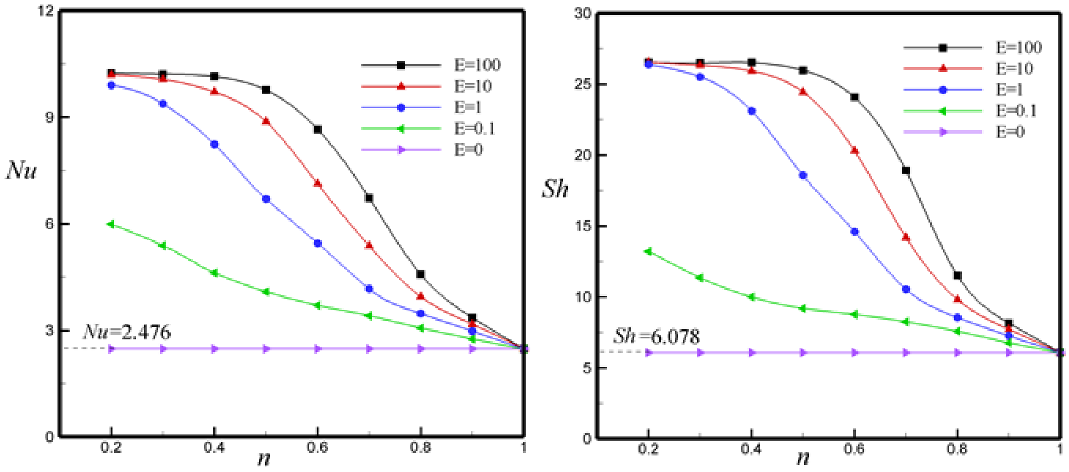


Fig. 4. Illustrates the influence of ratio of infinite-to zero-shear-rate viscosities, s, and power-law index, n, in the middle of the cavity while Nusselt and Sherwood numbers on the hot wall are also studied. The Nusselt and Sherwood numbers increase with decreasing of parameter, s. Furthermore, it is evident that the Nusselt and Sherwood numbers rise with decreasing of power-law index, n, gradually way. Moreover, it found that the convection heat and mass transfer increases with decreasing parameter, s, as the power-law index, n, decrease from Newtonian to shear thinning fluids.

Figure (4): Effect of ratio of infinite-to zero-shear-rate viscosities, s, and power-law index, n, on: Nusselt number, Nu, and Sherwood number, Sh, $Ra_T = 10^4$, $Le = 10$, $N = -0.5$, $E = 0.1$, $a = 2$, $\gamma = 0°$ and $S_T = D_f = 0$.

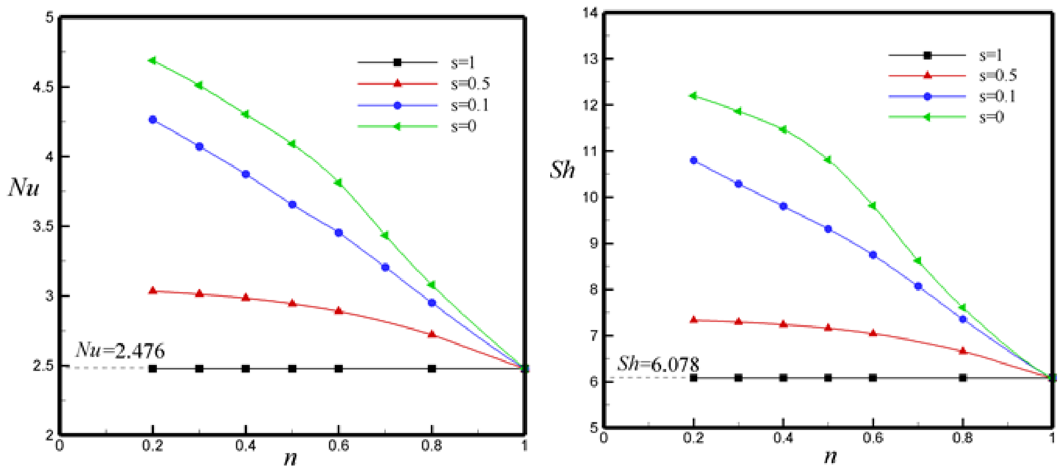


Fig. 5. Illustrates the Nusselt and Sherwood numbers for different parameter, a, and power-law index, n. It shows that the Nu and Sh number augment vastly when the parameter has decreased. Decreasing of the power-law index, n, increases the Nu and Sh in different parameters, a. Hence, the pattern clarifies that the augmentation of parameter, a, improves heat and mass transfer.

Figure (5): Effect of parameter, a, and power-law index, n, on: Nusselt number, Nu, and Sherwood number, Sh, fro $Ra_T = 10^4$, $Le = 10$, $N = -0.5$, $E = 0.1$, $s = 10^{-2}$, $\gamma = 0°$ and $S_r = D_f = 0$.

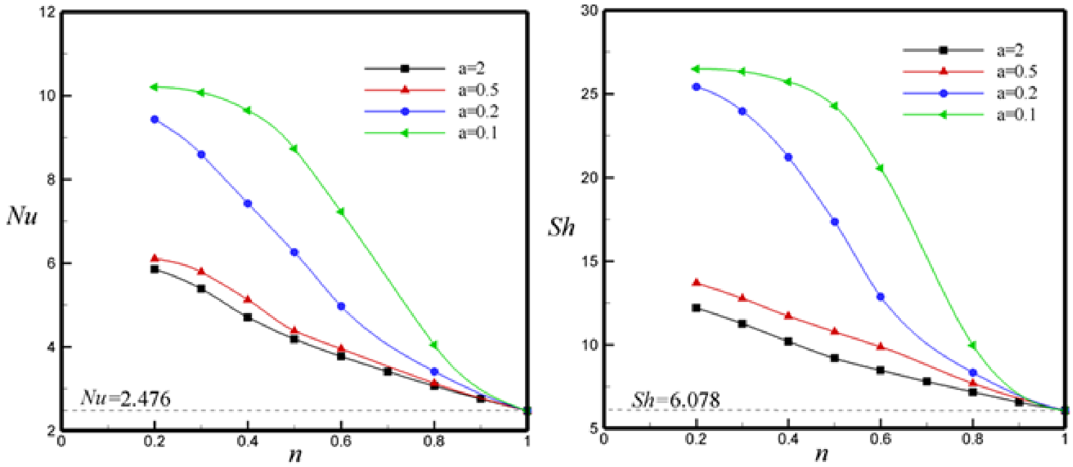
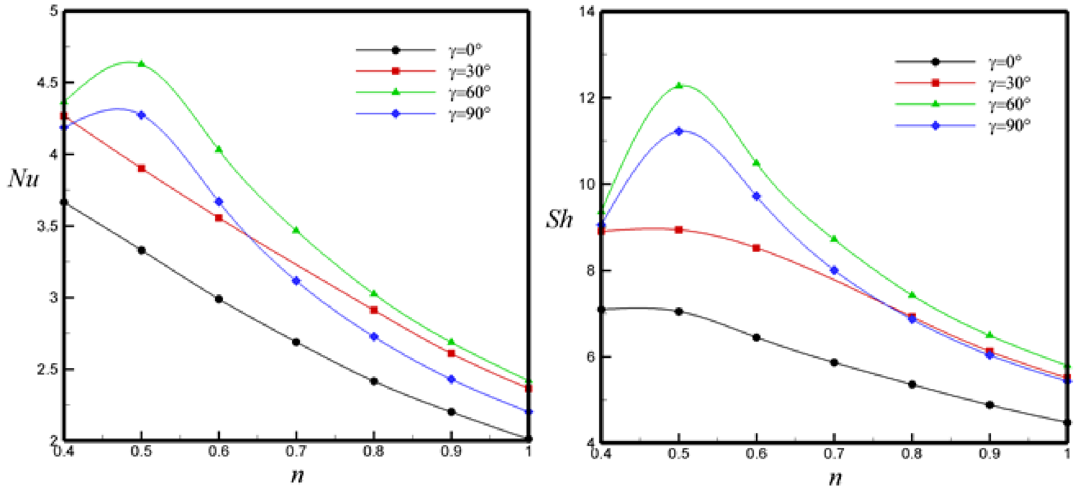


Fig 6. Illustrates the significant of distinct angle inclination,γ, on the Nusselt and Sherwood numbers. For a given values of power-law index,n, the Nusselt and Sherwood numbers augment significantly. At $0.4 \leq n \leq 0.65$ the Nusselt number increase from $\gamma = 0°$ to $\gamma = 30°$ and at $\gamma = 60°$ is more than$\gamma = 30°$, at $0.65 \leq n \leq 1$ the $\gamma = 30°$is more than $\gamma = 90°$. The Sherwood number increase with increasing of the inclined angle the $\gamma = 60°$ is more than $\gamma = 90°$

Figure (6): Effect of inclined angel, γ, and power-law index, n, on: Nusselt number, Nu, and Sherwood number, Sh, for $Ra_T = 10^4$, $Le = 10$ and $S_r = D_f = 0$.



3.CONCLUTION :

The Double diffusive natural convection the Carreau-Yasuda rheology effects and inclination angle in a square cavity saturated with non-Newtonian fluid has been studied while heat and mass transfer is analyzed. The governing parameters of the problem are the rheological parameters of the Carreau-Yasuda model, the Rayleigh number, and inclination angel. The results are obtained in terms of the Nusselt and Sherwood numbers as functions of the governing parameters. The main findings of this study are detailed below:

- Heat and mass transfer enhances with augmentation of thermal Rayleigh number in different power-law indexes, n.
- A decrease of the power-law index, n, enhanced the shear-thinning of fluid and promoted the early appearance of subcritical convection.
- The Nusselt and Sherwood numbers are sensitive to both of the power-law index, n, and the dimensionless time constant, E. Any decrease in, n, and increase in, E, increased considerably the convective heat transfer rate.
- The Nusselt and Sherwood numbers demonstrate that the heat and mass transfer increase with the decrease of power-law index, n, in various studied parameters.
- For a given set of Carreau-Yasuda parameters, it was found that the heat and mass increase with a decrease of, n.
- The effect of s parameter on heat and mass transfer rise as the power-law index decreases.
- The upper values of Nu and Sh increase as the angle increases from 60° to 90°.

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