The application of PDE in financial market

L'application du PDE au marché financier

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Abstract :

This paper explains the particular application of partial differential equations (PDE) generally used in physics (heat equation) and financial markets (Black-Scholes equation).

Keywords: Partial derivatives, Fourier series, the heat equation, Black-Scholes equation, pricing options (call-put**).**

Jel Classification Codes: **G10 ;G19**

الملخص

تناقش هذه الورقة نوع معين من المعادلات التفاضلية الجزئية والمستخدمة كثيرا في الفيزياء (معادلة الحرارة) والمطبقة في الاسواق المالية المتمثلة بمعادلة بلاك- شولز.

كلمات مفتاحية: التفاضل الجزئي ، سلاسل فوريي ، معادلة الحرارة، معادلة بلاك- شولز، ، تسعير خيارات (الشراء- البيع**)**.

تصنيف JEL:G10 ;G19.

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1.Introduction :

A major revolution took place over the last thirty years on Financial markets following a strong political deregulation. This new financial landscape is Born including imbalances and uncertainty over international economic relations since the early 1970s (debt burden of developing countries, volatility of exchange rates). The development of inflation and the volatility of interest rates have affected investor expectations. Furthermore, the internationalization of capital, technological advances in computing and communication have changed the relationships between different financial centers: New York, London, Tokyo, etc..: it is now possible to intervene at any moment in all markets **(Nicole El Karoui,2004,10).**

This paper focuses on a particular application of partial differential equations in mathematical finance. He considers the case study of the heat equation. This - is a homogeneous linear equation with partial derivatives of order 2 whose explicit solution has been demonstrated in mathematics and theoretical physics, particularly following the work of the French mathematician and physicist Jean - Baptiste Joseph Fourier **(1768-1830).**

As we will specify in more detail below, the heat propagation equation finds interesting applications in the economic analysis, particularly in the treatment of the famous equation Black - Scholes in finance. Indeed, the equation of **Black - Scholes** considers that, under certain conditions, the price **C = C (t, s)** of a call option (call) satisfies**(Boka David ,2013,108) ,** the homogeneous linear equation with partial derivatives of order 2 in The following**:**

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}+rS\frac{\partial V}{\partial S}-rS=0$$

2 . Derivatives Products:

A derivative or derivative contract or Product derivative is a financial instrument :

- Whose value changes in response to changes in the rate or the price of a product called underlying?
- Who requires no initial net investment or little significance?
- Of which payment is due at a future date.

This is a contract between two parties, a buyer and seller, which sets future cash flows based on those of an underlying asset, real or theoretical, usually financial.

Originally, derivatives were created to allow companies to hedge against various types of financial risks.

Transactions in derivatives are rapidly growing since the early 1980s and now account for the bulk of financial market activity **(Berruyer Olivier , 2015,5).**

3 – options, Hedge and pricing problem:

Options :

option is the right (but not the obligation) to buy (call option or call) or sell

(Option to sell or put) -Risques- asset at an agreed price (exercise price or Strike) and

For a specified period (exercise period) or until an agreed date (exercise date).

The underlying asset is related to a stock, a block of shares of a company, an index (Dow Jones, etc.), currency, commodities, etc.

An option is a contract between two parties on the trading of the asset at a future time **(Guibé Olivier, 2010,15).**

3-1- Definition 1 - (European Call) :

A European call option, call, entitles the holder to purchase a certain amount of underlying asset (St value at time t) a Some future date, called maturity, denoted **T**, and at a price fixed in the contract, denoted K, called Strike. The buyer, therefore having the right and not the obligation to exercise the option maturity, he will exercise his option if **ST > K** otherwise it does nothing. The real value therefore exchanged at maturity**,** Called **" pay-off " option** is therefore $(S_T - K)^+ = \max(S_T - K, 0)$

3-2- Definition 2 (Put European) :

A European put option, put, entitles its Holder to sell a certain quantity of underlying asset at a future date at a price set in the contract. With the same notation as for the call, you get the pay-off is the put

$(K - S_T)^+ = \max(K - S_T, 0)$ **(Salvarani francesco, 2011,4)**

3 –3- Hedge:

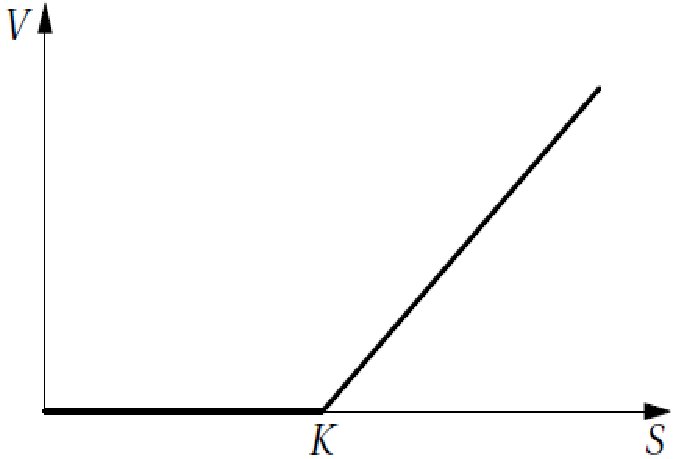
The coverage problem of an option is to find a financial strategy based on market assets whose value at each period t is equal to the pay-off option.

3- 4- pricing problem:

For call and put options appears a major problem. For example in the case of a call option the buyer is always protected because he chose the exercise of the option so he fears nothing.

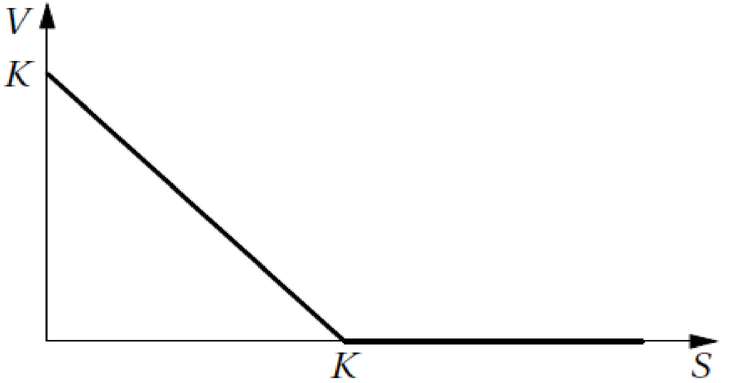
In contrast to the seller the risk is greatest because its loss is potentially infinite. So it is natural that the pricing of financial mathematics problem an option, that is to say, determining the value of the option on any date, or very important, and the coverage problem of option **(Salvarani francesco, 2011,4).**

FIGURE 1 Intrinsic value of a call (pay-off function)
The pay-off fonction



For a European call, if the date was **T**, we have **S < K** the option holder does not exercise it (otherwise it loses money), where **V = 0** if **ST < K**. If **ST > K** while the gain (excluding premium paid to acquire the option) is ST - K. so for a call $V(S_T, T) = \max(S_T - K, 0) = (S_T - K)^+.$

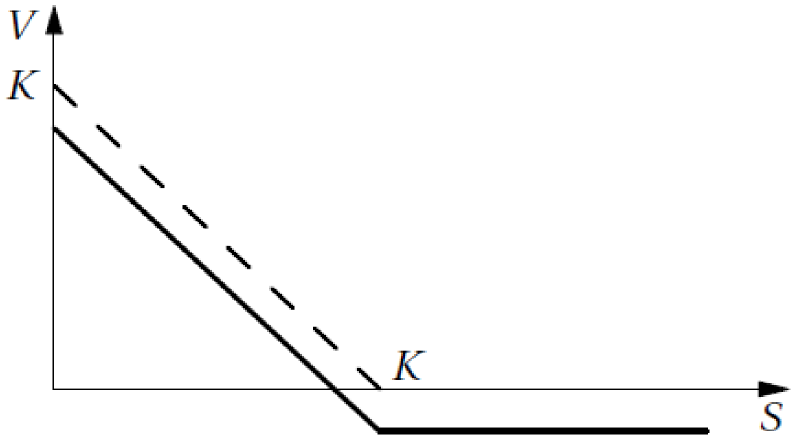
FIGURE 2 – intrinsic value of a put option (pay-off function)



For a European put, the exercise takes place only if **ST < K**, the pay-off function will be

$$V(S_T, T) = (K - S_T)^+.$$

FIGURE 3 – Diagram of profit



In Figures 1 and 2 we have not given the benefit. Indeed the owner bought
The option (premium) and possibly transaction costs. As these amounts are paid at **t0**, the update provides a multiplier of profits does not correspond to Figures 1 and 2 and induces **exp (r (t - t 0))**, where r is the average rate. Thus the diagram gives a negative profit for some values of **S**, see for example Figure 3.
For an American call was the same formula (but for everything **t ≤ T**),

$$V(S,t) = (S_t - K)^+$$

For a put American :

$$V(S,t) = (K - S_t)^+$$

4 - Analytical resolution of the heat equation:

The heat equation in one space dimension is given by the following partial differential equation:

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

Or **a² > 0** is a given constant**, u** is real unknown function of two real variables **x** and **t**. this function **u = u(x; t)** represents the temperature in a one-dimensional conductor. The value of **u(x; t)** depends on time **t** ¸ and the position **x**. The heat equation is the simplest example of parabolic equation **(Gisclon Marguerite, 1998 ,3).**

(with **a²=λ/ρc**) or **λ** is the conductivity of the rod, **ρ** its density and **c** specific heat). We will solve this problem in the case where the temperature is subject to the initial condition.

U(x, 0) = φ(x), or **φ** is a bounded function and integrable **IR**. We assume **u** in class **C²** compared x, and class **C1** compared a **t**.

- The Fourier transform was applied with respect to **x**, to both sides of the equation If the heat. Derive a differential equation satisfied by $\hat{\mathbf{u}}(\mathbf{x},\ \mathbf{t})$ the resolve.
- Were expressed **u(x, t)** as a **convolution product**.

We put :

$$\hat{\mathbf{u}}(\nu,\ \mathbf{t}) = \int u(\mathbf{x},\ \mathbf{t})e^{-2i\pi\nu x}dx$$ is then

$$\frac{\partial\hat{\mathbf{u}}}{\partial t}(\nu,\ \mathbf{t}) = \int \frac{\partial u}{\partial t}(\mathbf{x},\ \mathbf{t})e^{-2i\pi\nu x}dx = a^2\int \frac{\partial^2 u}{\partial x^2}(\mathbf{x},\ \mathbf{t})e^{-2i\pi\nu x}dx$$

$$= a^2([\frac{\partial u}{\partial x}(\mathbf{x},\ \mathbf{t})e^{-2i\pi\nu x}]_{-\infty}^{+\infty} + 2i\pi\nu\int \frac{\partial u}{\partial x}(\mathbf{x},\ \mathbf{t})e^{-2i\pi\nu x}dx)$$

$$= -4a^2\pi^2\nu^2\int u(\mathbf{x},\ \mathbf{t})e^{-2i\pi\nu x}dx = -4a^2\pi^2\nu^2\,\hat{\mathbf{u}}(\nu,\ \mathbf{t})$$

if we put اذا وضعنا

$g_v(t) = \hat{\mathbf{u}}(\nu,\ \mathbf{t})$, was therefore $\frac{g'_v(t)}{g_v(t)} = -4a^2\pi^2 v^2 = (Lng_v)'(t)$,

hence $Lng_v(t) = -4a^2\pi^2 v^2 t + c$ et $g_v(t) = Ke^{-4a^2\pi^2 v^2 t} = \hat{\mathbf{u}}(\nu,\ \mathbf{t})$ with,

for **t=0,** $\hat{\mathbf{u}}(\nu,\ \mathbf{t}) = K = \hat{\varphi}(v)$, so $\hat{\mathbf{u}}(\nu,\ \mathbf{t}) = \hat{\varphi}(v)e^{-2\pi^2(2a^2t)v^2}$

we Recognizer $e^{-2\pi^2\sigma^2 v^2}$ with $\sigma = a\sqrt{2t}$, and this is

$f\left(\frac{1}{2a\sqrt{\pi t}} e^{-\frac{x^2}{4a^2t}}\right)(v)$

Finally, it uses the property $f(F)f(G) = f(F*G)$

Then the Fourier transform of the inverse to conclude that:

$$u(\mathbf{x},\ \mathbf{t}) = \frac{1}{2a\sqrt{\pi t}} \int \varphi(s) e^{-\frac{(x-s)^2}{4a^2t}} ds$$

5- Analytical resolution of the Black – Scholes equation:

This section is an application in finance results established in the previous section, in particular Considering the case of the equation Black - Scholes ; equation named in honor to the American mathematician **Fischer Black (1938 - 1995)** and **Myron S. Scholes** American economist, recipient, **Robert C. Merton**, Nobel Prize in economics in 1997. It should not be confused model Black - Scholes equation and Black - Scholes. The latter formalizes an approach to the interaction between the implied price of the option and the price changes of the underlying asset – underlying (**(Boka David ,2013,123).**

5-1- The evaluation of formula:

Differentiating our formula of the value of an option in terms of share price, we will assume "ideal condition" in the market for the action and for option **(BLACK Fischer and SCHOLES Myrlon, 1973, 4).**

Fisher Black and Myron Scholes in 1973 suggested that now famous model. As it is a model they went assumptions:

- The underlying price follows a geometric Brownian motion;
- Volatility is known beforehand and is constant;
- It is possible to buy and sell the underlying at any time and without charge ;
- Short sales are permitted (a certain amount you borrow to buy
Underlying for sale);
- No dividend;
- The interest rate is known in advance and is constant;
- The exercise of the option cannot be done at the due date and not before (European option).

5 - 2- Notations :

One note V the value of an option (it will be limited to European options in this course). We can make the distinction between a **"call" C** and **"put" P** if necessary. These are functions of time t, and the current value of the underlying share is therefore noted S . **V = V (S, t)** Note also **(Auroux Didier, 2011, 17)**:

- **σ** volatility of stock price ;
- **E** the exercise price per option;
- **T** the time remaining to the option before maturity ;
- **r** the interest rate.

5 - 3 - Black-Scholes Equation:

This is a partial differential equation that calculates the value of a European option based on the time and the value of the underlying stock.

4-3- Case of European Call Option:

For European Call Option include the value **C (S, t),** which corresponds to **u** earlier, where **S** represents the underlying price, and **t ϵ [0, T], T** where **T** is maturity. We have the following equation:

$$\begin{cases} \frac{\partial C}{\partial t}(s,t) + \frac{\sigma^2}{2} S^2 \frac{\partial^2 C}{\partial S^2}(s,t) + rS\frac{\partial C}{\partial S}(s,t) - rC(S,t) = 0; \quad \forall t \in [0,T],, S \in R_+ \\ C(s,t) = (S-E)^+ = \max(S-E,0) \quad \forall s \in R_+ \end{cases} ....(1)$$

To solve this equation, we will make various changes of variable to take us a heat equation -type

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \\ u(\mathbf{x},\ 0) = \ u_0(x) \end{cases} .....(2)$$

To be able to bring this type of equation, first start by removing Coefficients **S** and **S²** the Black-Scholes equation. To do this, we put: $S = Ee^x$, $t = T - \frac{2\tau}{\sigma^2}$, $C(S,t) = Ev(x,\tau)$.

A change of this kind seems natural to be reduced to a condition at **t = 0** and

No more and **t = T**.

So we have:

$$v(x,\tau) = \frac{1}{E}C(Ee^x \quad , \quad T - \frac{2\tau}{\sigma^2}) = \frac{1}{E}C(S,t)........(3).$$

we derive (3) with respect to **x** :

$$v_x = \frac{1}{E}\frac{\partial C}{\partial S}\frac{\partial S}{\partial x} + \underbrace{\frac{1}{E}\frac{\partial C}{\partial t}\frac{\partial t}{\partial x}}_{0} = \frac{S}{E}\frac{\partial C}{\partial S}$$

Again we drive to **x** :

$$v_{xx} = \frac{\partial}{\partial x}(v_x) = \frac{\partial}{\partial x}(\frac{1}{E}S\frac{\partial C}{\partial S}) = \frac{\partial}{\partial S}(\frac{1}{E}S\frac{\partial C}{\partial S})\frac{\partial S}{\partial x}$$

$$= \frac{1}{E}(\frac{\partial C}{\partial S} + S\frac{\partial^2 C}{\partial S^2})S = \frac{S}{E}\frac{\partial C}{\partial S} + \frac{S^2}{E}\frac{\partial^2 C}{\partial S^2}$$

Now derive (3) relative to **τ**:

$$v_\tau = \frac{1}{E}\frac{\partial C}{\partial t}\frac{\partial t}{\partial \tau} = \frac{1}{E}\frac{\partial C}{\partial t}\frac{-1}{\frac{\sigma^2}{2}}$$

For clarity we introduce the following values :

$$C_t = -\frac{E}{2}\sigma^2 v_\tau ....(4).$$

$$SC_s = Ev_x ....(5).$$

$$S^2 C_{ss} = Ev_{xx} - SC_s = Ev_{xx} - Ev_x ....(6).$$

Recall the Black-Scholes equation:

$$C_t + \frac{1}{2}\sigma^2 S^2 C_{ss} + rSC_s - rC = 0........(7).$$

Then injected (4), (5) and (6) in (7), which yields:

$$-\frac{E}{2}\sigma^2 v_\tau + \frac{E}{2}\sigma^2 (v_{xx} - v_x) + rEv_x - rEv = 0.$$

Either by division (E/2) σ^2

$$-v_\tau + v_{xx} - v_x + kv_x - kv = 0. \text{ With } k = \frac{2r}{\sigma^2}$$

Or with the notations of departure:

$$\frac{\partial v}{\partial \tau} = \frac{\partial^2 v}{\partial x^2} + (k-1)\frac{\partial v}{\partial x} - kv........(8). \text{ With } k = \frac{2r}{\sigma^2}$$

So we as an initial condition (in τ = 0, since the condition is t = T) : $v(x,0) = \frac{1}{E}C(Ee^x \quad , \quad T) = \frac{1}{E}\max(Ee^x - E, 0) = \max(e^x - 1, 0).$

To arrive at an equation such as the heat equation, we then proceed to a second change of variables. We put

$$: v(x,\tau) = e^{\alpha x + \beta \tau} u(x,\tau)$$

Then it is reinjected into the equation (8):

$$e^{\alpha x+\beta\tau}(\beta u + \frac{\partial u}{\partial \tau}) = e^{\alpha x+\beta\tau}(\alpha^2 u + 2\alpha\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} + (k-1)(\alpha u + \frac{\partial u}{\partial x}) - ku).$$

or again:

$$\beta u+\frac{\partial u}{\partial \tau}=\alpha^2 u+2\alpha\frac{\partial u}{\partial x}+\frac{\partial^2 u}{\partial x^2}+(k-1)(\alpha u+\frac{\partial u}{\partial x})-ku.$$

By grouping the terms of same derived:

$$\frac{\partial u}{\partial \tau}=\frac{\partial^2 u}{\partial x^2}+(\alpha^2+(k-1)\alpha-k-\beta)u+(2\alpha+k-1)\frac{\partial u}{\partial x}$$

To reduce to the case of the heat equation, we need to eliminate the terms u and **du/dx** we must solve the following system:

$$\begin{cases}\beta=\alpha^2+(k-1)\alpha-k,\\ 2\alpha+k-1=0.\end{cases}\Leftrightarrow\begin{cases}\alpha=-\frac{1}{2}(k-1),\\ \beta=-\frac{1}{4}(k+1)^2.\end{cases}$$

So we have:

$$v=e^{-\frac{1}{2}(k-1)x-\frac{1}{4}(k+1)^2\tau}u(x,\tau).$$

And U then checks

$$\begin{cases}\frac{\partial u}{\partial \tau}=\frac{\partial^2 u}{\partial x^2} \quad , \quad \forall x\in\mathbb{R} \quad , \quad \forall \tau>0\\ u(x,0)=u_0(x)=e^{\frac{1}{2}(k-1)x}\max(e^x-1,0)=\max(e^{\frac{1}{2}(k+1)x}-e^{\frac{1}{2}(k-1)x},0).\end{cases}$$

u0(x) checks good. The solution is:

$$u(\mathbf{x},\ \tau)=\frac{1}{2\sqrt{\pi\tau}}\int_{-\infty}^{+\infty}u_0(s)e^{-\frac{(x-s)^2}{4\tau}}ds.$$

5-3-1- Evaluation The option price:

In this section, we'll get down to the evaluation of the integral in equation (9). It begins by putting **(Salvarani francesco, 2011,12)**

$$x'=\frac{s-x}{\sqrt{2\tau}}$$

$$:u(\mathbf{x},\ \tau)=\frac{1}{2\sqrt{\pi\tau}}\int_{-\infty}^{+\infty}u_0(s)e^{-\frac{(x-s)^2}{4\tau}}ds.$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}u_0(x'\sqrt{2\tau}+x)e^{-\frac{x'^2}{2}}dx',$$

$$=\underbrace{\frac{1}{\sqrt{2\pi}}\int_{-\frac{x}{\sqrt{2\tau}}}^{+\infty} e^{\frac{1}{2}(k+1)(x+x'\sqrt{2\tau})} e^{-\frac{x'^2}{2}} dx'}_{I_1}-\underbrace{\frac{1}{\sqrt{2\pi}}\int_{-\frac{x}{\sqrt{2\tau}}}^{+\infty} e^{\frac{1}{2}(k-1)(x+x'\sqrt{2\tau})} e^{-\frac{x'^2}{2}} dx'}_{I_2}$$

Calcul of **I1** Here we will try to bring back to normal distribution law centered reduced :

$$I_1=\frac{e^{\frac{1}{2}(k+1)x}}{\sqrt{2\tau}}\int_{-\frac{x}{\sqrt{2\tau}}}^{+\infty} e^{\frac{1}{4}(k+1)^2\tau} e^{-\frac{1}{2}(x'-\frac{1}{2}(k+1)\sqrt{2\tau})^2} dx'$$

$$=\frac{e^{\frac{1}{2}(k+1)x+\frac{1}{4}(k+1)^2\tau}}{\sqrt{2\tau}}\int_{-\frac{x}{\sqrt{2\tau}}-\frac{1}{2}(k+1)\sqrt{2\tau}}^{+\infty} e^{-\frac{1}{2}\rho^2} d\rho$$

With $\rho=x'-\frac{1}{2}(k+1)\sqrt{2\tau}$

On note then $N(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}S^2} ds$ which is the distribution function of the standard normal distribution.

It also poses $d_1=\frac{x}{\sqrt{2\tau}}+\frac{1}{2}(k+1)\sqrt{2\tau}$.

Is then obtained:

$$I_1=e^{\frac{1}{2}(k+1)x+\frac{1}{4}(k+1)^2\tau}(1-N(-d_1)).$$

Or, $\forall d\in R$, on a $N(d)+N(-d)=1.$ so eventually:

$$\begin{cases} I_1=e^{\frac{1}{2}(k+1)x+\frac{1}{4}(k+1)^2\tau}N(d_1). \\ d_1=\frac{x}{\sqrt{2\tau}}+\frac{1}{2}(k+1)\sqrt{2\tau}. \end{cases}$$

Calcul of **I2** By a similar calculation is obtained:

$$\begin{cases} I_2 = e^{\frac{1}{2}(k-1)x+\frac{1}{4}(k-1)^2\tau} N(d_2). \\ d_2 = \dfrac{x}{\sqrt{2\tau}} + \dfrac{1}{2}(k-1)\sqrt{2\tau}. \end{cases}$$

Now that we have the expression of u goes back to the expression of C, value of the Call. all

First, we have:

$$v(x,t) = e^{\alpha x+\beta\tau}u(x,\tau) \text{, et } C(S,t) = Ev(x,\tau)$$

It also recalls that:

$$k = \frac{r}{\frac{1}{2}\sigma^2} \qquad \tau = \frac{1}{2}\sigma^2(T-t) \quad x = Ln\frac{S}{E}, \text{ ou}$$

$$C(S,t) = Ee^{-\frac{1}{2}(k-1)x-\frac{1}{4}(k+1)^2\tau} \times (e^{\frac{1}{2}(k+1)x+\frac{1}{4}(k+1)^2\tau}N(d_1) - e^{\frac{1}{2}(k-1)x+\frac{1}{4}(k-1)^2\tau}N(d_2))$$

$$= Ee^{-\frac{1}{2}(k-1)Ln\frac{S}{E}-\frac{1}{4}(k+1)^2\frac{1}{2}\sigma^2(T-t)} \times (e^{\frac{1}{2}(k+1)Ln\frac{S}{E}+\frac{1}{4}(k+1)^2\frac{1}{2}\sigma^2(T-t)}N(d_1) - e^{\frac{1}{2}(k-1)Ln\frac{S}{E}+\frac{1}{4}(k-1)^2\frac{1}{2}\sigma^2(T-t)}N(d_2))$$

This simplifies to :

$$\begin{cases} C(S,t) = SN(d_1) - Ee^{-r(T-t)}N(d_2) \\ d_1 = \dfrac{Ln\frac{S}{E} + (r+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}} \\ d_2 = \dfrac{Ln\frac{S}{E} + (r-\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}} \end{cases} \qquad \text{...........(10)}$$

5-4- Case of a European put option:

We could do the same for a European put option, but we'll use the remark made in the first part: the Put -Call parity. P note on the price of European Put.

They had shown that between a put and a European Call, the same maturity and same E Strike,

Is a relationship between their price :

$$C(S,t)-P(S,t)=S-Ee^{-r(T-t)}.$$

Therefore:

$$P(S,t)=S(N(d_1)-1)-Ee^{-r(T-t)}(1-N(d_2))$$

This equation can be simplified using the fact that N(d) + N(−d) = 1. We get:

$$\begin{cases} P(S,t)=Ee^{-r(T-t)}N(-d_2)-SN(-d_1) \\ d_1=\dfrac{Ln\dfrac{S}{E}+(r+\dfrac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}} \\ d_2=\dfrac{Ln\dfrac{S}{E}+(r-\dfrac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}} \end{cases} \quad ....(11)$$

6-Conclusion:

In today's world, finance plays a most important role and is sometimes the cause of global crises. It then appears important that finance is based on solid models to assess the risks and prices. This requires the model and the Black-Scholes formula has become standard since 1973 in the calculation of option. Despite its flaws, this model knows this success because it has many advantages: ease of application and formula, its significant use by market participants but also and especially because it is used to calculate an important parameter in finance: **volatility**. Volatility measures the average change over time of a financial asset and therefore gives crucial information about risk.

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