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Maintenance Policy Optimization:

Exploration of Numerical Solutions for Complex Systems

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Abstract:

This paper specifically concerns the design of an optimal maintenance policy in order to minimize costs; this also allows reducing the frequency of system failures and increasing its operating time during its mission. An algorithm has been developed and the generated results are encouraging, this conception aims to identify potential problems or defects as early as possible ensuring that corrective actions can be taken to prevent or mitigate any negative consequences. An optimal maintenance procedure should be based on a thorough understanding of the system or process being inspected, including its critical components and failure modes. It should also take into account factors such as the frequency and duration of inspections. Ultimately, an optimal solution provided should strike a balance between detecting potential problems early enough to take corrective action while avoiding unnecessary downtime or inspection costs. We propose mathematical models based on different aspects. The first one allows for an optimal solution without renewal. Then, we generalize the model and finally extend it by introducing the cost of renewal. To demonstrate the use of the models in practical applications, a numerical example is provided. Solutions to optimal system parameters are obtained and the response of the model to these parameters is examined

Keywords: Optimal maintenance Policy, Failure, Renewal, Reliability Theory, Computational Algorithms.

JEL codes: C44, C54, C61

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1. Introduction

Maintenance management can be a complex and critical process for many companies. Based on mathematical models, this approach uses monitoring and analysis data to predict potential equipment failures with the aim of improving equipment availability and reliability while reducing maintenance costs.

Moreover, using mathematical models in maintenance management can also help companies reduce their maintenance costs. By predicting equipment failures before they occur, companies can plan maintenance activities in advance, allowing them to perform repairs during scheduled maintenance periods instead of during unplanned downtime. This approach helps to reduce the probability of costly emergency repairs and can help companies to optimize their maintenance budgets.

The maintenance policy problem refers to the challenge of developing a procedure for carrying out inspections in order to identify potential failures or defects in complex systems. This involves determining the most efficient and effective approach for detecting these issues while minimizing unnecessary downtime and costs. Such actions are necessary for certain complex systems in order to detect failures that would otherwise not be apparent.

Seeking an optimal solution based on inspections aims to minimize costs and maximize the safety and reliability of the system or process. This can be achieved through careful planning, execution, and evaluation of the inspection procedure, and by continuously improving the process based on data analysis.

The use of mathematical models will be the basis in developing the policy to determine the optimal timing for conducting inspections and repairs. This policy will take into account the costs of repair, inspection costs, replacement costs, and costs of lost production in the event of random equipment failure.

2. PRELIMINARY NOTATION

Let “ $w \succ 0$ a random variable with probability distribution

$$F(x)=P(w\leq x)\ -\infty \prec x \prec +\propto .$$

Also known as system reliability, we assume that all random variables are defined on the same probability space (Ω, F, P).

The survival probability of the system corresponding to a mission of duration x is by definition $R(x)=1-F(x)$ and if “ w ” has a density $f(x)=F'(x)$, then the conditional failure rate at time t is represented by : $r(t)=f(t)/R(t)$.

Or

$$r(t)=\lim_{x\to o}\frac{1}{x}\frac{F(t+x)-F(t)}{1-F(t)} \quad if\ R(t)>0 \qquad (2.1)$$

Given that there exists no failure before t, the quantity $r(t)dt + O(dt)$ represents the conditional probability of failure in the interval $(t + tdt)$.

If the conditional probability of failure during the next interval of duration x of a system at age t, then :

$$F(x/t) = \frac{F(t+x) - F(t)}{1 - F(t)} = 1 - R(x/t) \qquad (2.2)$$

Is such that $R(x/t) = R(x)$ for all $x,t \geq 0$ then $R(x) = e^{-\lambda x}$, the failure rate $r(t) = \lambda \succ 0$, and the mean time before failure $t_o = 1/\lambda$. It's the well known "memory less property". Certain systems improve their performance in time, in the sense that:

$$R(x/t) \text{ is increasing in } -\infty \prec t \leq +\infty \text{ for each } x \geq 0 \qquad (2.3)$$

If the density exists, then (2.3) holds if and only if $r(t)$ is decreasing in $t \geq 0$. We are concerned here by systems with an increasing failure rate. The state of such systems often can be known only by using some inspection policy.

3. Related Work

The problem of developing an optimal maintenance (inspections) for complex systems has been widely studied. One way to represent such systems is by using the parallel-series or series-parallel structure, which can be achieved using the coherent structures (Ahmadi, 2014). These structures provide a way to visualize the system and identify potential failure modes.

To use these structures, current tools require constructing a tree of errors, which is a graphical representation of all possible failure modes and their causes. Once the tree is constructed, the next step is to search for minimal ways and cuts. This means finding the shortest paths through the tree that lead to the potential failure modes, and identifying the components or sub-systems that are critical to the overall system's reliability.

The problem of developing an optimal inspection is complex and requires advanced tools and techniques to address it effectively. However, with the right approach and tools, it is possible to visualize and analyze complex systems and identify potential failure modes to develop an effective inspection strategy.

In the case whereby the laws of reliability and repair can be numerically expressed, we propose an algorithm for the optimal policy of inspection. Some problems are discussed in the book of (Wang, 2015a, Wu, 2015b). (Navon, 1985a, Nakach, 1985b) proposed a new cost model which is more adapted to the situation described above. We derive an algorithm for the computation of the optimal inspection policy relatively to the generalized Munford’s cost model over an infinite time span (Moon, 2010a, Misra, 2010b). Such algorithm is not stable numerically, but it is possible to improve the stability of the algorithm by defining rigorous criteria for its implementation. Some models and their results have been discussed in (Wang, 2021), (Jin, 2021). The authors aim to determine the optimal inspection interval and the number of inspections required for different failure modes. The objective is to minimize the expected total cost of inspection and repair while maintaining the system's reliability and safety. (Qiu, 2019) aim to develop a model that considers the effect of the two-stage deterioration process on the system's reliability and propose an inspection policy that balances the trade-off between inspection costs and system reliability. (Zhou, 2020), propose a novel method that takes into account both epistemic and aleatory uncertainties to determine the optimal inspection interval and inspection effort, while balancing the costs of inspection and potential downtime. The effectiveness of the proposed method is demonstrated through a case study of a pipeline system, showing significant improvements in both safety and cost-effectiveness. (El-Shafei, 2021), provides a review of existing literature in this area and highlights recent developments in the field. The authors explore various approaches for determining optimal inspection schedules and discuss their strengths and limitations. They also identify potential areas for future research and provide recommendations for practical application.

4. Building an optimal inspection

An inspection policy is a random or deterministic sequence: $S = \{x_n \quad , \ n = 0,1,......\}$, taking values in $\Re^+$, and such that

$$0 \le x_0 < x_1 < x_2 < .... < x_n < x_{n+1} < ......$$

We denote by $\Phi = \{S\}$ the space of such elements S. And we assume that the system cannot have any failure during the inspection,

Let C_1 denote the cost of conducting such an inspection.

Let C_2 be the cost per unit of sejourn time of the system in the state "undetected failure".

One can now express the system's life length in terms of the inspection policy.

$$w = \sum_{i=1}^{N(t)-1} \delta_i + X_{N(t)} - t \qquad (4.1)$$

Where $N(t) = max\{j : x_j \prec t; t \ge 0\}$ is the number of inspections up to time t of failure, and $\delta_i = x_j - x_{j-1} \quad ; j = 1,2,.......$, (Duration between two inspections)

This statement suggests that the paper focuses on inspection problems that are related to renewing a system after detecting a failure.

Let “w'” be the duration of such renewal with probability distribution.

$G(t) = P(w' \le t)$, and C_3, the cost of such operation.

The theoretical developments presented in this paper build upon the work previously presented by several researchers; we can extend their results to consider some hypothesis and generating an optimal inspection procedure

Let we assume that:

H1 : “w' “ ≡ 1 almost surely ;

H2 : The inspection policy stops at time of failure.

We can prove that the use of a deterministic policy is better to a random policy, relatively to the objective functions considered here.

We extend the argument to the domain of decision making under uncertainty, and demonstrate that a deterministic policy can lead to better outcomes than a random policy. Our results have important practical implications for a wide range of applications, from decision making to the design of autonomous complex systems.

Denote $D(t;S)$ the total cost involving by using the inspection policy $S=\{x_n\}$ given that the failure occurs at time t.

The inspection policy $S^*=\{x_n^*, n=0,1,\dots\}$ is optimal if :

$$\min_{S\in\Phi} E\{D(t;S)\} = E\{D(t;S^*)\} \qquad (4.2)$$

5. Inspection without renewal

Under the assumptions H1, the total costs may be written under the form:

$$D_1(t;S) = C_1 N(t) + C_2 (x_{N(t)} - t) \qquad (5.1)$$

Thus the Optimal inspection procedure S^* is such that (4.2) holds for the functional (5.1). If now $f(x)$ exists, then we can use the Kuhn-Tucker recurrent equation.

$$x_{n+1} = x_n + \frac{F(x_n) - F(x_{n-1})}{f(x_n)} - \frac{C_1}{C_2} \qquad (5.2)$$

It is clear that if $f(x)$ is Polya-Function II, then $\{\delta_n < 0\}$ is a non decreasing sequence. Moreover, for each policy $S=\{\delta_n\}$, for some $n > 0$ $\delta_n > \delta_{n-1}$ if $x_1 > x_1^*$, and $\delta_n < 0$ if $x_1 < x_1^*$. This theorem gives the well known algorithm for computation of the optimal inspection.

1. Choose the initial value x_1 ;
2. Compute x_2 , x_3,……using (5.2);
3. If $\delta_n > \delta_{n-1}$ for some n, then reduce x_1; return to step 2;

4. If $\delta_n < 0$ for some n, then augment x_1; return to step 2;

5. Repeat the process until the determination of the sequence $S^* = \{x_0 < x_1 < ....\}$ with the chosen precision. $|S_i - S_{i-1}| < \varepsilon$

6. Generalization of the model

The exact time of failure cannot be determined in the general case, so that the penality C_2 is actually extended to cover the entire interval between the current and the next inspection x_{n-1}, x_n.

We propose the following model of cost:

$$D_2\,(t;S) = C_1\,N(t) + C_2\,(x_{N(t)} - x_{N(t)-1}) \qquad (6.1)$$

Where "w" is exponentially distributed, the optimal policy is of the form:

$$x_n^* = n x_1^*, \; n = 1,2,\ldots..$$

Substituting this expression in (6.1) we obtain that for our model, x_1^* is given by the equation $exp(\lambda x_1^*) = \lambda x_1^* - 1 - \tau$, where $\tau = \lambda C_1 / C_2$.

7. Optimal policy with renewal

We will now make the assumption that:

H1. $P(w' > 0)$, so that $G(t)$ is a non degenerate distribution;

H2. The renewal cost is C_3. Once the renewal is completed, the system is considered to be in a new condition, and the inspection process starts over from the beginning.

We can do the generalization of our model under these assumptions with the objective function.

$$D_3\,(t;S) = C_1\,N(t) + C_2\,(x_{N(t)} - t) + C_3 \qquad (7.1)$$

This represents the total cost during a cycle. The duration of such elementary cycle for a policy $S = \{x_n\}$ is:

$$T_1(t;S) = w + x_{N(t)-1} - t + w' \qquad (7.2)$$

The algorithm gives the optimal policy S^* minimizing the expected total cost per unit of time in an infinite span.

$$R_1(S^*) = \min_{S \in \Phi} R_1(S) \qquad (7.3)$$

Where

$$R_1(S)=\frac{E\{D_3(t;S)\}}{E\{T_1(t;S)\}}$$

In the case of the possible renewal, we have to consider the objective function.

$$R_2(S)=\frac{E\{D_4(t;S)\}}{E\{T_2(t;S)\}} \qquad (7.4)$$

Where now

$$D_4(t;S)=C_1N(t)+C_2(x_{N(t)}-x_{N(t)-1})+C_3 \qquad (7.5)$$

The cycle duration is:

$$T_2(t;S)=w+x_{N(t)}-x_{N(t)-1}+w' \qquad (7.6)$$

The existence of the optimal policy can be proved in the usual way. In particular, it's sufficient that $F(x)$ is a continuous function and $min\{w,w'\}\prec\infty$

Now, assume that $f(x)$ is Polya Function 2, and denote.

$$H(y;S)=E\{D_4(t;S)\}-yE\{T_2(t;S)\} \qquad (7.7)$$

Then there exists y^* such that $H(y^*;S(y^*))=0$, and $S(y^*)$ minimize the objective function $R_2(S)$.

With (7.5) and (7.6) taken into consideration, we are able to formulate the algorithm for creating the optimal inspection. Furthermore, (7.7) can be expressed as :

$$H(y;S)=\sum_{n=1}^{\infty}\int_{x_{n-1}}^{x_n}[C_1n+(C_2-y)(x_n-x_{n-1})]dF(t)+C_3-y$$

$$\left\{\int_{\Re^+}tdF(t)+\int_{\Re^+}tdG(t)\right\} \qquad (7.8)$$

Or equivalently,

$$H(y;S)=C_1\sum_{n=0}^{\infty}R(x_n)+(C_2 - y)$$

$$\sum_{n=1}^{\infty}(x_n-x_{n-1}).(R(x_{n-1})-R(x_n))+C_3-y(t_0+t'_0)$$

Where

$$t_0=\int_0^{\infty}(1-F(t))dt \quad , \quad t'_0=\int_0^{\infty}(1-G(t))dt$$

The recurrent equation can be obtained from the necessary condition of extremum.

$$X_{n+1} - 2X_n + X_n = \frac{R(X_{n+1}) - 2R(X_n) + R(X_{n-1})}{f(x_n)} - \frac{c_1}{c_2 - y} \qquad (7,9)$$

7.1. Algorithm

1. Choose ‘ y ’ as "near" the optimal value y^* as possible.

2. Choose the initial value x_1.

3. Compute $x_2, x_3, ...$ using the recurrent formula (7.9).

If $H(y;S(y))>0$ then augment y, return to step 2

if $H(y;S(y))<0$ then reduce y, return to step 2

if $H(y;S(y))=0$ then $y^* = y$, and $S(y^*)$ is the optimal inspection policy. Else, choose another value of y and repeat the procedure from the step 2.

8. Results and discussion

We have considered the case of a complex system using the principle of reverse osmosis which is a technology commonly used in dialysis machines to produce pure water for the treatment of patients requiring 3 sessions per week. The complexity of the system requires the availability of the machines 7 days a week. We have collected statistical data on 2 months of operation, which allowed us to have failure and repair parameters, in addition to the costs incurred by the implementation of inspections and the replacement of components subject to wear.

The statistical analysis of data involves usual estimation techniques, and the tests were satisfactory for the application of the exponential distribution. We mainly used the maximum likelihood method, which yielded a Poisson distribution parameter with an empirical average $\lambda = 1.85$, a chi-square parameter equal to 6.62265, and a degree of error = 1%.

The cost of an optimal solution included here depends on several factors, including the type of maintenance and inspection being performed, the size and complexity of the system being inspected, the frequency of inspections, and the level of accuracy required.

In the second phase, we have implemented the algorithms in the C++ language with a user-friendly interface that allows us to input the data and choose the distribution to be used, namely the two distributions (Exponential, Weibull). A few numerical comparisons are available in this view, here we present some results obtained with algorithm mentioned before, and we used some statistical laws to compare the fitness of results.

First, the algorithm gives the sequences for optimal inspection, this policy propose a one week cycle for doing inspection, table 1 show these results with seven iterations, the algorithm stop at value 0.930716.

Table 1. Optimal Inspection policy without renewal

(Exponential law: $\lambda = 1.85$, error = 0.01, C_1, C_2)

Iteration	X_0	X_1	X_2
1	0.000	0.131	/
2	0.000	1.125	2.145572
3	0.000	1.0935	1.5699
4	0.000	9.8415	1.141
5	0.000	0.8875	0.9712
6	0.000	0.9349	0.971
7	0.000	0.92264	0.930716

Source: Obtained by program execution on the machine computer

Using the Weibull distribution and after two iterations of the algorithm, the second value was reduced to 0.500 and the process was stopped. The results are shown in Table 2.

Table2. Optimal Inspection policy without Renewal (Weibull law)

Iteration	X_0	X_1	X_2
1	0.000	0.500	0.750
2	0.000	0.500	/

Source: Obtained by program execution on the machine computer

In the other case where the procedure of the inspection is applied with renewal, the sequence is obtained for different criterion y^* and mentioned below:

This procedure (see table 3) required a cycle of six weeks (=5.991123) and 11 iterations taken into account repairs cost C_3. The implementation of the optimal policy with renewal must be effective over time, adjusting the inspection intervals as necessary based on new data and insights. This is the best policy obtained for exponential law.

Table 3. Optimal inspection policy with renewal for y =1, 2,…., 10,…..100.

$y=1$ **(31 iterations)**	$y=2$ **(31 iterations)**	$y=10$ **(10 iterations)**
$X_0 = 0.0000$	$X_0 = 0.00000$	$X_0 = 0.00000$
$X_1 = 0.1133$	$X_1 = 0.59985$	$X_1 = 1.01308$
$X_2 = 0.2666$	$X_2 = 0.61318$	$X_2 = 0.102641$
………..	………….	…………...
..……….	…………..	……….…..
$X_{31} = 0.41323$	$X_{31} = 0.41323$	$X_{10} = 0.13305$

$y=50$ **(20 iterations)**	$y=70$ **(34 iterations)**	$y=100$ **(11 iterations)**
$X_0 = 0.00000$	$X_0 = 0.00000$	$X_0 = 0.00000$
$X_1 = 1.14638$	$X_1 = 2.26610$	$X_1 = 0.96724$
$X_2 = 1.15971$	$X_2 = 2.27943$	$X_2 = 0.97532$
…..…………	……………..	…….…….
…………….	…………….	…….….…
$X_{20} = 1.39965$	$X_{34} = 3.09256$	$X_{11} = 5.5991123$

Source: Obtained by program execution on the machine computer

9. Conclusion

The study can contribute to the development of more efficient maintenance and inspection algorithms and decision-making systems. By analyzing large data sets and using machine learning techniques, researchers can identify relationships between maintenance actions and equipment performance. This can help to develop more accurate models of equipment degradation and failure, leading to more effective and efficient maintenance policies.

We believe that the main contribution of this work is a fresh way of thinking about the problem that, in our opinion, will bring a much deeper understanding of its combinatorial nature. This paper contributes to a better understanding of the problem in several ways following the steps of algorithms.

For the optimal policy, we have proposed classical models, we suggest also an extension of the model by introducing renewal, and the algorithms are encouraging.

The numerical instability of algorithms needs to be reduced, and to work to propose an extension to other statistical laws.

We can perform simulations to generate effective solutions to mitigate random maintenance actions repetitions. Table 3 shows the potential of using our methods to search for optimal solutions that guarantee a cost-effective policy within a regular time frame by identifying critical areas that require special attention, rather than inspecting everything randomly or systematically. Additionally, an optimal policy can help to maximize system reliability and availability by quickly detecting potential failures before they lead to costly downtime.

The practical implications are significant; they help organizations to optimize their maintenance processes, reducing downtime and repair costs.

The study can also contribute to a culture of continuous improvement within the organization. By optimizing their maintenance processes, organizations can gain a competitive advantage. Managers can use the study's results to reduce costs, improve efficiency, and deliver better quality products and services to customers.

Finally building a proposed software package is considered like a view of future. It is capable to recording all the iterations steps.

Bibliography List :

1. Raza. Ahmadi. (2014), Optimal maintenance scheduling for a complex manufacturing system subject to deterioration, Annals of Operations Research 217(1):1-29.

2. Wang, K., Si, S., & Wu, W. (2015), Optimal inspection policy for a repairable system with a preventive maintenance activity , IEEE Transactions on Reliability, 64(2), 754-763..

3. Navon, R., & Nakache, Y. (1985), On the optimal inspection policy , IEEE Transactions on Reliability, 34(1), 77-80.

4. Moon, K., and K. B. Misra. (2010), Generalized Munford's cost model for optimal preventive maintenance , International Journal of Production Economics, 123(2) , 229-236.

5. Wang, J., Tang, L., Wu, Y., & Li, Y. (2021), Optimal inspection policy for repairable systems with multiple failure modes and continuous monitoring, Reliability Engineering & System Safety, 213, 107736.

6. Jin, X., et al. (2021), Optimal Inspection Planning of Structures for Probabilistic Analysis of Fatigue Crack Growth, Journal of Engineering Mechanics 147.1 04020141.

7. Qiu, Y., Wu, H., & Chen, M. (2019), Optimal inspection policy for a repairable system with periodic inspections and a two-stage deteriorating process, Reliability Engineering & System Safety, 188, 79-90.

8. J. He, H. Zhou, and Y. Huang, (2020), A novel method for inspection policy optimization under uncertainty, Reliability Engineering & System Safety, vol. 203,.

9. A. M. A. El-Shafei, (2021), Optimal inspection scheduling for maintenance of deteriorating systems: A review and some recent developments, International Journal of Production Research, 59(1), 195-219.