

# Satellite Attitude Control Based Adaptive sliding Mode Method

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## Abstract

*This paper proposes an adaptive sliding-mode controller design for a three-axis stabilized rigid satellite attitude system with uncertain disturbances. The rigid satellite attitude control systems can be described by the dynamic equations and kinematic equations. This method combined sliding mode control and an adaptive algorithm, which is used to estimate the disturbances uncertainties. Tracking performance is guaranteed, as well as the stability of closed-loop attitude control systems is analysed by using Lyapunov approach stability, the simulations results demonstrate the effectiveness of the proposed control method.*

**Keywords:** Adaptive controller, Attitude control, Rigid spacecraft, Sliding mode.

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## I. Introduction

The attitude control of satellites has been a raising issue for decades [1-3]. Regarding the demand of future space mission, more and more accurate attitude controllers are required in order to meet the high-precision and high-stability of various spacecrafts. Moreover, unknown external and internal disturbances seriously affect attitude control performance. These problems pose a real challenge for attitude control system engineers, to improve robust performance and control accuracy a suitable control schemes are required to ensure the success of satellite mission.

Over the last years, many researchers have conducted extensive studies on the spacecraft attitude control system hence various controllers have been proposed to overcome this problem. These controllers include PID controller [4], an historical overview of optimal control theory in the design of aerospace systems has been presented in [5-7], while the closed loop stability analysis is performed for Lyapunov's method, sliding mode control results can offer high levels of robustness in the presence of parameter uncertainty and dynamic model errors, the first controller design of satellite attitude maneuvers using sliding mode theory using rigid body equipped with three axis external control torques has been proposed in [8], motivated by this work a

general sliding-mode scheme has been done in [9] based on nonlinear sliding manifolds and Gibbs vector parameterisation of spacecraft attitude, however this design procedure results in discontinuous controls where must be placed with saturated controls in order to avoid chattering. An important practical aspect of sliding mode is accounting for the control torque saturation in [10] where an asymptotically stable control method for robust attitude stabilisation which takes into account control input saturation explicitly. Nevertheless, this work provides stabilisation of spacecraft angular rate only, these limitations have been removed in subsequent by [11] such that a continuous version of the sliding mode control design of [12] is applied on a full state Lyapunov analysis.

An interesting approach to deal with complex plant dynamics that are changing or uncertain is the adaptive control where received an increased attention in aerospace engineering, in fact all spacecraft dynamics models are subject to uncertainty such as satellite inertia matrix uncertainty and environmental disturbances, an adaptive controller for the International Space Station (ISS) was performed in [13] where the spacecraft has been considered as a rigid body equipped with a momentum exchange devices and the controller is developed incorporating gain-scheduled adaptation of the attitude gains to ensure

acceptable attitude tracking performance. The inverse optimal theory and derive the adaptive control law have been employed in [14] to solve the attitude tracking control problem of a rigid spacecraft with external disturbances and an uncertain inertia matrix, also to compensate the parameter uncertainties and robust tracking performance a robust adaptive PID-type controller has been proposed in [15] by incorporating a fuzzy logic system and a sliding-mode control. A separation-type principle for observer-based adaptive attitude tracking control of a fully actuated rigid spacecraft has been considered in [16].

In this paper, an adaptive sliding mode control method is proposed for rigid spacecraft attitude control with uncertain disturbances. The method combines adaptive with sliding mode control to suppress uncertain disturbances, and the stability of the system under disturbance is proved by Lyapunov method. Finally the simulation results show that the proposed method can realize the attitude stabilization control of satellite with uncertain disturbances.

## II. Mathematical model of satellite attitude

Mainly there are two methods for describing satellite attitude motion: Euler angle and quaternion, which can be converted to each other. The dynamic and kinematic equation of satellite attitude can be expressed as follow: [17,18]

$$J\dot{\omega} = -\omega^\times J\omega + u + d \quad (1)$$

$$\dot{q}_v = \frac{1}{2}(q_v^\times + q_0 I_3)\omega \quad (2)$$

$$\dot{q}_0 = -\frac{1}{2}q_v^T \omega \quad (3)$$

Where  $J \in R^3$  denotes the moment of inertia matrix which is constant and symmetry.  $\omega = [\omega_1, \omega_2, \omega_3]^T$  denotes the angular velocity vector of body coordinate system relative to inertial coordinate system.  $u$  and  $d$  are respectively satellite's control force and disturbances.  $I_3$  is three unit matrix.  $q_v = [q_1, q_2, q_3]^T$  the vector part of  $q$ , the unit quaternion of satellite body coordinate system, and  $q = [q_0, q_v]^T$ . The relationship of them is:

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \quad (4)$$

$[a^\times]$  is an operator on any vector  $a = [a_1, a_2, a_3]^T$  such that :[19]

$$[a^\times] = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \quad (5)$$

Let  $q_e = [q_{ve}, q_{0e}]^T$  denote relative attitude error from a desired reference frame to the body-fixed reference frame of the satellite. Then we can have:

$$q_e = q \otimes q_d^{-1} \quad (6)$$

Where  $q_d^{-1}$  is the inverse of the desired quaternion and  $\otimes$  is the quaternion multiplication operator. Therefore, the relative attitude error is obtained by:

$$\begin{bmatrix} \dot{q}_{ve} \\ \dot{q}_{0v} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} q_{0v} I_{3 \times 3} + (q_{ve}^\times) \\ -q_{ve}^T \end{bmatrix} \omega_e(t) \quad (7)$$

and

$$\omega_e = \omega - \omega_d \quad (8)$$

Where  $\omega_d$  represents the desired angular velocity of the body, which is assumed to be equal to zero, therefore  $\omega_d = 0 \rightarrow \omega_e = \omega$ .

Hence, the rate of angular velocity can be obtained as follow:

$$\dot{\omega}_e = \dot{\omega} = -J^{-1}(\omega^\times)J\omega + J^{-1}u + J^{-1}d \quad (9)$$

The orbit of a small satellite, encompassing eccentricity, inclination, and altitude and the satellite in complex environments may be affected by aerodynamic moments  $d_p$ , gravity gradient moments  $d_g$ , geomagnetic moments  $d_m$  and solar pressure  $d_s$  [20]. The total environmental disturbance torque can be expressed as

$$d = d_p + d_g + d_m + d_s \quad (10)$$

These moments of external disturbance do not always exist and remain unchanged, but are related to the altitude of the satellite orbit, the distribution of the structure and the conditions of the space environment. Assumption: the total disturbance torque  $d$  is bounded and is slow-varying, thus it is reasonable that  $\dot{d} \approx 0$ .

### III. Sliding-mode adaptive attitude controller design

In this section an adaptive sliding mode attitude controller has been designed, which can compensate the effect of the variation of the spacecraft's disturbances, for this purpose, we will show the controller law in form Eq. (11) can stabilize the origin of the plant (Eq. (1), Eq (2) and Eq. (3)).

$$u = \omega^\times(J\omega) - C_1 J \dot{q}_{ve} - \hat{d} - k_1 \text{sat}(s) - k_2 s \quad (11)$$

where  $\hat{d}$  is the estimate of the disturbance  $d$ ,  $k_1$  and  $k_2$  are positive constant,  $\text{sat}(s)$  is the saturation function defined by following equation:

$$\text{sat}(s) = \begin{cases} \frac{s}{\|s\|} & \text{for } \|s\| \geq \varepsilon \\ \frac{s}{\varepsilon} & \text{for } \|s\| < \varepsilon \end{cases} \quad (12)$$

and  $s = [s_1, s_2, s_3]^T$  is a linear sliding surface in vector form which can be defined as:

$$\dot{s} = \dot{\omega} + C_1 \dot{q}_{ve} \quad (13)$$

Where

$$C_1 = \text{diag}[c_1, c_2, c_3] \text{ with } c_i > 0, i = 1, 2, 3 \text{ is scalar.}$$

The derivative of the sliding surface combined with Eq. (7) and Eq. (9) lead to the following:

$$\begin{aligned} \dot{s} &= \dot{\omega} + C_1 \dot{q}_v \\ &= -J^{-1}(\omega^\times)J\omega + J^{-1}u + J^{-1}d \quad (14) \\ &\quad + \frac{1}{2}C_1(q_{0e}I_{3 \times 3} + (q_{ve}^\times))\omega \end{aligned}$$

Then

$$J\dot{s} = -(\omega^\times)J\omega + u + d + JC_1\dot{q}_{ve} \quad (15)$$

In order to prove the stability of the system, a Lyapunov candidate function can be considered as

$$V = \frac{1}{2}s^T J s + \frac{1}{2}C_2 e^T e \quad (16)$$

Where  $e = d - \hat{d}$  is the disturbance estimation error, thus  $\dot{V}$  is given by

$$\begin{aligned} \dot{V} &= s^T J \dot{s} + C_2^{-1} e^T \dot{e} \\ &= s^T [-\omega^\times(J\omega) + u + d + C_1 J \dot{q}_{ve}] - C_2^{-1} e^T \dot{\hat{d}} \quad (17) \\ &= s^T e - k_1 \|s\| - k_2 \|s\|^2 - C_2^{-1} e^T \dot{\hat{d}} \end{aligned}$$

The derivative of the disturbance observer can be designed as follow

$$\dot{\hat{d}} = C_2 s \quad (18)$$

Hence (17) leads to

$$\begin{aligned} \dot{V} &= -k_1 \|s\| - k_2 \|s\|^2 \\ &\leq 0 \end{aligned} \quad (19)$$

Since  $\dot{V}$  is negative definite then the asymptotic stability has been proved.

### IV. Numerical simulations

To demonstrate the effectiveness of the proposed control schemes, numerical simulations results have been carried out under the Matlab/Simulink platform. These results are obtained using the following parameters.

Table1. Satellite simulation parameters

Parameter	Value
Inertia [ kg.m <sup>2</sup> ]	$\begin{bmatrix} 152.9 & 0 & 0 \\ 0 & 152.5 & 0 \\ 0 & 0 & 4.91 \end{bmatrix}$
Orbit [ km ]	686
Inclination [ deg ]	98
Initial attitude [ deg ]	[-0.15 0.2 0.4]
Initial attitude rate [ deg/ s ]	[0 -0.06 0]
External Torques [ N.m ]	$(\ \omega\ ^2 + 0.005) \begin{bmatrix} 0.05 \sin(0.8t) \\ 0.05 \cos(0.5t) \\ 0.05 \cos(0.3t) \end{bmatrix}$

Parameters are selected as  $k_1 = 0.112$ ,  $k_2 = 8.5$ ,

$C_1 = \text{diag}(0.75, 0.85)$  and

$C_2 = \text{diag}(10e3 * (3.75, 1.96, 1.85))$

The control objective is to transfer the system from the initial attitude to the desired attitude, the simulation results are shown in figure 1-7.

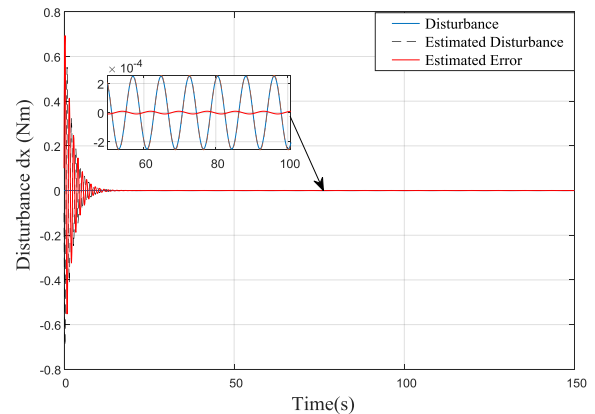


Figure. 1, Time responses of Disturbance dx

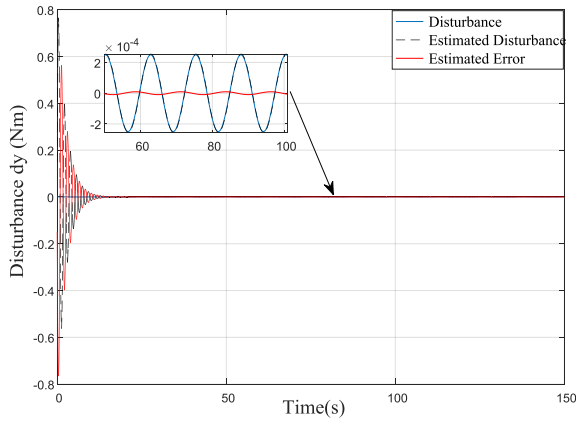


Figure 2. Time responses of Disturbance dy

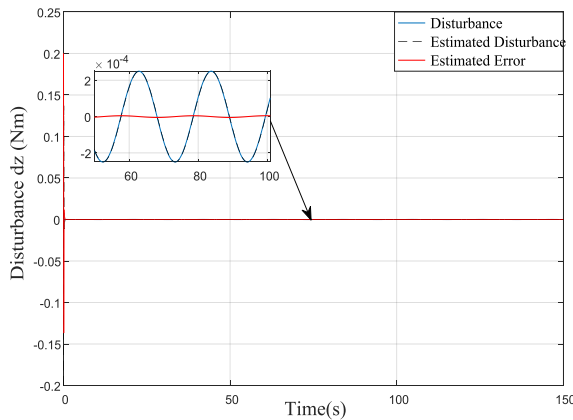


Figure 3. Time responses of Disturbance dz

Figures 1-3 show the time evolutions of disturbances, disturbances estimations and estimation error according the three axes respectively, it is clear that the proposed controller can effectively estimate the uncertain disturbances. The control torques of three axes are given in Fig. 4. It is seen that at beginning of the simulation the control torque are larger, the reason is that the proposed controller has better dynamic response than a classical control and can estimate compensate for the disturbances. Furthermore, after 7.2s, no excessive control energy are required to reject the disturbances.

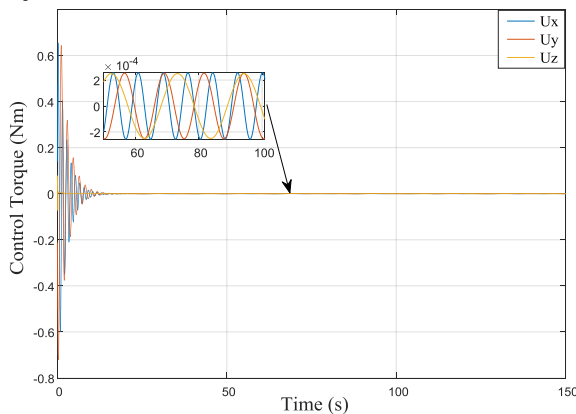


Figure 4. Time response of control Torque

Fig. 5 and Fig. 6 illustrate the attitude control error and the error quaternions where the attitude control accuracy has been improved and the partial amplifications demonstrate that the error has been reduced to the lowest.

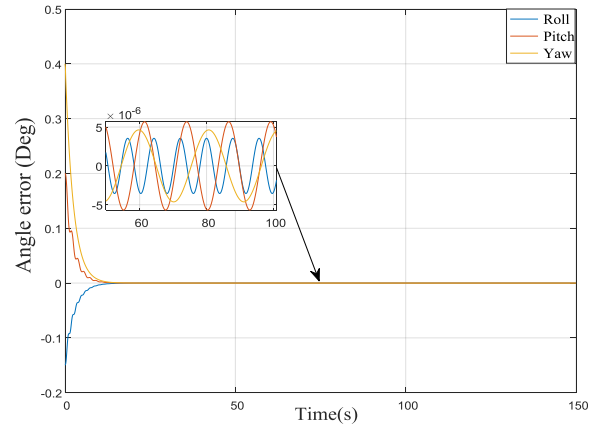


Figure 5. Attitude angle error

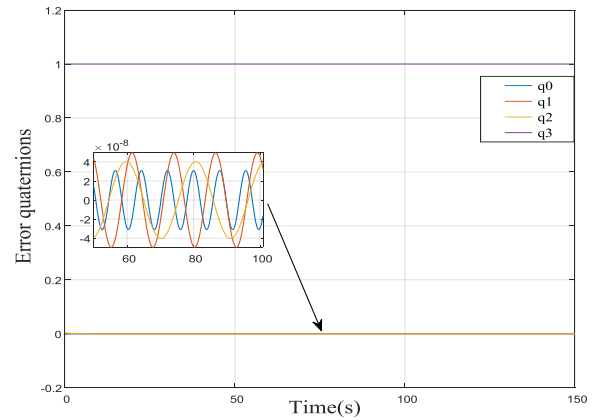


Figure 6. Error quaternions

The responses of the spacecraft error angular velocity components are depicted in Fig 7. It can be seen that the stabilization is obviously improved using the proposed controller.

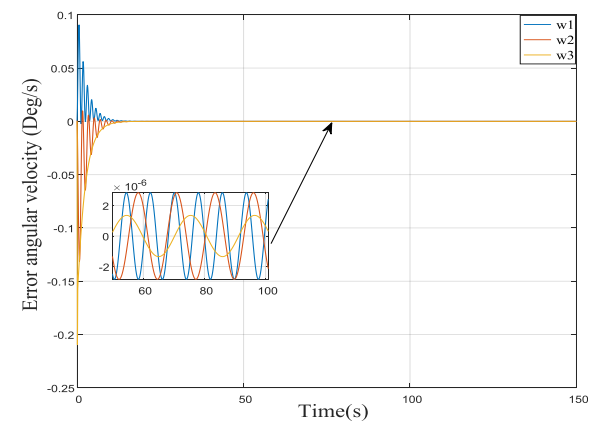


Fig. 7. Error angular velocity

## V. Conclusions

The main goal of this paper is focused on the adaptive sliding mode controller design of microsatellite attitude control with uncertain disturbances, so that the controller can successfully improve the pointing accuracy and the stability of the system is proved theoretically based on Lyapunov theory and it is verified by simulation. Numerical simulations have shown that the controller designed in this paper achieve better the pointing accuracy and the stabilization of the spacecraft. Moreover, the error can reduced to the lowest by the proposed controller. This method provides a useful and promising way for the attitude control of rigid spacecraft.

For the future researches, it is required to taking into account the model with the flexible vibrations and design a new controller to achieve better control effect.

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