POVERTY MEASUREMENTS IN AN IMPRECISE ENVIRONMENT

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Abstract:
The traditional approach is to define poverty with respect to a single indicator of well-being, such as income or expenditure. But such approaches reflect merely one specific aspect of poverty and are not able to represent all of the elements brought to bear. Multidimensional approaches have been proposed which take several aspects of poverty into consideration. Despite its drawbacks, the monetary approach remains unbeatable for quantification of poverty and for tracking its evolution over time. Nonetheless, when one envisions integration of the many different facets of poverty within a multidimensional analysis, it is generally agreed that the transition from a condition of utter privation to a condition of well-being occurs in a gradual manner. Among the current possible approaches that have the potential to reflect this specificity is the use of fuzzy set theory. This theory is, in fact, an extremely powerful tool for modeling vague concepts such as poverty. The aim of this paper is to use fuzzy set theory and the concept of membership functions to measure poverty.

1. Introduction

It is not an easy task to fully comprehend poverty. There are two fundamental issues to be resolved, the first being the identification of those who are poor, and the next being their aggregation. The identification problem is a problem with the definition of poverty, where criteria must be established allowing one to distinguish between the poor and non-poor. The solution to the second problem consists of deriving a poverty index summarizing the information about the number of poor persons, their income levels and possibly how they spend their income. Let us examine the problem of identification first.

The definition of poverty has evolved and grown broader over time. A survey of the literature on the subject demonstrated that studies on poverty have been dominated by two primary approaches: the absolute approach and the relative approach. In the absolute approach, poverty is considered strictly in terms of subsistence. A purely absolute threshold would in this case correspond to a shopping basket of those goods considered indispensable (Rowntree 1901, Booth 1969, Orshanski 1965 and Watts 1967). In the other case, the relative approach defines poverty and poverty thresholds with respect to quality of life or the prevailing social standards in a given country, at a given moment. The purely relative thresholds are based on a notion that the poverty is defined as not having certain of the goods considered basic in a given society (Townsend, 1979). In addition to these two approaches, there is another, called "subjective", where impoverishment is based on an assessment of the degree of needs satisfaction expressed by the interviewee (Kilpatick 1973, Rainwater 1974, Goedhart *et al.* 1977 and Van Praag 1971).

The common thread to these approaches is that they define poverty with respect to a single indicator, reflecting resources or social status (income or expenditure). "Poor" then refers to those persons or families whose income or expenditure lies below a certain threshold. These

approaches share a common weakness in that they fail to consider more than one aspect of reality. It is certainly true that several aspects of well-being may be expressed objectively by assigning a monetary value to them, that is, representing them using income or expenditure, which offers the benefit of being relatively exhaustive in the statistical sense of the word, and is unidimensional and continuous. Nonetheless, it is also true that certain fundamental aspects of well-being cannot be dealt with so simply. The limits of one or the other of the two indicators are clear. Regarding income, while the determination of a person's income level may allow one to know whether the former possesses the necessary means to satisfy his essential needs, this does not allow one any glimpse into whether or not it truly satisfies those needs, in the sense that income levels provide no indication whatsoever of how the money is actually spent. Furthermore, income levels do not provide us with sufficient information on the person's living conditions. The same remark holds true for expenditure data. Despite the fact that this can be measured with greater precision than income, expenditure data does not, unfortunately, allow us to know the level of (consumption: spending of disposable income). A person with low (consumption: spending of disposable income) can not be considered poor if that person is able to select goods and services at the best price available.
The dissatisfaction researchers have felt regarding the application of income and expenditure as the sole indicators of well-being has led to the examination of alternative indicators. Thus certain authors such as Travers and Richardson (1993) propose recourse to the concept of "full income" to establish a person's quality of life. In addition to the components usually included in definitions of income, full income also includes qualitative indicators such as state of health, level of education, time and how it is spent. In order to integrate these qualitative criteria with the full income concept, a monetary value was assigned to each criteria based on the market opportunity cost, or the price that individuals must pay for such services. Travers and Richardson also propose to use direct measurements of poverty in tandem with the concept of full income. These measurements are completed by an interviewee self-assessment, in terms of nourishment, clothing, transportation, etc. In the end, this of course leads to a purely quantified definition of poverty, and thus a poverty threshold.

The existence of such overlap between definitions only highlights the difficulty of illuminating the reality of poverty, which can never be captured in a simple, unidimensional definition. Accordingly, researchers have made many attempts to go beyond the definition of poverty based on a monetary approach, by proposing other supplementary and complementary approaches (Ruggles 1990). This has given way to new, multi-dimensional approaches.
Fundamentally, beyond its strictly comparative surface appearance, poverty is multidimensional. This affects our apprehension of the phenomenon and the implementation of strategies designed to combat poverty. One of the benefits of a multidimensional approach is achieved when one combines financial circumstances and the general living conditions under which individuals subsist. As Whelan (1993) points out, a global poverty indicator based on a set of privation indicators provides superior measurements of a certain permanent level of poverty than those indices based solely on income or expenditure. This indicator takes into consideration basic needs (food, clothing, heating and lighting, household appliances) and certain variables linked with social participation, and which occasionally exercise a constraint on the latter (working conditions, health status, level of education, leisure activities, environment, family and social activities).

Several authors have tried to integrate various aspects of poverty not taken into account by the monetary approach to poverty measurement. Among these, Townsend (1997) proposed a list of

60 indicators that he felt could summarize the normal activities within a given society – that is, which could reflect the capacity of an individual to satisfy certain social standards. Townsend then derived an index of privation based on which it should be possible to define the poverty threshold. Other authors such as Mack and Lansley (1985) or Gordan and Pantaziz (1998) expanded on this proposal and chose to establish poverty measurements based exclusively on those goods considered necessary to the majority of the population. More recently yet, Gordan *et al.* (2000) expanded the list of goods considered necessary to the population, introducing a range of goods, activities, circumstances and opportunities that better reflect the range of resources that should be taken into consideration when developing a relative notion of poverty, or social exclusion.

In the unidimensional approach, the process for measuring poverty is based on the definition of a poverty threshold allowing one to distinguish between the poor and non-poor. In addition to the fact that there is a lack of consensus regarding the position of this threshold, it is questionable that establishment of an exact value for such a threshold is a valid aim. Certain authors such as Cerioli and Zani (1990) point out that a strict division of the population into poor and non-poor is unrealistic. Moreover, it results in a loss of information (Betti and Cheli 2001). Consider how difficult it would be to argue that two individuals who have only the same income share the same living conditions. Consider the even greater challenge of arguing that two individuals whose incomes differ by only a few dinars situating them just above and below any given poverty threshold should be considered poor and non-poor, respectively, on the strength of a few dinars. In general it is agreed that moving from a state of privation to a state of well-being is, far from a sudden and obvious change, rather a gradual change (Lellis 2000). To avoid this over-simplification of reality and to take the continuous nature of this transition into account, one could try to apply the theory of fuzzy sets. First introduced by Zadeh (1965) and now widely used in many areas of research, this theory has also recently won considerable attention for its application into the analysis of income inequality, well-being and poverty measurement. According to Chiappero-Martinetti (2000), fuzzy set theory represents a very interesting tool for handling inexact knowledge and probabilistic reasoning.

Recently Cerioli and Zani (1990) proposed a statistical method for measuring poverty that takes into account its multidimensional nature and is based on fuzzy set theory, with their application being a study of living conditions in the province of Parma (Italy). Since that time, other studies have been carried out, investigating further a few of the theoretical aspects linked to the use of fuzzy sets in the multidimensional analysis of poverty (*cf.* Cheli and Lemmi, 1995, Chiappero Martinetti, 1994, 1996 and 2000, Betti and Verma, 1999), with applications to certain countries such as Italy (Dagum *et al.*, 1992, Betti and Cheli, 2000), Poland (Cheli *et al.*, 1994), Great Britain (Betti and Cheli, 2001) or Switzerland (Miceli, 1998).

In the first section below, we will provide a short definition of a fuzzy set (section 2.1) referring to the membership function, and will provide a few of the possible forms it may take when using privation indicators (section 2.2). In section 2.3, we will approach the problem of aggregating various indicators in such a way as to fold membership degrees into a single dimension. Finally, in section 2.4, we present a general poverty index.

2. Fuzzy Sets Theory

The theory of fuzzy sets represents a very interesting tool for handling those problems lacking specific criteria for assessing the degree to which a person or household belongs or does not belong to a given group. A result of this is that the theory of fuzzy sets allows one to solve the problem of identifying a poor individual. Another benefit of this type of approach is that one is not forced to establish a poverty threshold. In the following section (2.1) we will provide the definition of a fuzzy set using the membership function. In section 2.2 we will present a few interesting forms of the membership function. Then, in section 2.3, we will begin to tackle the issue of aggregating the various privation indicators. Finally, in section 2.4, we will present a general poverty index.

2.1 Definition of a Fuzzy Set

Consider a set X and let x be an element of X. A sub-set A of X is defined as the set of couples:

(1) $A = \{x, \gamma_A(x)\}$;

for each $x \in X$, where γ_A is an application of the set X within the closed interval [0,1], known as the membership function of the fuzzy sub-set A. In other words, each set or sub-set A or X is characterized by a membership function $\gamma_A(x)$ which associates a real number lying in the interval [0,1] to each point in X, with the value of $\gamma_A(x)$ thus representing the degree of membership of the element x in set A. More formally expressed, if A were an ordinary set, then the membership function associated with it could only take on two values: 0 and 1. In that case, one would have:

(2) $$\gamma_A(x) = \begin{cases} 1, \text{if } x \in A; \\ 0, \text{if } x \notin A. \end{cases}$$

Taking that definition into account, $\gamma_A(x) = 0$ would imply that x does not belong to the fuzzy set A, and $\gamma_A(x) = 1$ would imply that x belonged to the fuzzy set A, and an intermediate value, between 0 and 1 would denote partial membership of x in A. In the latter case, the closer the value of $\gamma_A(x)$ to 1, the greater the degree of membership of x in A.

The same procedure may be applied to define a fuzzy set of poor individuals. Given N, a set of n individuals or households, and P, a fuzzy sub-set of N defined by the couples:

(3) $P = \{i, \gamma_P(i)\}$;

where $i=1,...,n$ and $\gamma_P(i)$ represent the degree of membership of each individual i in the fuzzy sub-set of the poor of the population. As seen previously, three cases will then be possible:

(4) $$\begin{cases} \gamma_p(i) = 0, \text{ if the individual } i \text{ absolutely does not belong to the set of poor individuals;} \\ \gamma_p(i) = 1, \text{ if the individual } i \text{ belongs entirely to the set of poor individuals;} \\ 0 < \gamma_p(i) < 1, \text{ if the individual } i \text{ belongs only in part to the set of poor individuals.} \end{cases}$$

2.2 Membership Functions

Taking the multidimensional character of poverty into account leads to the requirement of choosing the pertinent privation indicators for an analysis of living conditions. This consists of assessing the degree of membership of each individual or household in the fuzzy set of poor persons, based on indicators which are developed to reflect quantitative variables as well as qualitative variables. Each variable is linked to a specific aspect of poverty, reflecting a privation with respect to a good or activity. The problem, then, is to choose from among all of the possible membership functions, that one function that is the most appropriate for each of the privation indicators. At least two categories of variables may be considered: continuous variables, which are quantitative, and discrete variables (polytomic or dichotomic). A membership function may be specified for each of these two categories. In the case of discrete variables, we may encounter two conditions: a) dichotomic variables and b) polytomic variables.

a) Dichotomic variables

Let us consider a set of living condition indicators, $\varphi = [\varphi_1, ..., \varphi_k]$. Consider I_j those individuals or households enduring a certain level of privation with respect to indicator φ_j, with j=1,...,k.

The simplest membership function is the one associated with the dichotomous variable indicating the possession of certain durable consumer goods. In this case, the sub-set I_j of the population is not a fuzzy set, since the membership function may be written:

$$(5) \qquad \gamma_{I_j}(i) = \begin{cases} 1, \text{ if } \varphi_{ij}=0; \\ 0, \text{ if } \varphi_{ij}=1. \end{cases}$$

Thus, whether the individual did not possess the consumer good j ($\varphi_{ij}=0$) or did possess the consumer good j ($\varphi_{ij}=1$), the function output is clear. In the first case, the privation is absolute while in the second case, there is absence of privation.

b) Polytomic variables

Among the pertinent types of poverty indicator that allow one to reflect the multidimensional aspect of the problem, we find qualitative variables. These are presented as several different criteria, each of which corresponds to a certain degree of privation. These criteria may be ranked according to their decreasing risk of poverty (Cerioli and Zani 1990) or by increasing risk of poverty (Miceli 1998). Among these variables (for example, low level of education, gender, employment situation), one could consider, for example, the variable which indicates the subjective perception that households have of their own situation.

Let us assume that these criteria may be ranked by decreasing risk of poverty. The possible values may then be: very well, fairly well, average, fairly badly, very badly. With m criteria, the indicator φ_j takes its values from the set $\{\varphi_j^{(1)}, ..., \varphi_j^{(m)}\}$. Let us assume that these criteria are ranked in such a manner that the increasing values of the upper index denote accentuation of the state of privation. According to the specification offered by Cerioli and ZANI (1990), IT IS possible to have scores of $\zeta^{(l)}{}_j$, with l=1,...,m for different criteria. The relationship between these different scores may be represented by:

$$(6) \qquad \zeta^{(1)}{}_j < \cdots < \zeta^{(l)}{}_j < \cdots < \zeta^{(m)}{}_j .$$

In most cases, these scores are defined using the first m integers:

$$(7) \qquad \zeta^{(l)}{}_j = l \text{ (where } l=1,...,m).$$

When we allow one of the privation indicators to be an individual's subjective opinion of his own situation, from which we derive assessments categorized by risk of increasing poverty, the definition (7) implies that there is the same distance between a situation considered very poor and a poor situation, as separates an average situation from a poor situation. Given the ordinal nature of the qualitative variable φ_j, it is possible to choose a criteria corresponding to a sufficiently favorable state serving to exclude poverty entirely, or alternatively, to choose a criteria that clearly indicates a degree of poverty. If ζ_j^{min} and ζ_j^{max} are scores corresponding to two limits, then the membership function proposed by Cerioli and Zani (1990) is the following:

(8)
$$\gamma_{ij}(t)=\begin{cases}0, \text{ if } \zeta_{ij}\le\zeta_j^{min};\\ \dfrac{\zeta_{ij}-\zeta_j^{min}}{\zeta_j^{max}-\zeta_j^{min}}, \text{ if } \zeta_j^{min}\le\zeta_{ij}\le\zeta_j^{max};\\ 1, \text{ if } \zeta_{ij}\ge\zeta_j^{max}.\end{cases}$$

With this specification, the membership function increases linearly as the risk of poverty increases. We should point out that linear membership functions are the most popular, in empirical analysis, since they are easy to specify, to interpret and to represent graphically (Lelli, 2000).

c) Continuous variables

Among the indicators of privation or poverty represented by a continuous variable we find income and expenditure. Certain authors have proposed alternative methods to specification of a poverty threshold. For example Kakwani (1995) chose to focus on the uncertainty associated with any specification of a poverty threshold and proposed using a privation coefficient to measure it. His procedure resulted in the definition of a new class of poverty measurement. Along the same lines, Atkinson (1987) and Foster and Shorrocks (1988 a,b,c) highlighted the impossibility of determining a unique poverty threshold and proposed an ordinal approach linked with stochastic predominance. These methods both establish an interval that, it is presumed, contains a poverty threshold. Adopting a similar procedure, Cerioli and Zani (1990) propose the establishment of two thresholds, the first to be denoted $\chi^{\min}$, and corresponding to the value of the continuous variable chosen as the poverty indicator beneath which an individual or household can be considered to be poor; the second to be denoted $\chi^{\max}$, and corresponding to the value of the chosen continuous variable above which an individual or household can be considered extremely poor (Cerioli and Zani 1990). For values of the variables contained between these two thresholds, the membership function takes on values within the interval [0;1]. For variables such as income or expenditure, for which an increase is reflected in an improvement in well-being, the membership function is continuous and decreasing. Making the assumption that the risk of poverty varies in a linear manner between the two thresholds $\chi^{\min}$ and $\chi^{\max}$, Cerioli and Zani (1990) propose to define the membership function as follows:

(9)
$$\gamma_{ij}(t)=\begin{cases}0, \text{ if } 0\le\chi_{ij}\le\chi_j^{min};\\ \dfrac{\chi_j^{max}-\chi_{ij}}{\chi_j^{max}-\chi_j^{min}}, \text{ if } \chi_{ij}\in[\chi_j^{min},\chi_j^{max}];\\ 1, \text{ if } \chi_{ij}\ge\chi_j^{max}.\end{cases}$$

It is a given that the only real condition that must be fulfilled by a membership function is that it must take its values only from the interval between 0 and 1. The membership function can however take on several forms, other than those presented until now. In a particular case, the researcher may legitimately choose whatever type of curve whose form can be defined, based on which he considers a desirable combination of simplicity, appropriateness and efficiency. Consider also, that the membership function should be selected relative to the context to which it refers, and the type of indicator that one wants to describe (Chiappero-Martinetti, 2000).

Certain authors have attempted to improve on or generalize the membership functions proposed by Cerioli and Zani (1990) (Cheli *et al.* 1994, Cheli and Lemmi 1995, Betti and Cheli 2000, 2001). Cheli and Lemmi (1995) qualify their approach as entirely fuzzy and relative. This approach offers several advantages. It is completely congruent with the fact that the concept of poverty is itself a fuzzy concept, and also avoids the use of a poverty threshold. In addition, it can be used to provide a multidimensional perspective of the poverty analysis, using different types of living condition indicators.
For the polytomic qualitative variable, Cheli and Lemmi (1995) suggest the following membership function:

(10) $$\gamma_{\Xi_j}(i)=\begin{cases}0, \text{ if } \chi_{ij}=\chi_j^1;\\ \gamma_{\Xi_j}(\chi^{(k-1)})+((H_j(\chi_j^{(k)})-H_j(\chi_j^{(k-1)}))/(1-H_j(\chi_j^{(k)})), \text{ if } \chi_{ij}=\chi_j^{(k)},\ k=2,\dots,m.\end{cases}$$

and where $\gamma_{\Xi_j}(\chi^{(k-1)})$ represents the degree of belonging to the set Ξj of an individual exhibiting the criteria $k-1$ for variable χ_j, and where the criteria are ordered in increasing rank with respect to the risk of poverty. Note that H_j represents the cumulative distribution function of the variable χ ranked starting at k. Thus, $\chi_j{}^{(1)}$ denotes minimal risk of poverty, while $\chi_j{}^{(m)}$ denotes maximal risk of poverty. For this membership function, the zero value is always associated with a criteria (or value) corresponding to a maximal risk of poverty while the value of 1 is associated with a criteria corresponding to a maximal risk of poverty. Between these two extremes, the degree of poverty ranges between 0 and 1 and is an increasing function for the risk of poverty. Cheli and Lemmi (1995) underline that with this type of specification, one avoids all a priori or arbitrary choices, and that such membership functions are the mirror of the sample distribution: in this sense, the approach may be considered as "Totally relative", and a perfect reflection of reality (Cheli and Lemmi, 1995).

In the case of continuous variables, Cheli and Lemmi (1995) define the membership function in two fashions. In the case where risk of poverty or privation would increase when the value taken by the continuous variable χ_j increased, the membership function is expressed:

(11) $$\gamma_{\Xi_j}(i)=H_j(\chi_{ij}).$$

While in the case where risk of poverty or privation would increase when the value taken by the continuous variable χ_j decreased, the membership function is expressed:

(12) $$\gamma_{\Xi_j}(i)=1-H_j(\chi_{ij}).$$

These two functions comply perfectly with the conception of a relativistic notion of poverty. In fact, they are actually defined using the relative position of each individual with respect to the set of individuals. Note that in the case when the analysis is based on income, the membership function takes the form proposed by equation (12). In this case $H_j(\chi_{ij})$ represents the income distribution function. This distribution may be empirical, or a theoretical model estimated using a sample set (for instance, the model proposed by Dagum 1977). This way of handling the issue allows one, according to Cheli and Lemmi (1995), to avoid the requirement of establishing the thresholds χ^{min} and χ^{max}, which would certainly be somewhat tarnished by arbitrariness. Moreover, this specification by Cheli and Lemmi (1995), in contrast with that of Cerioli and Zani (1990), is based upon a theoretical basis, according to the two authors (as it is congruent with the notion of relative poverty) and upon empirical verification (since $H_j(\chi_{ij})$ is estimated on the basis of a sample set). However, the formulation proposed by Cheli and Lemmi (1995) is not as appealing as that of Crioli and Zani (1990), for a number of reasons (Miceli 1998). Consider that according to the Cheli and Lemmi (1995) approach, only the relative position of each individual with respect to other individuals is pertinent in attribution of the degree of membership. Thus, significance is granted only to the percentage of individuals who have a higher income than the individual under consideration, without attaching any significance to the absolute level of income, nor to the magnitude of income gaps between individuals. The problem is that this can yield the same system of degrees of membership, especially when an empirical distribution is used, even if two very different income distribution systems are being considered, with concentrated incomes typical of one and dispersed incomes characteristic of the other. On the other hand, the value of the Cerioli and Zani (1990) formulation is that it takes into account all of these elements.

2.3. Aggregation of Fuzzy Sub-Sets

Having evaluated the degree of membership of each individual or household in the impoverished fuzzy set, in the previous section, based on the set of privation indicators, a means must be identified to reduce the membership degrees obtained to a single dimension, for each of the various indicators. This consists, then, of determining the degree of membership $\gamma_p(i)$ of each individual i in the impoverished fuzzy set P.

In general it is possible to define an aggregation operation by a function: $h:[0;\ 1]^k \rightarrow [0;\ 1]$, for $k \geq 2$ (Chiappero-Martinetti, 1994). For example, let us consider k fuzzy sets $\Xi_1, \Xi_2, \ldots, \Xi_k$ defined over the set N of individuals and the function h which serves to indicate degrees of membership $\gamma_{\Xi_j}(i)$ for each individual i belonging to N, for each privation indicator; this determines a new set P, whose degrees of membership are given as:

(13) $$\gamma_p(i) = h(\gamma_{\Xi 1}(i),\ \gamma_{\Xi 2}(i), \ldots, \gamma_{\Xi k}(i)).$$

It remains to be known how the function h may be defined. Several methods are possible. According to Zadeh (1965), there are two possibilities. The first consists of allowing the function h to correspond to the maximal function $\max(\gamma_{\Xi 1}(i),\ \gamma_{\Xi 2}(i), \ldots, \gamma_{\Xi k}(i))$, defined by the union operation over these fuzzy sets. The second possibility is to define the function h as the minimal function $\min(\gamma_{\Xi 1}(i),\ \gamma_{\Xi 2}(i), \ldots, \gamma_{\Xi k}(i))$. Neither of these two solutions are particularly desirable. In the first case, a person or household is considered completely poor if they find themselves in a situation of total privation with respect to at least one of the privation indicators. This would have the effect of judging two individuals in the same way, even if one experienced this privation with respect to only one of the indicators, and the other person experienced this privation with respect to the entire set of indicators. In the second case, an individual is considered completely poor only if he experiences total privation with respect to all of the privation indicators. This is not an ideal situation, either, since individuals whose lives may vary enormously from the point of view of living conditions, would find themselves attributed the same degree of poverty.

According to Chiappero-Martinetti (1994), the function h must be able to take on intermediary values between maximum and minimum, reflecting the potential for interaction between the various privation indicators. Thus, poverty is comprehended as accumulated disadvantages. This requirement may be taken into consideration nsing the aggregation of calculations of the mean, in such a way that the following relation holds true:

(14) $\min(\gamma_{\Xi 1}(i), \gamma_{\Xi 2}(i), \ldots, \gamma_{\Xi k}(i)) \leq h(\gamma_{\Xi 1}(i), \gamma_{\Xi 2}(i), \ldots, \gamma_{\Xi k}(i)) \leq \max(\gamma_{\Xi 1}(i), \gamma_{\Xi 2}(i), \ldots, \gamma_{\Xi k}(i))$.

A minimal axiomatic structure is generally associated with the function h and verifies the axioms of monotony, continuity and symmetry. A class of operators that satisfies this axiomatic structure may be expressed as the generalized mean of the degrees of membership:

(15) $$h_\alpha(\gamma_{\Xi 1}(i), \gamma_{\Xi 2}(i), \ldots, \gamma_{\Xi k}(i)) = \frac{\sum_{j=1}^{k} (\gamma_{\Xi j}(i))^{\alpha})^{1/\alpha}}{n}$$

where $\alpha \neq 0$ is an integer allowing one to specify different types of means. For example, when $\alpha \rightarrow 0$, we obtain a geometric mean; when $\alpha = 1$, an arithmetic mean is obtained and when $\alpha = -1$, we obtain a harmonic mean. If one uses an aggregation operator selected from the class of operators expressed by equation (15), one is agreeing that privation indicators have the same significance in a global assessment of living conditions. Yet it is fully evident that certain privation indicators are more significant than others. As a result, the more significant indicators must be granted addition weight in the aggregation process, which means that the symmetry axiom must be suppressed, by the introduction of an appropriate weighting scheme. Thus, the function h_α will be redefined and expressed as a generalized weighted mean of degrees of membership:

(16) $$h_\alpha(\gamma_{\Xi 1}(i), \gamma_{\Xi 2}(i), \ldots, \gamma_{\Xi k}(i)) = \frac{\sum_{j=1}^{k} \omega_j (\gamma_{\Xi j}(i))^{\alpha})^{1/\alpha}}{n}$$

where the weighting scheme, expressed as $\omega_j \geq 0$ and $\sum_{j=1}^{k} \omega_j = 1$ specifies the relative significance granted each privation indicator.

Cerioli and Zani (1990) define the degree of membership of each individual i to the impoverished fuzzy sub-set as an arithmetic mean of the degree of membership in the impoverished set, according to each of the privation indicators.

(17) $\gamma_p(i) = \sum_{j=1}^{k} \omega_j (\gamma_{\Xi j}(i))$.

The principal characteristic of this aggregation procedure is the weighting scheme, which is chosen, like the membership functions themselves, according to the context of the analysis and the judgment of the researcher. In fact, the selection of an appropriate weighting scheme is a critical step in determination of the impoverished fuzzy set. One of the possible specification of this weighting has been suggested by Cerioli and Zani (1990) :

(18) $\omega_j = \ln(1/\bar{\gamma}_{\Xi_j}) / \sum_{j=1}^{k} \ln(1/\bar{\gamma}_{\Xi_j})$

where $\bar{\gamma}_{\Xi_j} = (1/n)\sum_{j=1}^{k} \gamma_{\Xi_j}(i)$ represents the fuzzy proportion of poor households as per the privation indicator φ_j. In this specification, the choice of the logarithmic form is entirely justifiable, as one grants greater significance to those privation indicators which reflect the less frequent symptoms of poverty.

It must be pointed out that the weighting scheme corresponds to a relative poverty situation. So individuals or households are defined as more poor, the less they conform to the usual lifestyle of the society in which they live. Note that we will be unable to identify those cases in which individuals are deprived of a certain good or activity by preference, or as a result of an active choice in the matter. Therefore the greater the importance of careful selection of those indicators likely to successfully encapsulate the living conditions of an individual.

In a similar manner, but with direct bearing on the membership function (10), Cheli and Lemmi (1995) propose a specification defined as follows:

(19) $\omega_j = \ln(1/n \sum_{j=1}^{k} \gamma_{\Xi_j})$

This expression coincides with that suggested by Cerioli and Zani (1990) in the instance of dichotomic variables.

Now we shall examine a problem that is particularly significant in those cases where the analysis is performed on the basis of a continuous variable (income or expenditure). This problem may arise at the level of the fuzzy proportions $\bar{\gamma}_{\Xi_j}$ of poor households, so defined by the privation indicator φ_j, when the latter refers to a continuous variable and when using membership function (12). Two cases may occur. If one uses a distribution function $F_j(\varphi_j)$ estimated on the basis of a sample from a Dagum *et al* (1992) density function, the sample mean $\bar{\gamma}_{\Xi_j}$ can be expressed as:

(20) $E[\gamma_{\Xi_j}(i)] = 1 - F_j(\bar{\varphi}_j) = 0.5$;

where E(.) designates the expected value. To put it another way, the sample mean $\bar{\gamma}_{\Xi_j}$ tends to coincide with the value of the distribution function that corresponds with mean income. In a certain sense, this means that poverty will always be deemed to affect half the population (Cheli, 1995).

In the case where the empirical distribution function is used for $F_j(\varphi_{ij})$, the degree of membership for each individual i in the set of those impoverished, according to income, equals:

(21) $\gamma_{\Xi_j}(i) = 1 - i/n$.

This is true in the case where the incomes of the various individuals or households are different, which is nearly always the case if one uses sample date. If the sample size is sufficiently large, a value of 0.5 is obtained. So, this is always the value obtained if one uses membership function (12). In other words, application of function (12) always results in assigning the same absolute weight to indicators such as income or expenditure, which is unrealistic according to Miceli (1998), as it assigns the same mean degree of membership to very different situations.

3. Fuzzy Measurement of Poverty: Construction of a General Index of Poverty

Up to this point, we have considered how to measure the degree of poverty of each individual or household in relation to several privation indicators. If one were to proceed to aggregate the various measurements, a poverty index could be constructed for all households. To this end, Cerioli and Zani (1990) propose defining this index as the arithmetic mean of the household membership functions:

(22) $P = 1/n\sum_{i=1}^{n}\gamma_{\Xi_j}(i)$; with $P \in [0;\ 1]$.

The parameter P represents that proportion of households that belong to the fuzzy sub-set of households. The proportion P will be equal to zero if and only if $\gamma_{\Xi_j}(i) = 0$ for all households, that is, in the complete absence of poverty, whatever the privation index under consideration. The parameter P will be equal to 1 if and only if $\gamma_{\Xi_j}(i) = 1$ for all households, that is, under conditions of extreme difficulties for all honseholds, and according to all the privation indices. However, the most common case is that $0<P<1$, with P a monotone function, increasing with respect to the degree of poverty of each household. This means that when there is deterioration in the living conditions of an individual or a household belonging to the impoverished fuzzy set, P will increase. It must be noted that the index P possesses two interesting properties. The first is that it is considered a generalization of the "headcount ratio" (in cases where limited to a single privation indicator, income) or as per Cerioli and Zani (1990) as a generalization of other poverty indices (in cases where one chooses a membership function defined as a generalization of that provided by equation 9). The second property is that P is decomposable, and belongs to the additively decomposable class of poverty indices, which can take on a form similar to that proposed by Chakravarty (1983), Foster, Greer and Thorbeck (1984) and Foster and Shorroks (1991).

4. Conclusion

In this paper, we have introduced a multidimensional measurement for poverty based on the theory of fuzzy sets. It is clear that this is not the only possible method, but it does offer the advantage of handling certain issues such as poverty, for which there are no specific criteria that allow one to assess the degree to which a person or household belongs or does not belong to a particular group, or set. Furthermore, this method offers congruency with the fact that poverty is by its nature a multifaceted, fuzzy phenomenon. We venture to suggest that the failure of the many anti-poverty policies implemented in many different countries, is almost certainly due to inaccurate measurements of poverty. In fact, it must be stated that the measurements used were based almost exclusively upon a single privation indicator. Such approaches represent a radical simplification of a phenomenon that is in contrast, very complex. Because of this, the method introduced in this paper may serve as a more appropriate tool for measuring poverty. In ending, we wish to point out that empirical studies carried out in several countries have shown that such measurements provide a far more complete overview of living conditions, since they take several privation indicators into consideration.

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