Submitted on: 13/09/2018
Revised form accepted on: 15/08/2019
Corresponding author: cheikhabdelbakki@yahoo.fr

Nature et Technologie
Nature & Technology
http://www.univ-chlef.dz/revuenatec
ISSN: 1112-9778 – EISSN: 2437-0312

Dynamic Behavior Analysis of Functionally Graded Nanoplates Based on Elastic Foundations

CHEIKH Abdelbaki

Civil engineering Department, Applied Sciences Faculty, Ibn Khaldoun University of Tiaret, Algeria

Abstract:

An analytical study to predict the behavior of FGM Nano-plates supported by Pasternak elastic foundations based on a theory of hyperbolic shear strain. Nonlocal elasticity theory is used to introduce the effect of small scale. The influence of the parameters of the geometry, the foundation stiffness, and the material property are presented. Hence it is unnecessary to use shear correction factors. The governing equations are derived from the principle of virtual work. The free vibration solutions are finally presented for the nonlocal higher order plate models. The numerical results obtained in the present study for several examples are presented and compared to other models available in the literature.

Keywords: Pasternak elastic foundations, Hyperbolic shear strain, Nano-plates

1. Introduction

Functionally graded materials are a new type of composite structures that are of great interest in the design, use, and manufacture of engineering. Functionally graded structures allowed the material properties to be graded continuously through the thickness and to avoid abrupt changes in stress and displacement distributions. Functional materials (FGM) are classified as new composite materials widely used in the aerospace, nuclear, civil, automotive, optical, biomechanical, electronic, chemical, mechanical and shipbuilding industries. FGM has attracted the attention of several researchers in recent years such as [1-22], and these may have a number of advantages such as high resistance to temperature gradients, a significant reduction in residual and thermal stresses and resistance high wear. In this work using a high order shear deformation theory to predict the vibrational behavior of simply supported nanoplates (SS) and the results of nondimensional frequencies are in excellent agreement with literature.

2. Mathematical formulation

By using the non-local elasticity theory, it is assumed that the stress tensor at a point depends on the strain tensor at all the points of the continuous medium; the nonlocal constitutive relationships of a Hookean nanomaterial can be represented by the following differential constitutive relationships [23]:

$$(1-\mu^2)\sigma_{ij}^{NL} = \sigma_{ij}^{L} \tag{1}$$

with $\mu = e_0.a$ is the nonlocal parameter represents the small-scale effect.

The nonlocal constitutive equations of an FGM nonlocal plate can be written as [23]:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix} - (e_0 a)^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix} = \begin{bmatrix}Q_{11}(z) & Q_{12}(z) & 0 & 0 & 0\\ Q_{12}(z) & Q_{22}(z) & 0 & 0 & 0\\ 0 & 0 & Q_{66}(z) & 0 & 0\\ 0 & 0 & 0 & Q_{44}(z) & 0\\ 0 & 0 & 0 & 0 & Q_{55}(z)\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{2}$$

where: ***a*** is an internal characteristic length; ***e*₀** a constant; $(\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{yx})$ and $(\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{yx})$ are the stress and strain components, respectively. The elastic constants Q_{ij} in terms of Young's modulus E and Poison's ratio ν are:

$$Q_{11}(z) = Q_{22}(z) = \frac{E(z)}{1-\nu^2};$$
$$Q_{12}(z) = Q_{21}(z) = \frac{\nu E(z)}{1-\nu^2}$$
$$Q_{44}(z) = Q_{55}(z) = Q_{66}(z) = \frac{E(z)}{2(1+\nu)}$$

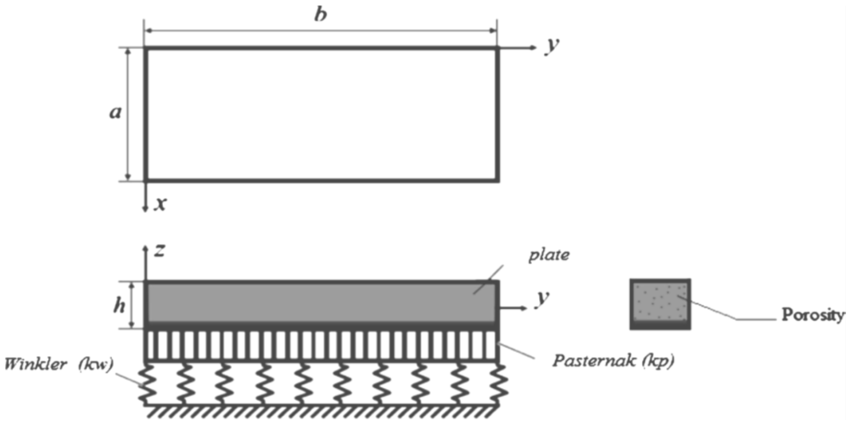

Figure 1. The geometry of the FGM plate resting on elastic foundations [23].

Young's modulus (E) and material density (ρ) equations of the FG plate can be expressed by the Power-law distribution as:

$$\begin{cases} E(z) = (E_C - E_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p E_m \\ \rho(z) = (\rho_C - \rho_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p \rho_m \end{cases} \quad (3)$$

where:

E_c and E_m are the corresponding properties of the ceramic and metal, respectively.

ρ_c and ρ_m are the Material density of the ceramic and metal, respectively.

The displacement field of the present model can be given as:

$$\begin{cases} u(x,y,z) = u_0(x,y) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \\ v(x,y,z) = v_0(x,y) - z\frac{\partial w_b}{\partial y} - f(z)\frac{\partial w_s}{\partial y} \\ w(x,y,z) = w_b(x,y) + w_s(x,y) \end{cases} \quad (4)$$

$f(z)$ is defined by [24]:

$$f(z) = \frac{h \cdot \sinh(10 \cdot \frac{z}{h})}{10 \cdot \cos(5)} - \frac{h}{100} - z \quad (5)$$

where u_0, v_0, w_b, w_s are four unknown displacements of the mid-plane of the plate, $f(z)$ denotes shape function representing the variation of the transverse shear strains and stresses within the thickness.

The strain field is calculated by:

$$\begin{cases} \varepsilon_x = \varepsilon_x^0 + zk_x^b + f(z)k_x^s \\ \varepsilon_y = \varepsilon_y^0 + zk_y^b + f(z)k_y^s \\ \gamma_{xy} = \varepsilon_{xy}^0 + zk_{xy}^b + f(z)k_{xy}^s \\ \gamma_{xz} = g(z)\gamma_{xz}^s \\ \gamma_{yz} = g(z)\gamma_{yz}^s \\ \varepsilon_z = 0 \end{cases} \quad (6)$$

where:

$$\begin{cases} \varepsilon_x^0 = \frac{\partial u_0}{\partial x} & k_x^b = -\frac{\partial^2 w_b}{\partial x^2} & k_x^s = -\frac{\partial^2 w_s}{\partial x^2}, \\ \varepsilon_y^0 = \frac{\partial v_0}{\partial y} & k_y^b = -\frac{\partial^2 w_b}{\partial y^2} & k_y^s = -\frac{\partial^2 w_s}{\partial y^2}, \\ \gamma_{xy}^0 = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} & k_{xy}^b = -2\frac{\partial^2 w_b}{\partial x \partial y} & k_{xy}^s = -2\frac{\partial^2 w_s}{\partial x \partial y} \\ \gamma_{yz}^s = \frac{\partial w_s}{\partial y} & \gamma_{xz}^s = \frac{\partial w_s}{\partial x} & \\ g(z) = 1 - f'(z) & \text{et} & f'(z) = \frac{df(z)}{dz} \end{cases} \tag{7}$$

By using the principle of virtual displacements. The principle of virtual work is presented in the form:

$$\int_{-h/2}^{h/2} \int_{\Omega} [\sigma_X \delta\varepsilon_X + \sigma_y \delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}]\, d\Omega dz - \int_{-h/2}^{h/2} \int_{\Omega} [\rho \ddot{U}\delta U + \ddot{V}\delta V + \ddot{W}\delta W]\, d\Omega dz \tag{8}$$

Using the integral by part and after simplification, the equilibrium equations associated with the present formulation for the nonlocal plate:

$$\begin{cases} \delta_u: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_1 \ddot{u} - I_2 \frac{\partial \ddot{w}_b}{\partial x} - I_4 \frac{\partial \ddot{w}_s}{\partial x} \\ \delta_v: \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_1 \ddot{v} - I_2 \frac{\partial \ddot{w}_b}{\partial y} - I_4 \frac{\partial \ddot{w}_s}{\partial y} \\ \delta w_b: \frac{\partial^2 M_x^b}{\partial\ x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial\ y^2} = I_1(\ddot{w}_b + \ddot{w}_s) + I_2\left(\frac{\partial \ddot{u}}{\partial x} + \frac{\partial \ddot{v}}{\partial y}\right) - I_3\left(\frac{\partial^2 \ddot{w}_b}{\partial x^2} + \frac{\partial^2 \ddot{w}_b}{\partial y^2}\right) + I_5\left(\frac{\partial^2 \ddot{w}_s}{\partial x^2} + \frac{\partial^2 \ddot{w}_s}{\partial y^2}\right) \\ \delta w_s: \frac{\partial^2 M_x^s}{\partial\ x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial\ y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} = I_1(\ddot{w}_b + \ddot{w}_s) + I_4\left(\frac{\partial \ddot{u}}{\partial x} + \frac{\partial \ddot{v}}{\partial y}\right) - I_5\left(\frac{\partial^2 \ddot{w}_b}{\partial x^2} + \frac{\partial^2 \ddot{w}_b}{\partial y^2}\right) + I_6\left(\frac{\partial^2 \ddot{w}_s}{\partial x^2} + \frac{\partial^2 \ddot{w}_s}{\partial y^2}\right) \end{cases} \tag{9}$$

where:

$$(N_x, N_y, N_{xy}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) dz$$

$$(M_x^b, M_y^b, M_{xy}^b) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) z dz$$

$$(M_x^s, M_y^s, M_{xy}^s) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) f dz$$

$$(S_{xz}^s, S_{yz}^s) = \int_{-h/2}^{h/2} (\tau_{xz}, \tau_{yz}) g dz$$

3. Analytical solutions for vibration problems non-local plates

The solution that checks the equation of equilibrium is in form

$$\begin{Bmatrix} u_o \\ v_o \\ w_b \\ w_s \end{Bmatrix} = \begin{Bmatrix} U\, cos(\lambda x)\, sin(\beta y).\, e^{i\omega t} \\ V\, sin(\lambda x)\, cos(\beta y).\, e^{i\omega t} \\ W_b\, sin(\lambda x)\, sin(\beta y).\, e^{i\omega t} \\ W_s\, sin(\lambda x)\, sin(\beta y).\, e^{i\omega t} \end{Bmatrix} \tag{10}$$

4. Numerical results and discussion

The material properties used in the present study are as follows [25]:

E_m = 70 *GPa*, ρ_m = 2 707 kg/m^3 for aluminum
E_c = 380 *GPa*, ρ_c = 3 800 kg/m^3 for alumina
$\nu_c = \nu_m$ = 0.3 kg/m^3 for aluminum
where:
E, ν and ρ are respectively Young's modulus, Poisson's ratio, and plate density.

The parameters of the foundation are given in the dimensionless form as:

To cite this article: CHIKH Abdelbaki, Dynamic Behavior Analysis of Functionally Graded Nanoplates Based On Elastic Foundations, Nature & Technology Journal, 11 (2) (2019): 07-11. https://www.asjp.cerist.dz/en/Articles/47

$K_w = \frac{k_w a^4}{D_c}$; $K_p = \frac{k_p a^2}{D_c}$;

$D_c = \frac{Eh^3}{12(1-\nu^2)}$ and $\bar{\omega} = \omega a^2 \left(\frac{\rho_c h^3}{D_c}\right)^{0.5}$

with D_c the Flexural Rigidity and $\bar{\omega}$ the non-dimensional frequency

The Winkler stiffness constant k_w is defined by the stiffness of the linear springs. Distinct from the Winkler model, there is an additional shear layer in the Pasternak model, which is characterized by the Pasternak stiffness constants k_p. The influence of the rigidity of the foundation is presented in tabular form and graphically by taking the two parameters of the foundation different from zero ($k_w \neq 0$ and $k_p \neq 0$).

Table 1:
Comparison of free vibration $\bar{\omega}$ of a simply supported homogeneous square nanoplate ($a/h = 1, P = 0,\ \mu = 0{,}5\ , k_p = 10$) resting on Pasternak's elastic foundations.

m	n	a/h	k_w	Sobhy [26]	Present
1	1	100	100	2.6551	2.6553
			500	3.3400	3.3401
		10	200	2.7842	2.7988
			1000	3.9806	3.9901
2	1	100	100	5.5718	5.5731
			500	5.9287	5.9300
		10	200	5.3051	5.4050
			1000	6.0085	6.0947
2	2	100	100	8.5405	8.5441
			500	8.7775	8.7810
		10	200	7.7311	8.6039
			1000	8.2237	9.0683

(m, n) represents the vibration modes

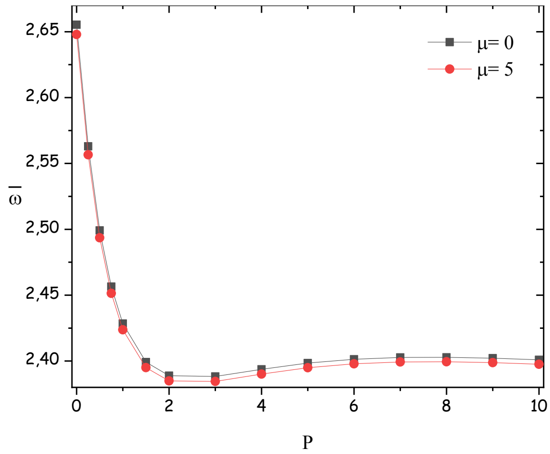

Figure2.The effect of P on the non-dimensional fundamental frequency of (SSSS) square nanoplate resting on elastic foundations. $a/h = 100, k_w = 100, k_p = 10$

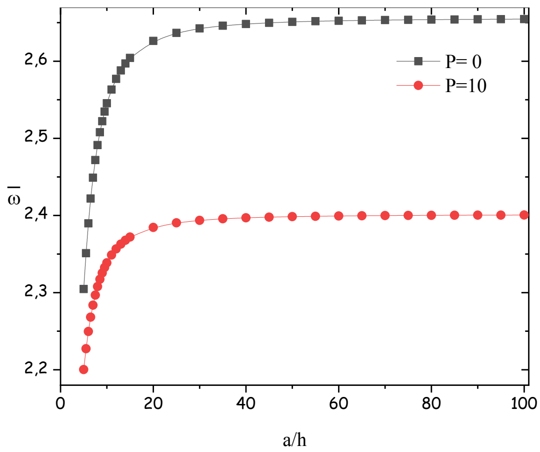

Figure3.The effect of thickness ratio ***a/h*** on the non-dimensional fundamental frequency of (SSSS) square nanoplate resting on elastic foundations. $\mu = 0.5, k_w = 100, k_p = 10$

The numerical comparison of this model and other formulation is presented in table 1. The numerical comparison of the present model and other formulation obtained by Sobhy [23] is presented in table 1.

The results of the dimensionless frequencies obtained by the present formulation for homogeneous isotropic plates (P = 0) resting on elastic foundations of Pasternak are identical to that of Sobhy [23].

Figure 2 shows the variation of the fundamental frequencies with the exponent of the power law for an FGM square nanoplate. It can be observed that the non-dimensional frequency decreases when P increases.

Figure 3 shows the variation of the fundamental frequencies as a function of thickness ratio a/h for an FGM square nanoplate. We can observe that the non-dimensional frequency increases when the ratio a/h

increases in the interval zero to the twenty and almost constant from a/h = 20.

3. Conclusions

In this study, a nonlocal elasticity model for the free vibration of FGM nanoplates on elastic foundations was developed using a high order shear deformation theory. The results obtained show that the frequency values decrease for each increase of (P) and the frequencies of the plates increase considerably if passing from a thick plate to a thin plate.

References

[1] Abdelbaki C., Tounsi A., Adda Bedia E-A,Thermal Stability Of FGM Rectangular Plates Using A New Hyperbolic Shear Deformation Theory, J Mater. Process. Environ., 5 (1) (2017): 34–45. https://www.asjp.cerist.dz/en/article/25822. Accessed 15 June 2018

[2] Barati MR, Zenkour AM., Analysis of postbuckling behavior of general higher-order functionally graded nanoplates with geometrical imperfection considering porosity distributions, Mech. Adv. Mater. Struct., 26 (12) (2018): 1081-1088. https://doi.org/10.1080/15376494.2018.1430280.

[3] Chikh A, Tounsi A, Hebali H and Mahmoud Hassan S.R., Thermal buckling analysis of cross-ply laminated plates using a simplified HSDT. Smart Struct. Syst. 19 (3) (2017): 289–297. https://doi.org/10.12989/sss.2017.19.3.289 .

[4] Chikh A., Bakora A., Heireche H., Houari Mohammed S-A., Tounsi A., Adda Bedia E-A.,Thermo-mechanical postbuckling of symmetric S-FGM plates resting on Pasternak elastic foundations using hyperbolic shear deformation theory. Structural Engineering and Mechanics, 57 (4) (2016): 617-639. https://doi.org/10.12989/sem.2016.57.4.617

[5] Ebrahimi F, Barati M.R., Size-dependent thermally affected wave propagation analysis in nonlocal strain gradient functionally graded nanoplates via a quasi-3D plate theory, Proc. Inst. Mech. Eng. Part C, J. Mech. Eng. Sci., 232 (2018): 162–173. https://doi.org/10.1177%2F0954406216674243

[6] Bellifa H, Bakora A, Tounsi A, *et al.* An efficient and simple four variable refined plate theory for buckling analysis of functionally graded plates, Steel and Composite Struct., 25 (3) (2017): 257–270. https://doi.org/10.12989/scs.2017.25.3.257.

[7] Frankland S.J.V., Harik V.M., Odegard G.M., Brenner D.W., Gates T.S., The stress-strain behavior of polymer-nanotube composites from molecular dynamics simulation, Compos. Sci. Technol., 63 (2003): 1655–61. https://doi.org/10.1016/S0266-3538(03)00059-9

[8] Fujii M, Zhang X, Xie H, *et al.*, Measuring the thermal conductivity of a single carbon nanotube, Phys. Rev. Lett., 95 (6) (2005): 065502. https://doi.org/10.1103/PhysRevLett.95.065502

[9] Karami B, Janghorban M, Li L., On guided wave propagation in fully clamped porous functionally graded nanoplates. Acta Astronaut., 143 (2018): 380–390. https://doi.org/10.1016/j.actaastro.2017.12.011. Erratum: https://doi.org/10.1016/j.actaastro.2017.12.042

[10] Karami B, Shahsavari D, Li L., Temperature-dependent flexural wave propagation in nanoplate-type porous heterogenous material subjected to in-plane magnetic field, J. Therm. Stress, 41 (4) (2018): 483–99. https://doi.org/10.1080/01495739.2017.1393781

[11] Karličić D, Adhikari S, Murmu T, Cajić M., Exact closed-form solution for non-local vibration and biaxial buckling of bonded multi-nanoplate system, Composites, Part B, Eng., 66 (2014): 328–39. https://doi.org/10.1016/j.compositesb.2014.05.029

[12] Lin M.X., Lai H.Y., Chen C.K., Analysis of nonlocal nonlinear behavior of graphene sheet circular nanoplate actuators subject to uniform hydrostatic pressure, Microsyst. Technol., 24 (2) (2018): 919–28. https://doi.org/10.1007/s00542-017-3406-9.

[13] Lu P., Dynamic analysis of axially prestressed micro/nanobeam structures based on nonlocal beam theory, J. Appl. Phys., 101 (7) (2007): 073504 (1-10). https://doi.org/10.1063/1.2717140.

[14] Murmu T, Adhikari S., Nonlocal elasticity based vibration of initially pre-stressed coupled nanobeam systems, Eur. J. Mech. A/Solids, 34 (2012):52–62. https://doi.org/10.1016/j.euromechsol.2011.11.010.

[15] Murmu T, Adhikari S., Nonlocal vibration of bonded double-nanoplate-systems, Compososites Part B Eng., 42 (7) (2011): 1901–1911. https://doi.org/10.1016/j.compositesb.2011.06.009

[16] Murmu T, Sienz J, Adhikari S, Arnold C., Nonlocal buckling of double-nanoplate-systems under biaxial compression. Compososites, Part B Eng., 44 (1) (2013): 84–94. https://doi.org/10.1016/j.compositesb.2012.07.053

[17] Shahverdi H, Barati M.R., Vibration analysis of porous functionally graded nanoplates. Int. J. Eng. Sci., 120 (2017): 82–99. https://doi.org/10.1016/j.ijengsci.2017.06.008

[18] Zenkour A.M., Sobhy M., Nonlocal piezo-hygrothermal analysis for vibration characteristics of a piezoelectric Kelvin–Voigt viscoelastic nanoplate embedded in a viscoelastic medium, Acta Mech., 229 (2018): 3–19. https://doi.org/10.1007/s00707-017-1920-6

[19] Zhan J, Bando Y, Hu J., Kurashima K., Fabrication of ZnO nanoplate-nanorod junctions, Small, 2 (2006): 62–5. https://doi.org/10.1002/smll.200500219

[20] Zhao J, Wang F, Zhang X, *et al.* Vibration Damping of Carbon Nanotube Assembly Materials. Adv Eng Mater 2018;20. https://doi.org/10.1002/adem.201700647

[21] Guenfoud H., 2019, « *Specials finite elements modelisation of structures with functionally graded materials*» (In french). May 8, 1945 University of Guelma, (Thesis University May 8, 1945 Guelma, Algeria. http://dspace.univ-guelma.dz:8080/xmlui/handle/123. Accessed August 15, 2019

[22] Guenfoud H, Ziou H, Himeur M, et Guenfoud M., Analyses of a composite functionally graded material beam with a new transverse shear, Journal of Applied Engineering Science & Technology, 2 (2) (2016): 105-113. http://revues.univ-biskra.dz/index.php/jaest/article/view/1898. Accessed June 10, 2019

[23] Mechab I, Mechab I., Benaissa S, Ameri M., Serier B., Probabilistic analysis of effect of the porosities in functionally graded material nanoplate resting on Winkler – Pasternak elastic foundations, Appl. Math. Model, 40 (2) (2016): 738-749. https://doi.org/10.1016/j.apm.2015.09.093

[24] Mechab I., Mechab B., Benaissa S., Serier B., Bouiadjra B., Free vibration analysis of FGM nanoplate with porosities resting on Winkler Pasternak elastic foundations based on two ‑ variable refined plate theories, J. Brazilian Soc. Mech. Sci. Eng., 38 (8) (2016): 2193-2211. https://doi.org/10.1007/s40430-015-0482-6

[25] Sobhy M., Buckling and free vibration of exponentially graded sandwich plates resting on elastic foundations under various boundary conditions, Compos. Struct., 99 (2013): 76–87. https://doi.org/10.1016/j.compstruct.2012.11.018