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Numerical Modeling of an Elastic Spherical Contact under Combined Normal and Tangential Loading

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Abstract

The Contact between two surfaces with normal and tangential force involve friction dissipation phenomenon .The friction phenomenon can be formulated as a constitutive relation in a similar form to that of the elasto-plastic constitutive equation of materials, the present paper studies of sliding and slip rules of elastic frictional contact by using the formalism of plasticity, for regularized passage from stick to sliding in contact of solids. It purpose is to present new approach for regularized constitutive model for interface contact elastic with friction, this model is implemented infinite element code ABAQUS by user subroutine Vfric.

Keywords: Slip rules, frictional contact, sub-loading friction model, multi-surface plasticity

1. Introduction

Frictional contact between surfaces is an area of intense research, with several applications in mechanics. Many phenomena occur in the interface, and modeling of them is quite difficult. The difficulty in analyzing contact friction problems and poor understanding of frictional phenomena ,a lot of studies investigate the describing friction phenomena in contact solids, in most of case interface friction is modeled using The classic friction model incorporate the penalty parameters representing fictious spring between contact surface Constitutive equations of friction within the framework of elasto-plasticity were formulated first as rigid-plasticity by Michalowski and mroz , Subsequently, they were extended to elasto-perfect-plasticity by curnier, Wriggers, Giannakopoulos and others authors which incorporates the penalty parameters representing fictitious spring between contact surface, However, these equations fall within the framework of conventional elasto-plasticity, which assumes a friction-yield surface enclosing a purely elastic domain, kiruchi developed a subloading friction model describing the smooth transition from static to kinetic friction. In the present paper we shall discuss an improvement the stick slips rules at elastic plastic frictional contact.

2. Analytical formulation of elastic frictional contact

Hertz considered the normal contact of two spheres 1 and *2*, as shown in figure1, For sphere 1 we consider R_1 to be the radius, υ_1 the Poisson's ratio, and E_1 the Young's modulus. Similarly, R_2, υ_2, E_2 are the properties of sphere *2*. We define for the contact the equivalent elastic modulus E and the equivalent contact curvature R as:

$$\frac{1}{E} = \frac{1-\upsilon_1^2}{E_1} + \frac{1-\upsilon_2^2}{E_2} \tag{1}$$

and

$$\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \tag{2}$$

According to Hertz theory for the elastic contact of two spheres in the normal direction, the radius of the circular contact area a is expressed as

$$a = (KNR)^{\frac{1}{3}} \tag{3}$$

with $\quad K = \dfrac{3(1-\upsilon^2)}{4E}$

The distribution of normal tractions at the contact plane is described by the relation:

$$p(r) = \frac{3N}{2\pi a^3}(a^2 - r^2)^{\frac{1}{2}} \qquad (4)$$

The approach of the two sphere centers due to normal loading N, can be calculated using

$$\alpha = 2\left(\frac{KN}{\sqrt{R}}\right)^{2/3} \qquad (5)$$

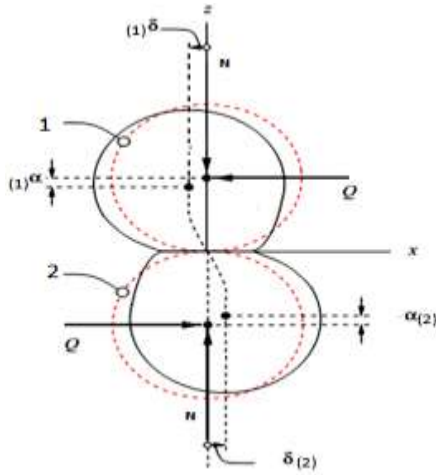Figure 1: Two spheres in contact and subjected to normal and tangential forces.

Application of the tangential force Q under constant normal force induces development of slip with a circle domain expanding from the outer contact boundary r = a, the tangential traction distribution in the contact zone is:

$$\tau = \frac{3\mu N}{2\pi a^3}(a^2 - r^2)^{\frac{1}{2}} c \ \leq r \leq a$$

$$\hspace{6cm} (6)$$

$$\tau = \frac{3\mu N}{2\pi a^3}\left\{(a^2 - r^2)^{\frac{1}{2}} - (c^2 - r^2)^{\frac{1}{2}}\right\} r \leq c$$

The tangential displacement of the center of spheres is expressed as follows:

$$\delta = \frac{3(2-\upsilon)\mu N}{8Ga}\left(1 - \frac{c^2}{a^2}\right) = \frac{3(2-\upsilon)\mu N}{8Ga}\left[1 - \left(1 - \frac{Q}{\mu N}\right)^{2/3}\right] \qquad (7)$$

where μ is coefficient of friction.

3. Application Model of Friction to FEM

3.1. Classical friction law

General FE contact modelling is often based on a master-slave approach, where the nodes on the slave surface are not allowed to penetrate the segments of the master surface, a variational formulation of the contact between two deformable bodies constitutes a good basis for the development of a constitutive law of friction including an impenetrability condition. To this end consider two bodies, one called the master and the other the slave, bound to contact one another within a surface A characterized by the unit outward normal n to the target .The gap g_n represents the distance of contact separating each point from master and slave body , the contact condition is given by

$$g_n \geq 0 \qquad f_n \geq 0 \qquad f_n \cdot g_n = 0 \qquad (8)$$

Coulomb's friction law states that friction force is proportional to the normal force through the global following relation

$$Q \leq \mu. N \qquad (9)$$

The contact zone may be divided into two parts (Figure 2), sticking and slipping parts are defined according to the relation between the tangential friction and normal stress as

$q < \mu. f_n$ $\qquad$ Sticking and $\qquad u_t = 0$

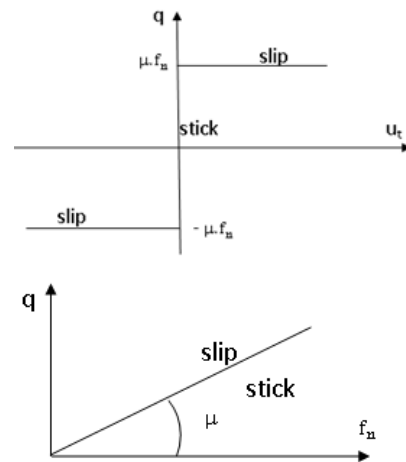$q \geq \mu. f_n$ $\qquad$ Slipping and $\qquad u_t = -\gamma. \tau \qquad (10)$

Figure 2: Representation of the coulomb friction

ABAQUS used for numerical analyses, Coulomb friction model , by assumed that no relative motion occurs if the equivalent frictional stress $q_{eq} = \sqrt{q_1^2 + q_2^2}$ is less than the critical stress($\mu.f_n$), where q_1 and q_2 are frictional stresses in the two orthogonal directions on a contact surface, and μ and f_n are the friction coefficient and the contact pressure, respectively. Slip can occur if $q_{eq} = \mu.f_n$ of the slip and the frictional stress q_j coincide like that:

$$\frac{q_j}{q_{eq}} = \frac{v_j}{v_{eq}} \tag{11}$$

where v_j slip velocity in direction j, and $v_{eq} = \sqrt{v_1 + v_2}$ is the magnitude of the slip velocity. In friction formulation with Lagrange multiplier, tangential behavior model, Lagrange multipliers are used to enforce exact sticking conditions. The rate of virtual work with a constraint term enforced with Lagrange multipliers can be written for a contact surface. For the stick condition, where k0 and **u**j are a reference stiffness internally selected by ABAQUS and a tangential slip in direction j, respectively:

$$d\delta\pi_c = \int_s (k_0\delta u_j du_j + \delta u_j dq_j + k_0\delta q_j du_j + \tau_j d\delta u_j)dS \tag{12}$$

for the slip condition, where n_j and n_k are the normalized slip directions, t is the time step, and is the δ_{jk} Kronecker delta. The slip/ stick status of an element is updated in ABAQUS as follows: if an element is currently in the stick condition and satisfies, then it is updated to the slip condition. If an element is currently in the slip condition and satisfies at the end of the iteration, then it is updated to the stick condition.

3-2 formulation of regularized friction law

Another possibility to formulate tangential constitutive equations in the contact interface is given by a regularization of the stick-slip behavior. Such a formulation is used to avoid the non-differentiability of Coulomb's law at the onset of sliding.

The interface relative displacement increment may be represented by means of additive decomposition

$$\Delta u = \Delta u_n.\vec{n} + \Delta u_t.\vec{t}$$

$$\Delta u_t = \Delta u^e + \Delta u^s \tag{13}$$

The elastic part is given:

$$\Delta f_n = K_n.\Delta u_n$$

$$\Delta q = K_t.\Delta u_t \tag{14}$$

where Δf_n and Δq are the normal and tangential increment components of traction applied of contact area, And $\Delta u = \bar{v}.\Delta t$ is the relative total increment displacement, with normal component $\Delta u_n = \bar{v}_n.\Delta t$ and $\Delta u_i.t_i = \Delta u - \Delta u_n.n$, i=1,2 in local basis $< n, t_1, t_2 >$.

Relative velocity between the counter (slave) bodies to the main (master) body $\bar{v} = v_2 - v_1$

The contact stress vector f acting on the main body orthogonally splits into the normal traction vector f_n and the tangential traction vector q

$$\vec{f} = \vec{f_n} + \vec{q} = -f_n.n + q.t$$

$$f_n = (n.f)n = (n\otimes n)f = -f_n.n \tag{15}$$

$$q = f - f_n = (I - n\otimes n)f = q.t$$

with $t_f = \frac{q}{\|q\|}.$

The elastic relations are given by:

$$\dot{q} = -K_t.\bar{v}_t^e$$

and

$$\dot{f_n} = -K_n.\bar{v}_n^e$$

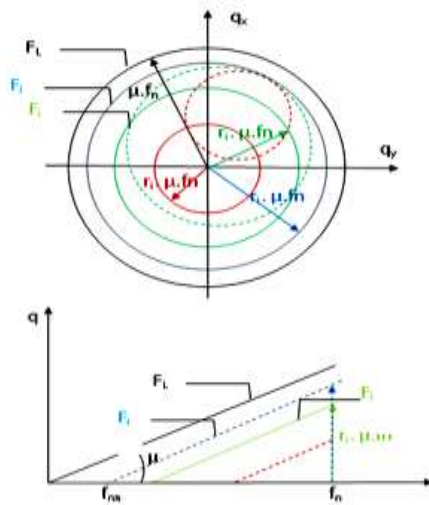where K_n, K_t are normal and tangential stiffness in adherence at contact surface.

Figure 3: Elastic friction contact for active loading surface and limit surface

Interface slipping initiates when loading of the tangential stress, for an elastic friction contact problem (Figure 3), the sliding rules were associated with limit friction surface $F_L(q_1, q_2, q_n) = 0$. The transition from adherence condition to the slipping condition were generated by active loading surface F_i, moving inside the interior domain bounded by limit surface F_L and given respectively by this equations :

$$F_i(q, \alpha, r) = q_e - \mu \cdot f_n \cdot r_i \tag{11}$$

with

$$q_e = \sqrt{(q_x - \alpha_x)^2 + (q_y - \alpha_y)^2}$$

where q_e is the equivalent relative tangential traction α_x and α_y are locates the centre of the surface in the traction space. The stress states corresponding to the interior of the yield surface correspond to elastic response, where $0 \leq r \leq 1$ is the size of the slipping condition for active loading surface (Figure 4).
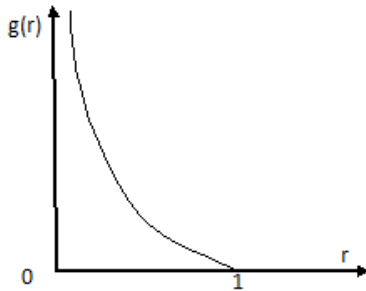

Figure 4: Evolution rule of loading

The evolution rule of active surface loading ratio can be assumed that the sliding ratio increases with sliding velocity ,the plastic sliding process be formulated as

$$\dot{r} = g(r) \cdot \|v^s\| \quad \text{for} \quad v^s \neq 0 \tag{12}$$

where $g(r)$ is a monotonically decreased function of fulfilling the following conditions

$$g = +\infty \qquad \text{for} \quad r = 0$$

$$g = 0 \qquad \text{for} \quad r = 1 \tag{13}$$

$$g < 0 \qquad \text{for} \quad r > 1$$

Let the function $g(r)$ satisfying Eq. (13) be simply given by

$$g(r) = -u \cdot \cot\left(\frac{\pi}{2} r\right) \tag{14}$$

Analytically integrated in the case of a monotonic sliding process as:

$$r = \frac{\pi}{2} \cos^{-1}\left\{\cos\left(\frac{\pi}{2} r_0\right) \exp\left[-\frac{\pi}{2} u\left((u^s - u_0^s)\right)\right]\right\} \tag{15}$$

where $u^s = \int \|v^s\| \, dt$ is the accumulated plastic sliding displacement, and r_0 and u_0^s are the initial values of r and u^s ,respectively. The relationships of contact traction rate and sliding velocity is based on the elasto-plastic theory, we assume the following sliding flow rule.

$$v_i^s = \lambda \frac{\partial F}{\partial q_i} = \lambda \cdot t \tag{16}$$

with $\|t_f\| = 1$, where λ is a positive proportionality factor.

Let us assume the actual slipping surface with isotropic cinematic hardening describes similar way as incremental in the theory of multi surface plasticity given by equation (11), and the consistence condition leads to

$$\frac{\partial F}{\partial \|q\|} t \cdot \dot{q} = \frac{\partial F}{\partial \|N\|} n \cdot \dot{N} + \dot{R}_i F \tag{18}$$

$$dF = \frac{\partial F}{\partial q_1} dq_1 + \frac{\partial F}{\partial q_2} dq_2 + \frac{\partial F}{\partial R_i} dR_i \tag{19}$$

$$\frac{\partial F}{\partial q_i} = \frac{q_i}{q_{eq}} = n_i \tag{20}$$

Substituting this equations give the proportionality factor λ derived as follows:

$$\lambda = \frac{\frac{\partial F}{\partial \|q_e\|} . t . \dot{q} - f_n . R_i - \dot{f_n}}{\mu . U(R_i)} \tag{21}$$
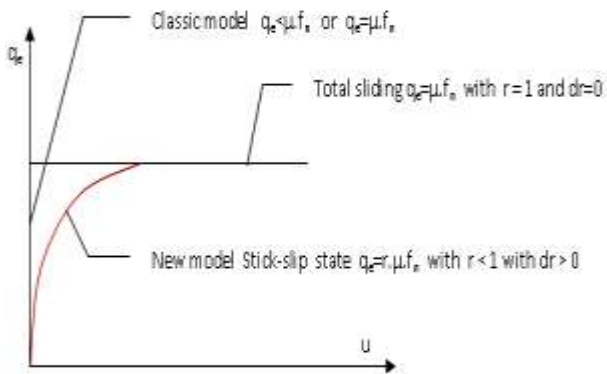

Figure 5 : Relationship traction versus tangential displacement

The integration procedure under consideration, falls within the category of return mapping algorithms and follows in a straightforward manner from the theory of operator splitting applied to elasto-plastic type of constitutive relations:

1 – Elastic predictor computation

$$\|q_e^{tr}\|_{n+1} = \|q_e\|_n + K_t . \Delta u$$

2 – Verification of slip /stick condition

If $\|q_e^{tr}\|_{n+1} \leq r_n$ sticking occurs, otherwise slipping occurs. Slip corrector is given by:

$$q_{n+1} = q_{n+1}^{tr} - K_t . \lambda \frac{\partial F}{\partial q}$$

4. Result of finite element model

In order to validate the model, an application on the case of a sphere-plane contact, the schematic diagram of the contact between a deformable block and a deformable hemisphere of a radius R, under combined normal and tangential loads. The loading process is described as follows: First, an initial normal preload N is applied to the top of sphere and leads to an initial interference, a tangential force Q is added to the top of sphere keeping the interference constant, study non-linear FE code

ABAQUS is used to analyze stress distributions with a quasi-static explicit scheme. The finite element model used in simulations is shown in figure 6 .The radius of the sphere is 10 mm, with elastic material property of steel with Young's modulus (E = 210 kN/mm^2), Poisson's ratio (ν = 0.3).
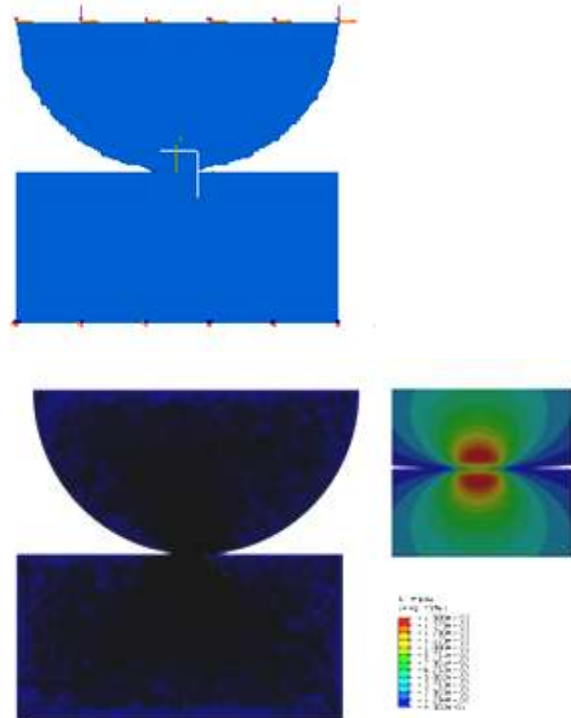

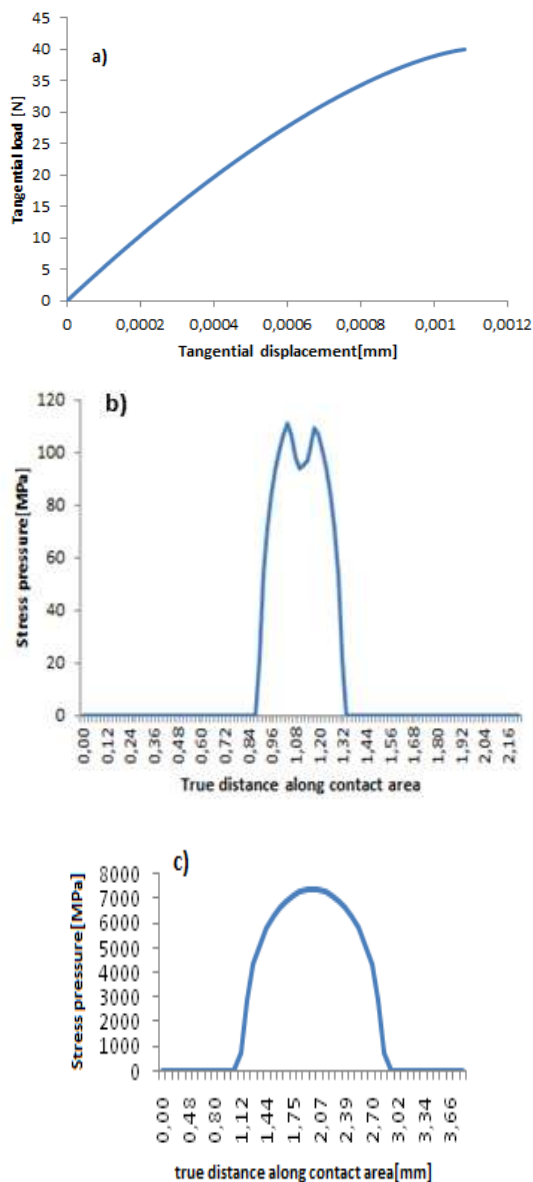Figure 6 : FEM of an elastic sphere in contact with a elastic block under combined normal and tangential loading

Figure 7 : a) Relationship tangential vs. displacement. b) Shear traction c) Pressure distribution

like that continuity and differentiation equation between normal and tangential force.

A finite element analysis program, is capable of implementing an arbitrary friction constitutive equation via user-subroutines VFRIC, implemented in ABAQUS. The curves for shear stress and load displacement given in figure 7 produces by using the present model were the same as given found in work of Nelias [3].

3. Conclusion

A constitutive model for the description of friction phenomena is formulated by incorporating the new approach model with multi-surface concept, and the following improvements are attained by the present model

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