Frequency analysis of the hydrological drought regime. Case of oued Mina catchment in western of Algeria

A. Yahiaoui [1], B. Touaïbia [2], C. Bouvier [3]

(1) Chargé de Cours. Département Hydraulique. Université de Bechar. B.P. 417 Bechar. Algérie.
email : yahiaoui_halim@yahoo.com

(2) Professeur. Ecole Nationale Supérieure de l'Hydraulique Abdellah Arbaoui. B.P. 31 Blida. Algérie.
email: touaibia@yahoo.fr

(3) Directeur de Recherche IRD, HydroSciences Montpellier, BP 64501, F-34394 Montpellier Cedex 5. France.
email: bouvier@msem.univ-montp2.fr

ABSTRACT. *Hydrological drought is associated with the effects of periods of precipitation shortfalls on surface (stream-flow level) or subsurface water supply (ground-water level). The frequency and severity of hydrological drought is often defined on a catchement. As an important random phenomenon in hydrology, the frequency analysis is necessary in the aim to know about the drought's regime. In order to study the stream flow regime of oued Mina catchment in western of Algeria. Frequency analysis of the drought stream flow is based on analysis of the deficit volume and the corresponding duration, where the basic data is obtained from defined a specific threshold as considered an index of the drought regime. Representative sample of stream flow, allow doing the frequency analysis with the probability distributions: Weibull, Generalized Pareto and Log-Normal for the PDS which will be combined to the probability of the occurrence in threshold level method.*

Keywords: Drought, Estimation, Distribution, Threshold, Return Period, PDS.
.

1. Introduction

Drought is a major natural hazard having severe consequences in regions all over the world. The range of drought impacts is related to drought occurring in different stages of the hydrological cycle and usually different types of droughts are distinguished. The origin is a meteorological drought, which is defined as a deficit in precipitation. A meteorological drought can develop into a soil moisture drought, which may reduce agricultural production and increase the probability of forest fires. It can further develop into a hydrological drought defined as a deficit in surface water and groundwater, e.g. reducing water supply for drinking water, irrigation, industrial needs and hydropower production, causing death of fish and hampering navigation in some countries.

A general definition of drought is given by Tallaksen and van Lanen (2004), who define drought as "a sustained and regionally extensive occurrence of below average natural water availability". This definition relative to normal implies that droughts can occur in any hydroclimatological region and at any time of the year. In response to the different impacts of drought in different regions, a large number of quantitative drought characteristics have been developed. Recently published summaries can be found in, e.g. Heim (2002), Hisdal et al. (2004), Smakhtin and Hughes (2004) and Hayes (2005).

Expressed as a single number, drought characteristics are often referred to as drought indices or drought statistics. The choice of a suitable drought characteristic for a specific study depends on the hydroclimatology of the region, the type of drought considered, the vulnerability of society and nature in that region, the purpose of the study and the available data. Due to the lack of a unique standard definition, this choice is subjective and a large number of different characteristics are used to describe and quantify droughts. In case of stream flow drought two main approaches of deriving drought characteristics can generally be distinguished (Hisdal et al., 2004).

One is to analyze low flow characteristics such as a time series of the annual minimum n-day discharge, the mean annual minimum n-day discharge or a percentile from the flow duration curve (FDC). These characteristics describe the low flow part of the regime and characterize droughts according to their magnitude expressed through the discharge (Tallaksen et al., 1997). The development in time of a drought event is not considered. In the second approach, discharge series are viewed as a time dependent process, and the task is to identify the complete drought event from its first day to the last. In this way a series of drought events can be derived from the discharge series, and droughts can be described and quantified by several properties, such as drought duration or deficit volume. These so-called deficit characteristics are commonly derived by the threshold level method.

In this study the threshold level method is evaluated for its applicability to daily discharge series for streams in different climate zones and with different hydrological regimes. The methods are applied to a global data set of daily discharge series from a wide range of flow regimes including perennial as well as intermittent streams. Stream flow deficit characteristics, such as deficit volume and duration are derived for all series and the methods are evaluated based on the following criteria:

The evaluation focuses on within-year droughts time series with respect to the discharge series. A frequency analysis of PDS of drought deficit characteristics is conducted, focusing on the choice of extreme value distributions.

The frequency analysis is ported on the catchment of oued Mina, which is situated in the west of Algeria, it is characterize by It's surface of 4285 ckm, and the station where obtaining all the flow data is called oued El Abtal

2. Flow duration curve

The flow duration curve (FDC) plots the empirical cumulative frequency of stream flow as a function of the percentage of time that the stream flow is equaled or exceeded. The curve is constructed by ranking the data, and for each value the frequency of exceedance is computed using a probability plotting position formula. The empirical FDC for oued Mina is shown in Figure 2.
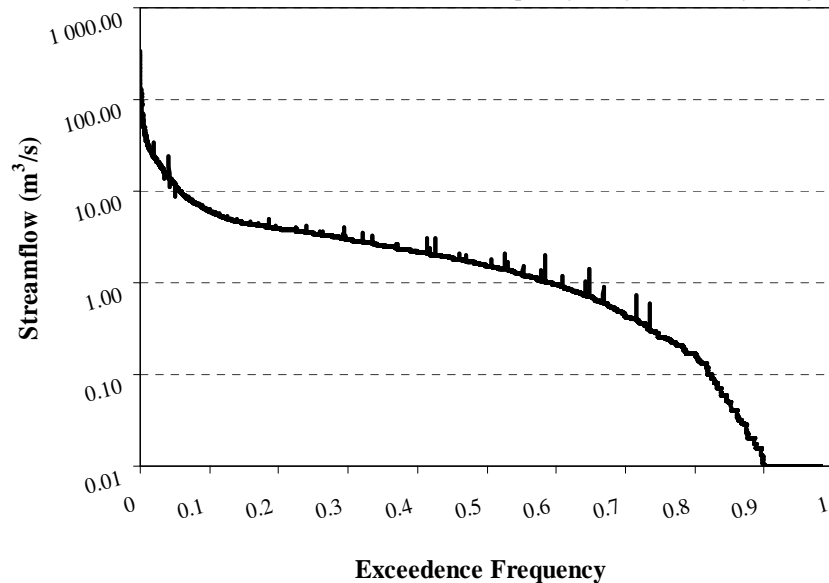
Figure 1. Flow Duration Curve FDC of oued Mina catchment.

Traditionally, low flow indices are obtained from FDCs based on the total period of record. From the FDC. Threshold levels are chosen to represent the range of commonly used. Q90 and Q70, are used for perennial streams. Based on the experience form previous studies (Tallaksen et al., 1997; Hisdal et al., 2002; Engeland et al., 2004) threshold levels in the range between Q90 and Q70 for perennial streams are considered reasonable also for an extreme value analysis of droughts.

In the case of the oued Mina catchment the following indices can be obtained from the graph (Fig. 1) : Q70 = 0.45 cms, Q80 = 0.09 cms and Q90 = 0.01 cms. Although the Q70 will be considered as the threshold level in this study.

3. Threshold level method

The threshold level method originates from the theory of runs introduced by Yevjevich (1967), who originally defined droughts as periods during which the water supply does not meet the current water demand. Both the water supply, S(t), as well as the water demand, D(t), are expressed as time series, and a drought event is defined as an uninterrupted sequence of negative values in the supply-minus-demand series, Y(t)=S(t)−D(t). Later, Yevjevich (1983) simplified the concept by applying a constant demand. The demand is represented by a threshold level, Qt, and droughts are defined as periods during which the discharge is below the threshold level. Common deficit characteristics are the start of the drought, ti , drought duration, di , deficit volume or severity, vi and the minimum flow occurring during the drought event, Qmin , as illustrated in Fig. 1. Additional deficit characteristics can be defined, such as drought intensity, which is the ratio of deficit volume and duration, and recovery time. The latter is defined, e.g. by Correia et al. (1987) as the time it takes to compensate a certain fraction of the deficit volume by excesses of water above a certain recovery level (Fleig et al., 2006).

In general, the threshold level can either represent a certain water demand, for example for power plants or water supply, or the boundary between normal and unusually low stream flow conditions. The threshold level might be fixed or varying over the year to reflect, e.g. seasonally different water demands. However, not all periods with relatively low flow compared to a varying threshold are considered a drought, such as relative low flow periods due to a delayed onset of the snowmelt flood. Stahl (2001) and Hisdal et al. (2004) therefore used

the terms stream flow deficiency or anomaly when defining deficit periods (periods with discharge below the threshold level) using a varying threshold level.

When the threshold level is set to represent the boundary between normal and unusually low stream flow, it is chosen based on the characteristics of the stream flow regime. In this case low flow indices, such as percentiles from the flow duration curve (FDC), are frequently applied for both perennial and intermittent streams. For regional studies these were found to give more consistent results than percentages of the mean (Tallaksen et al., 1997). Also linear combinations of the mean flow and the standard deviation have been applied for regional studies (e.g. Ben-Zvi, 1987). The choice of threshold level influences both the number of events and the presence of multi-year droughts in the derived drought series. When focus is, as in this study, on within-year droughts neither a large amount of multi-year droughts nor a large number of years without any droughts should be included in the series as these can complicate an extreme value analysis (Tallaksen et al., 1997). The threshold level has to be chosen as a compromise between these two features. For short data series the use of very low threshold levels can be problematic, as the derivation of statistical properties of droughts requires a certain minimum number of events.

These considerations do not reveal a single preferable threshold level, and its selection, and hence the definition of drought, remains a subjective decision. For perennial streams threshold levels between the 70-percentile flow (Q70) and the 95-percentile flow (Q95) from the FDC are frequently applied, which are the flows that are exceeded 70–95 percent of the time. For intermittent streams lower exceedance percentiles have to be chosen, depending on the percentage of zero flow. The threshold level method was developed for discharge series with a time resolution of one month or longer, but it has also been applied to daily discharge series, e.g. Zelenhasić and Salvai (1987) and Tallaksen et al. (1997). When the time resolution is short in comparison with the droughts to be studied two problems have to be considered in particular: the occurrence of minor droughts and mutually dependent droughts (Fig. 2). Minor droughts are events of short duration and small deficit volume. A high number of minor droughts in the sample may disturb an extreme value analysis and the number of minor droughts should thus be reduced. Mutually dependent drought events can occur during a prolonged period of low discharge when short excess periods with discharge above the threshold level divide the period of low discharge into several drought events. When the excess periods are of short duration, , and small excess volume, if, one would generally consider the whole period of low discharge to be one drought event. Short excess periods can be caused by short rainfall events or artificial influences. The split drought events are called mutually dependent droughts. They cannot be considered independent of one another, and e.g. for an extreme value analysis it is recommended to combine these into larger independent events.

This can be done by so called pooling procedures, of which three common ones are described in details in the next section. In a regional study pooling is further recommended due to differences in catchment responses. For example in a slowly responding groundwater-fed catchment a short rainfall event during a prolonged dry period will lead to a much smaller rise in stream flow as compared to a fast responding neighboring catchment. As a result, a drought might be split in one catchment but not in the other one.
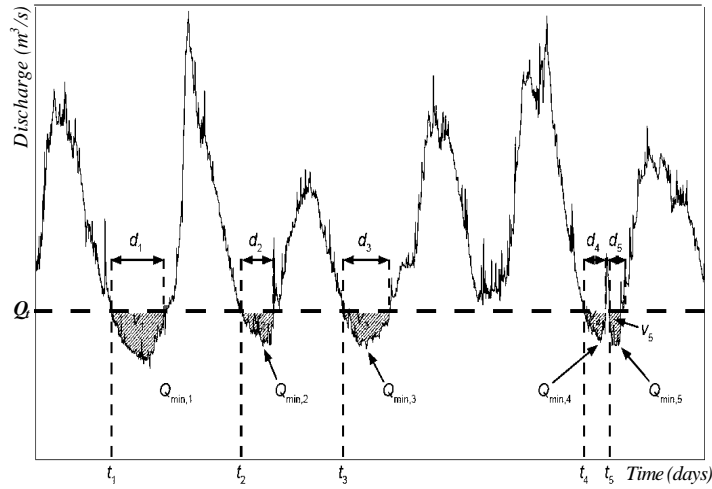
Figure 2. Possible ways of deriving drought characteristics (Hisdal et al., 2004).

So, the following table (Tab. 1) gives the statistical characteristics of the deficit volume and the corresponding stream flow duration samples of oued Mina catchment as the threshold Q70 considered.

	Deficit (cms.day)	Duration (days)
Mean	2.70	18.76
Median	1.37	8.00
Standard Deviation	4.04	26.49
Sample Variance	16.32	701.63
Kurtosis	20.93	9.51
Skewness	3.85	2.52
Range	30.49	147.00
Minimum	0.27	2.00
Maximum	30.76	149.00

Table 1. Statistical characteristics.

4.Frequency analysis

Frequency analysis can be conducted in classical manner for deficit volume characteristics as well as for duration deficit characteristics, where the largest events are of interest, but, when there are insufficient data (below 30 years) like in this study, which we contain just 19 years of observed stream flow drought under the considered threshold (Fig. 3 & 4), we must to use another analysis based on the occurrence of events.
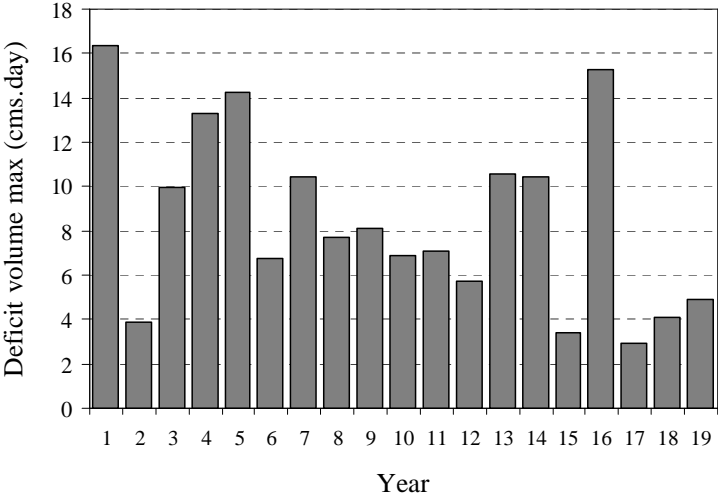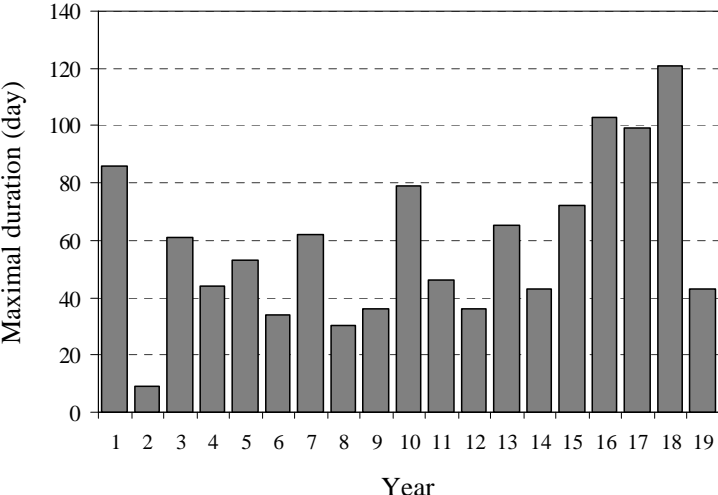
Figure 3. Largest deficit volume.

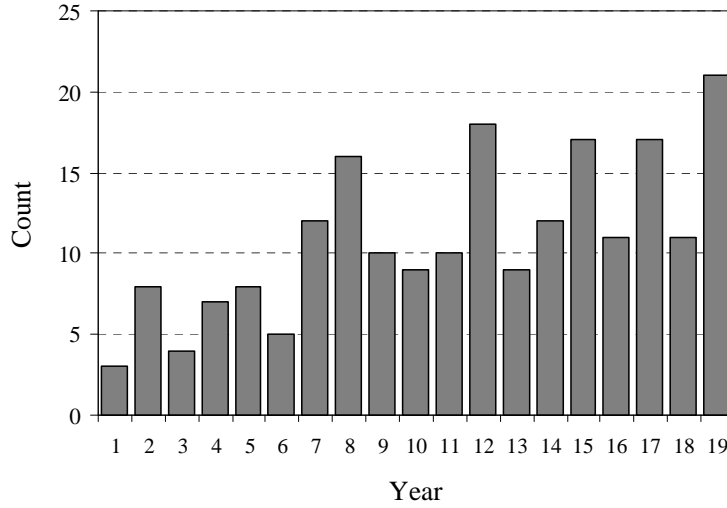Figure 4. Maximum stream flow duration.

Figure 5. Number of events under the threshold level for each year.

So for the predefined upper threshold, the number of exceedances becomes a random variable and the location parameter can be set equal to u (Fig. 5) . When the drought events are selected by the threshold level method. Zelenhasić & Salvai (1987) suggested to derive the cumulative distribution function of the largest stream flow drought occurring in a given time interval from a PDS of drought events. The method works on daily discharge data for drought events lasting less than one year and characterizes droughts either in terms of their deficit volume or their duration. As such the method consists of two parts. The first one is to estimate the probability of the number of events occurring during the chosen time interval. The second part is to estimate the distribution function of the chosen deficit characteristic of all drought events occurring in the chosen time interval. From that Zelenhasić & Salvai (1987) calculated the distribution function of the largest drought event in the following way:

$$F(x) = \sum_{k \geq 0} G^k(x) P(N = k)$$

Where:

$F(x)$ distribution function of the largest drought event,

$G(x)$ distribution function of all drought events,

$P(N = k)$ the probability that k drought events occur during the time, it can be expressed by :

$$P(N = k) = \frac{(\lambda t)^k}{k!} \exp(-\lambda t)$$

Where t is the total number of time blocks or years in the record (equals 19 for the case of oued Mina). The parameter λ of Poisson equals the expected number of exceedances in each year and is estimated as :

$$\widehat{\lambda} = \frac{Mean \ of \ events \ under \ threshold}{t}$$

In case of oued Mina catchment $\widehat{\lambda} = 0.58$

For estimation of the T_p-year event, a probability distribution G is fitted to the PDS of the exceedance series $(x_i - u)_{i=1,\dots,n}$ and an estimate given by :

$$\widehat{x}_{T_p} = u + G^{-1}\left(1 - \frac{1}{\widehat{\lambda} t T_p}\right)$$

Annual exceedance probabilities can be estimated from the PDS provided the average number of events per year. As λ is larger than the upper limit considered u (Stedinger et al., 1993), so the probability $F(x)$ of the largest drought event in the year can be expressed as:

$$F(x) = \exp\left(-\lambda t(1 - G(x))\right)$$

The annual exceedance probability is correspondingly $1 - F(\widehat{x}_T)$ so:

$$\frac{1}{T} = 1 - F(\widehat{x}_T) = \exp\left(-\lambda t(1 - G(\widehat{x}_T))\right)$$

or,

$$G(\widehat{x}_T) = 1 - \frac{1}{\lambda t T_p}$$

Then,

$$\frac{1}{T} = 1 - \exp\left(-\frac{1}{T_p}\right)$$

The average return period for x_T in the PDS can be obtained by solving for T_p:

$$T_p = -\frac{1}{\ln\left(1 - \frac{1}{T}\right)}$$

The expression of $F(x)$ can be transformed as:

$$F(x) = \left(\exp\left[-\left(1 - G(x)\right)\right]\right)^{\lambda t}$$

or $0 \leq 1 - G(x) \leq 1$ so: $\exp\left[-\left(1 - G(x)\right)\right]$ can be approached by $G(x)$

then:

$$F(x) = \left(G(x)\right)^{\lambda t}$$

4.1. Case of Pareto distribution

The Pareto distribution (GP), is given by :

$$G(x) = 1 - \left[1 - \kappa \frac{x}{\alpha} \right]^{1/\kappa}$$

where α a scale parameter and κ a shape parameter.

For given samples of the deficit volume and the duration, the parameters estimates by the methods of moments conduct to :

- For deficit volume: $\hat{\alpha} = 1.960$ and $\hat{\kappa} = -0.276$,
- For duration: $\hat{\alpha} = 14.086$ and $\hat{\kappa} = -0.249$.

4.2. Case of Weibull distribution

The Weibull distribution with scale α and shape κ parameters, is given by:

$$G(x) = 1 - \exp\left(\frac{x}{\alpha} \right)^{\kappa}$$

For given samples, the parameters estimates by the classical method of moments conducts to:

- For deficit volume: $\hat{\alpha} = 2.199$ and $\hat{\kappa} = 0.722$,
- For duration: $\hat{\alpha} = 15.251$ and $\hat{\kappa} = 0.722$.

4.3. Case of Log-Normal distribution

If X is distributed according to a log-normal distribution, then $Y = \mathrm{Ln}X$ is normally distributed. The parameters ξ and α are the population mean and variance of Y, the probability density function is given as:

$$g(x) = \frac{1}{x\alpha\sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{\ln x - \xi}{\alpha} \right)^2 \right)$$

ξ and α can be estimated by the maximum likelihood from the sample of the logarithmic transformed data $\left(y_i = \ln x_i, \quad i = 1,2,...,n \right)$: So the parameters estimates for the case of oued Mina are:

- For deficit volume: $\hat{\xi} = 0.438$ and $\hat{\alpha} = 0.964$,
- For duration: $\hat{\xi} = 2.222$ and $\hat{\alpha} = 1.155$.

In the below figures (Fig. 6 & 7) an illustration of the comparison between the three types of fitting.
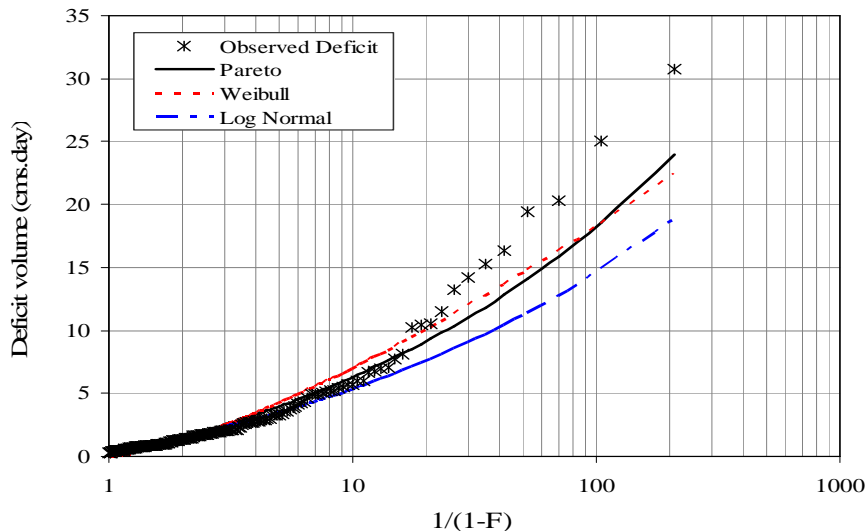
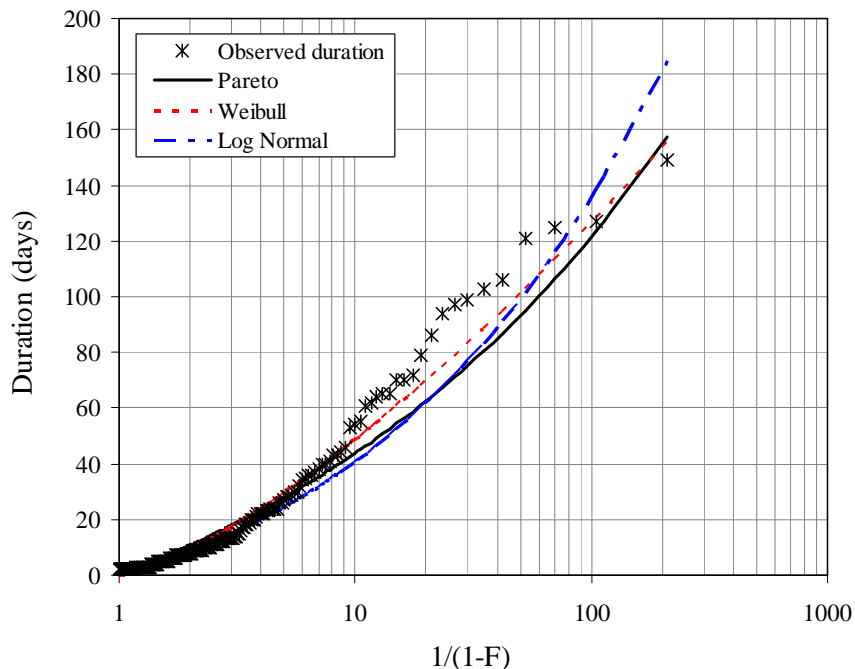Figure 6. Comparison of different fitting of the deficit volume.

Figure 7. Comparison of different fitting of the stream flow drought duration

We can remark from this comparisons, the presence of outliers in the observed samples either in deficit volume or in duration, although all the fittings of the considered probability distributions are practically the same. The following table (Tab. 2) gives the results of the goodness of fit analyzing with Probability Plot Correlation PPC, Root Mean Square Deviation RMSD and the Kolmogorov-Smirnov KS test.

	PPC		RMSD		KS	
	Deficit	Duration	Deficit	Duration	Deficit	Duration
Pareto	0.983	0.984	0.316	0.315	0.155	0.126
Weibull	0.972	0.991	0.424	0.349	0.228	0.201
Log Normal	0.981	0.975	0.167	0.214	0.085	0.096

Table 2. Goodness of fit test results.

From this table, the goodness of fit of the Log Normal distribution is the best. Lets we note $\phi(\ln x, \xi, \alpha)$ the probability distribution function of Normal distribution, with the parameters $\phi(\ln x, \xi, \alpha)$ for the random variable X distributed as Log Normally, so:

$$P(X \le x) = \phi(\ln x, \xi, \alpha) \equiv \phi(\ln x) = \frac{1}{\alpha\sqrt{2\pi}} \int_0^x \frac{1}{v} \exp\left(-\frac{1}{2}\left(\frac{\ln v - \xi}{\alpha}\right)^2\right) dv$$

5. Distribution of the largest deficit and the largest duration

The central part of the analysis of deficits and durations is the distribution function $F(x)$ of the largest deficit and the largest duration. Since the first steps towards this goal are completed, it is now a simple thing to write down the expression for $F(x)$. On the basis of the results obtained, the distribution function of the largest deficit or duration is:

$$F(x) = \left[\phi(\ln x)\right]^{\lambda t} \qquad x > 0$$

For a given T-year the corresponding x_T can be obtained directly from the equation:

$$x_T = \exp\left[\phi^{-1}\left((1 - 1/T)^{1/\lambda t}\right)\right]$$

The following table gives for different return period's quartiles for the corresponding largest deficit volume and largest stream flow drought duration of oued Mina catchments.

T (years)	Deficit (cms.day)	Duration (days)
5	29.87	263
10	40.00	373
20	51.47	504
50	69.22	718
100	84.98	918
500	130.96	1540
1000	155.48	1892

Table 3. Quartiles of the largest stream flow deficit volume and duration.

6. Conclusion

Droughts are natural hazards which can cover large regions and last for long periods of time. This implies that robust drought characteristics applicable in regions with different hydroclimatology and hydrogeology are needed. In this study the threshold level method is evaluated to derive stream flow deficit characteristics from

series with a daily time resolution. The pooling procedures are designed to overcome the problem of mutually dependent droughts. The threshold level method proved to be a suitable method for perennial and intermittent streams and useable both for all-year. It allows defining droughts depending on the purpose of the study as the threshold level can be chosen.

A frequency analysis requires that the events are iid, which in this case is difficult to fulfil. Regional drought studies require a consistent set of drought characteristics that can be applied across the region.

Deficit characteristics derived by the threshold level method proved to give comparable results for different kinds of streams provided that comparable threshold levels are chosen in accordance with the stream flow regimes. This is an advantage when estimates of design events are derived across a larger, often heterogeneous region. It should be emphasised that a methodology suitable for application in large regions, adapting to streams with widely differing flow regimes, would not necessarily imply the best choice for individual sites. In general, the choice of drought definition is a subjective choice that is made based on the purpose of the study, the hydrological regime, the type of drought considered, the demand and vulnerability of nature (and society) in that region and the available data. In addition, there are in most cases subjective elements inherited in the procedures themselves. For the threshold level method these include the choice of time resolution, threshold level, pooling procedure, criteria to exclude minor droughts, and parameters of the pooling criteria and criteria to exclude minor droughts. It was further found that the Generalized Pareto model is a good choice for the distribution of the magnitudes of drought events (PDS of deficit volume and duration) for most streams, thus supporting the theoretical base of extreme value modelling. There are large uncertainties related to fitting distributions based on observations only, in particular in the tail of the distributions. It is therefore recommended to let the choice of distribution function be guided by extreme value theory as this will likely give better predictions of the most extreme events.

It was further found that the Log Normal model is a good choice for the distribution of the magnitudes of drought events (PDS of deficit volume and duration) for most streams, thus supporting the theoretical base of extreme value modeling. There are large uncertainties related to fitting distributions based on observations only.

REFERENCES

Ben-Zvi, A.: Indices of hydrological drought in …, J. Hydrol., 92, 179–191, 1987.

Correia, F. N., Santos, M. A., and Rodrigues, R.: Engineering risk in regional drought studies, in: Engineering, Reliability and Risk in Water Resources, edited by: Duckstein, L. and Plate, E. J.,NATO ASI Series E: Applied Sciences No. 124 Martinus Ninjhoff Publ., Dordrecht, 61–86, 1987.

Engeland, K., Hisdal, H., and Frigessi, A.: Practical extreme value modeling of hydrological floods and droughts: a case study, Extremes, 7, 5–30, 2004.

Fleig, A. K., L. M. Tallaksen, H. Hisdal and S. Demuth. : A global evaluation of streamflow drought characteristics. Hydrol. Earth Syst. Sci., 10, 535–552, 2006.

Hayes, M. J.: Drought Indices, available at: http://www.drought.unl.edu/whatis/indices.htm, 2005.

Heim Jr., R. R.: A review of twentieth-century drought indices used in the United States, BAMS, 83(8), 1149–1165, 2002.

Hisdal, H., Tallaksen, L. M., and Frigessi, A.: Handling nonextreme events in extreme value modelling of streamflow droughts, in: FRIEND 2002 – Regional Hydrology: Bridging the Gap between Research and Practice, IAHS Publ. no. 274, 281–288, 2002.

Hisdal, H., Clausen, B., Gustard, A., Peters, E., and Tallaksen, L. M.: Event Definitions and Indices, in: Hydrological Drought – Processes and Estimation Methods for Streamflow and Groundwater, edited by:

Smakhtin, V. U. and Huges, D. A.: Review, automated estimation and analyses of drought indices in South Asia,Working Paper 83, International Water Management Institute, Colombo, Sri Lanka, 2004.

Stahl, K.: Hydrological Drought – a Study across Europe. PhD Thesis Albert-Ludwigs-Universit¨at Freiburg, Freiburger Schriften zur Hydrologie no. 15, Freiburg, Germany, also available at: http://www.freidok.uni-freiburg.de/volltexte/202, 2001.

Stedinger, J. R., Vogel, R. M., and Foufoula-Georgiou, E.: Frequency analysis of extreme events, in: Handbook of Hydrology, edited by: Maidment, D. R., McGraw-Hill, Inc., New York, NY, 18.1–18.66, 1993.

Tallaksen, L. M. and van Lanen, H. A. J., Developments inWater Science, 48, Elsevier Science B.V., Amsterdam, 139–198, 2004.

Tallaksen, L. M. and van Lanen, H. A. J. (Eds.): HydrologicalDrought – Processes and Estimation Methods for Streamflow and Groundwater, Developments inWater Sciences 48, Elsevier B.V., Amsterdam, 2004.

Tallaksen, L. M., Madsen, H., and Clausen, B.: On the definition and modelling of streamflow drought duration and deficit volume, Hydrol. Sci. J., 42(1), 15–33, 1997.

Yevjevich, V.: An objective approach to definitions and investigations of continental hydrologic droughts, Hydrology Papers 23, Colorado State University, Fort Collins, USA, 1967.

Yevjevich, V.: Methods for determining statistical properties of droughts, in: Coping with droughts, edited by: Yevjevich, V., da Cunha, L., and Vlachos, E., Water Resources Publications, Colorado, 22–43, 1983.

Zelenhasic, E. and Salvai, A.: A Method of Streamflow Drought Analysis, Water Resour. Res., 23(1), 156–168, 1987.