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IMPORTANT NOTICE

Multi-model system design using H∝ Multi-controllers for a robotic wrist

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Abstract: In this paper we present a novel approach of multi-model system design used multi-controllers of H∝ control type applied to control robotic wrist. we have presented modeling of our system(nonlinear and linear model) follows by the principal of our approach of control. we have presented H∝ control principal used for control nonlinear system and linear local models around each operating points choose . Different simulations are realized in matlab-simulink software improve efficiency of our approach. Finally we have presented conclusion and discussed of obtained results followed by some perspectives for future work

Keywords: Modeling, Manipulateur robot, H∝ Control, multi-model approach, simulation.

1. INTRODUCTION

Precise, optimal and robust control of manipulators arm in the face of uncertainties and variations in their environments is a prerequisite to feasible application of robot manipulators to complex handling and assembly problems in industry and space [1]. An important step toward achieving such control can be taken by providing manipulator hands with sensors that provide information about the progress of interactions with the environment. But more important is the lack of adequate controller architectures and computing techniques needed to take advantage of such sensory information, where it available.

Different architectures and techniques are used to control the manipulator arms [1], like multi-controller approach developed by Narandra & balakrishnan [2] base in RST controller or fuzzy controller with frank switching system and fuzzy switching system [3]. Other approach of control used same approach with PID controller, Fractional order PID controller and PSO-PID controller [4]. Other approach in litterateur, used nonlinear controller [5], adaptive controller [6]. The mechanical design of the manipulator arm has an influence on the choice of control type. The physical process (robot arm) behavior has generally many non-linearity [5] that are not taken into account in the modeling process. In the each operating point (equilibrium point) of the physical process we can develop a local linear model. In this work we used multi-model approach [2]. Then the objective of this approach [1] is to control the process in operational space using the local information [2][3]. We have proposed same modification in this approach and the diagram block of the multi-controllers approach modified is represented as follows:
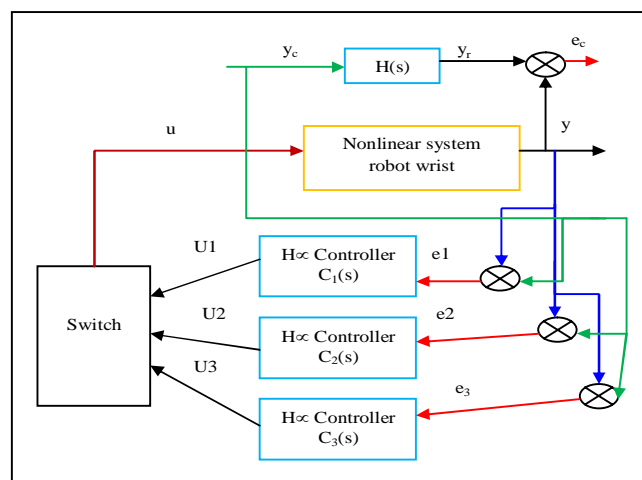

Fig.1. Structure of multi-model approach of control.

In our work, we have chosen the use optimal H∝ controller works in around each operating point with three linear local models around each operating point.

2. PROCESSUS MODELING

The manipulator Stâubli Robot Rx-90 has coupling between axis 5 and 6. The actuators are brushless motors and the engine control uses the rotor position to magnetic flux rotate to achieve desired torque value and generally this motor as a DC motor behave[5-9]. Our process corresponds to a robot wrist (axis 6) can be represented by the following figure:
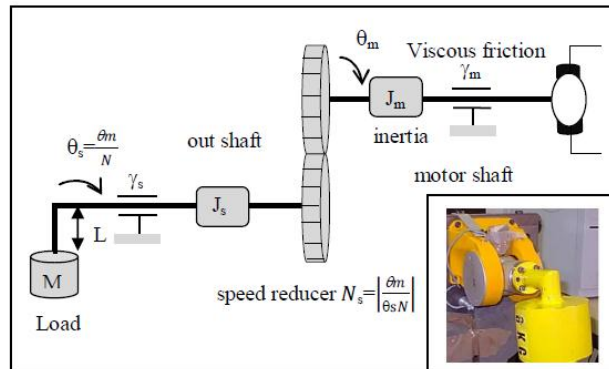

Fig.2.Process(Robot wrist) model.

Mathematic Process dynamic model is given by the following equations:

$$\Gamma_m - \Gamma_s = \left(J_m + \frac{J_S + M.L^2}{N^2}\right) \cdot \ddot{\theta}_m + \left(\gamma_m + \frac{\gamma_S}{N^2}\right) \cdot \dot{\theta}_m \qquad (1)$$

With :

$$J_T = \left(J_m + \frac{J_S + M.L^2}{N^2}\right) \text{ and } \gamma_T = \left(\gamma_m + \frac{\gamma_S}{N^2}\right) \qquad (2)$$

Jm, Js: Inertia moment applied in the motor shaft and the output shaft (output shaft with mass) respectively.

γm,γs: Viscous friction applied in the motor shaft and the output shaft respectively.

The motor torque is given by:

$$\Gamma_m = K_e \cdot u(t) \qquad (3)$$

Ke : is the torque constant and u(t)the voltage applied in process. To find the linear structure of local parametric models, we applied the tangent linearization methods and the linear local model is as follows [11]:

$$G(s) = \frac{-K_p}{s^2 + a_{p1} \cdot s + a_{p2}} \qquad (4)$$

After identification of the linear system near operating point θs0=0 [10]. The corresponding continuous linear model is as follows:
operating points, θs0=0 :

$$G_1(s) = \frac{-111.5}{s^2 + 11.25 \cdot s + 79.14} \qquad (5)$$

operating points, $\theta_{s0}=\pi/3$ and $\theta_{s0}=2\pi/3$ respectively:

$$G_2(s) = \frac{-111.5}{s^2 + 11.25 \cdot s + 39.57}; \qquad G_3(s) = \frac{-111.5}{s^2 + 11.25 \cdot s - 39.57} \qquad (6)$$

Reference model is :

$$H(p) = \frac{\gamma^2}{s^2 + \lambda_1 \cdot s + \lambda_0} \qquad (7)$$

With: $\gamma=10$; $\lambda_0=\gamma^2$; $\lambda_1=2.\gamma$

3. OPTIMAL CONTROL WITH H∝

Several representations can be used for control problems of closed loop systems, such as H∞ and H2 optimization problems. It is therefore practical to have recourse to a general formulation, in order to have a "standard problem" for this type of controls. The configuration of the closed loop system with the various specifications (weighting functions) is shown in Figure (3).
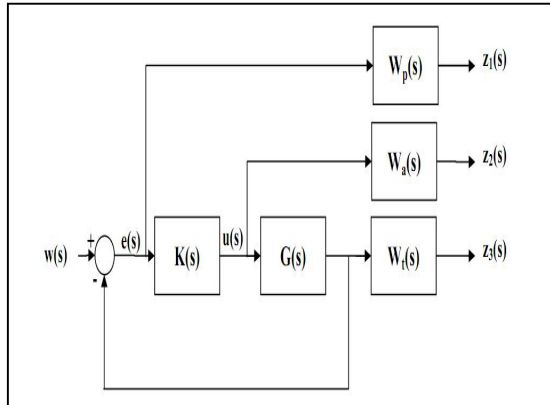

Fig.3. Problem formulation Standard

Where: Wt(s): transfer matrix of the stability specification.

 Wa(s): transfer matrix relating to the additive error.

Wp(s): matrix for transferring the performance specification.

Note: In the following, we are only interested in the case where the uncertainties are of unstructured type.

The general configuration of the standard problem [12-18] is presented in Figure (4) (LFT, Linear Fractional Transformations representation [12-18]).
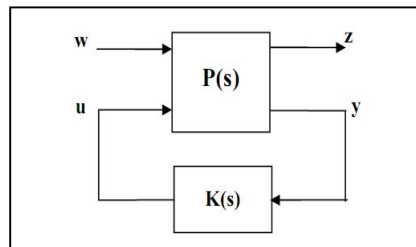

Fig.4. Standard problem (LFT representation)

Where: u: system commands (dimension "m")

w: perturbed inputs (dimension "l")

y: measurements on the system (outputs) (dimension "q")

z: controlled outputs (dimension "p")

x: state vector (dimension "n")

The solution of the standard problem (generalized mixed sensitivity problem) is found by finding a control law u - delivered by a controller K(s) - such that: u = K(s).y minimizing the influence of the perturbation signal w on the output signal z, namely:

$$\left\| \begin{bmatrix} W_p S \\ W_a R \\ W_t T \end{bmatrix} \right\|_{\infty} < 1 \qquad (8)$$

T(s): Complementary Sensitivity defined by

$$T(s) = L(s)(I + L(s))^{-1} \qquad (9)$$

L(s): is the Open loop L(s) = G(s) K(s)
R(s): Transfer to Control defined by

$$R(s) = K(s)(I + L(s))^{-1} \tag{10}$$

S(s): Sensitivity defined by:

$$S(s) = (I + L(s))^{-1} \tag{11}$$

The different matrices are enclosed in a single system, called the augmented plant P(s). It is defined by the following equations of state ([5], [12]):

$$\begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = C_1 x + D_{12} u \\ y = C_2 x + D_{21} w \end{cases} \tag{12}$$

The advantage of using these state equations is that we have a complete knowledge of the system and the weighting functions (Wt(s), Wa(s) and Wp(s)). In the form of a LFT representation:

$$P(s) = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \tag{13}$$

In the form of a transfer matrix:

$$P(s) = \begin{bmatrix} W_p & -W_p G \\ 0 & W_a \\ 0 & W_t G \\ I & -G \end{bmatrix} \tag{14}$$

We associate with the standard problem the following cost function Tzw:

$$T_{zw}(s) = P_{11}(s) + P_{12}(s)K(s) + [I - P_{22}(s)K(s)]^{-1} P_{21}(s) \tag{15}$$

$$With: P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}$$

From where: $\quad z(s) = T_{zw}(s)w(s) \tag{16}$

In the following we are interested in the problem H∞ based on Riccati equations resolution ([4], [6]). The solution of the H∞ problem is based on the verification of the following hypotheses [11-17]:
(H1) : The pair (A, B2) is stabilizable and the pair (A, C2) is detectable.
(H2) - D12 and D21 : are of full rank.

$$(H_3) - rank \begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} = n + m \tag{17}$$

$$(H_4) - rank \begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = n + q \tag{18}$$

We will illustrate the steps for obtaining the K(s) controller by solving the problem H∞. The problem of optimization by H∞ is to find a controller K(s) stabilizing the process, so as to minimize the transfer between the inputs w and the outputs z, namely:

$$\|T_{zw}(j\omega)\|_\infty = \max_\omega \bar{\sigma}(T_{zw}(j\omega)) \tag{19}$$

To obtain the structure of the controller K(s), we are interested in the problem H∞ "suboptimal", where we try to reduce the norm H∞ below a positive threshold γ. For the standard problem of figure (4) defined by equations (2) to (6) and verifying the hypotheses (H1) to (H4), there exists a controller K(s) which ensures internal stability [6] such that:

$$\|T_{zw}(j\omega)\|_\infty \leq \gamma \quad for \ \gamma > 0 \tag{20}$$

If and only if ([12-18]):

$$H\infty \in dom(Ric) \ and \ X\infty = Ric(H\infty) \geq 0 \tag{21}$$

$$J\infty \in dom(Ric) \ and \ Y\infty = Ric(J\infty) \geq 0 \tag{22}$$

$$max \ |\lambda(X\infty \ Y\infty)| < \gamma 2 \tag{23}$$

Such that: X∞ and Y∞ are the solutions of the Hamiltonians below:

$$\mathbf{H}_\infty \colon \begin{bmatrix} \mathbf{A} & \gamma^{-2}\mathbf{B_1B_1'} - \mathbf{B_2B_2'} \\ -\mathbf{C_1'C_1} & -\mathbf{A'} \end{bmatrix} \tag{24}$$

$$\mathbf{J}_\infty \colon \begin{bmatrix} \mathbf{A'} & \gamma^{-2}\mathbf{C_1'C_1} - \mathbf{C_2'C_2} \\ -\mathbf{B_1B_1'} & -\mathbf{A} \end{bmatrix} \tag{25}$$

And their corresponding Riccati equations below:

$$\mathbf{A'X + XA + C_1'C_1 + X(\gamma^{-2}B_1B_1' - B_2B_2')X = 0} \tag{26}$$

$$\mathbf{AY + YA' + B_1B_1' + Y(\gamma^{-2}C_1'C_1 - C_2'C_2)Y = 0} \tag{27}$$

In this case, the controller K(s) satisfying the condition: $\|Tzw(j\omega)\|\infty \leq \gamma$ Is expressed as the following LFT representation: K(s) = Fl (M∞ , Q) with:

$$M_\infty = \begin{bmatrix} A_\infty & -Z_\infty B_1 & Z_\infty B_2 \\ F_\infty & \mathbf{0} & I \\ -C_2 & I & \mathbf{0} \end{bmatrix} \tag{28}$$

With :

$$\begin{cases} A_\infty = A + \gamma^{-2}B_1B_1'X_\infty + B_2F_\infty + Z_\infty L_\infty C_2 \\ F_\infty = -B_2X_\infty \\ L_\infty = -Y_\infty C_2' \\ Z_\infty = (I - \gamma^{-2}Y_\infty X_\infty)^{-1} \end{cases} \tag{29}$$

Q(s) is any stable transfer function of norm H∞ less than γ, namely: $\|Q\|\infty < \gamma$. A special case is the central controller, it is obtained if: Q(s) = 0. The central controller K(s) is then written in this way:

$$K(s) = \begin{bmatrix} A_\infty & -Z_\infty L_\infty \\ F_\infty & \mathbf{0} \end{bmatrix} \tag{30}$$

$$K(s) = -Z_\infty L_\infty (sI - A_\infty)^{-1} F_\infty \tag{31}$$

The mixed sensitivity problem is a special case of the standard H∞ problem. It consists in finding a robust controller K(s) capable of maintaining the closed-loop stability and of ensuring the required performances ([14]) such that:

$$\|T_{zw}(j\omega)\|_\infty = \left\| \begin{bmatrix} W_pS \\ W_tT \end{bmatrix} \right\|_\infty < 1 \tag{32}$$

Several necessary criteria must be ensured in closed-loop systems control: attenuation and rejection of disturbances, limitation of the energy delivered to the system, and of course robustness [8]. By including the sensitivity S (s) in the synthesis, this will result in the attenuation of the effect of the perturbations, while the complementary sensitivity T(s) will have the pursuit problem of the output z at the input w [16]. The association of the sensitivity function S(s) will give rise to a controller which ensures closed-loop stability and attenuates the resonance peaks on the maximum singular value of the sensitivity S(s) [18][19]. In this case, the standard H∞ problem becomes:
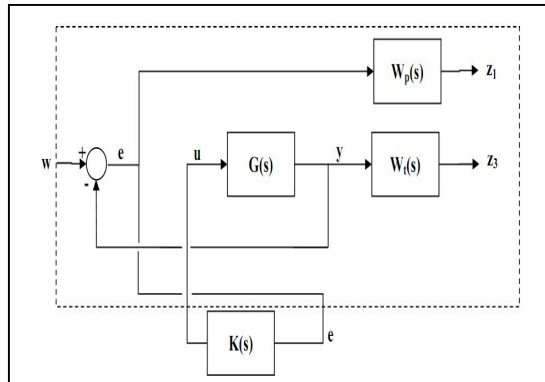
Fig.5. Mixed sensitivity problem in standard form

The solution to the problem of optimization by H∞ previously stated will be realized by the iteration on the parameter γ and the optimal robust controller K(s) will have to satisfy the condition: ||Tzw(jω)||∞ ≤ γ . Thus, the parameter γ will satisfy the compromise "Stability / Performance".

We presented the problem H∞ with the steps for the determination of the robust controllers. All these calculation steps can be considered long before obtaining controller structure, because they must be carried out for each value of the parameter γ. It is therefore preferable to use a calculation algorithm, which will make it possible to obtain the robust controller in a faster and more precise manner. A computational algorithm for the determination of the robust controller is presented by:

1. *Choice of specifications Wt, Wp and Wa.*

2. *Realization of the augmented plant P(s).*

3. *Take γ = 1, synthesize controller $H_∞$.*

4. *Calculation of the cost function Tzw.*

5. *If $||T_{zw}(jω)||_∞$ ≤ γ go to 7.*

6. *Otherwise adjust γ and go to 2.*

7. *Evaluation of frequency and temporal results.*

8. *If the results are satisfactory go to 10.*

9. *Otherwise adjust γ and go to 1.*

10. *End.*

Fig.6. Algorithm of H∝ controller

Thanks to this algorithm, it will be faster to arrive at the controllers' structure K(s), in addition to having the possibility to refine the results of the synthesis with adjustment parameter γ.

The implementation of the controller will be obtained by software of MATLAB via Robust Control.

4. SIMULATION

The object of this simulation is the illustration of H∝ controller efficiency and the stability of closed loop control. The simulation is done in continuous time around the following operating points θs0=0rad, θs0=π/3rad and θs0=2π/3rad.

The parameter of H∞ controller :

a=W=10 : Acceleration parameter

τ=11.65e-3 : System time constant in open loop

Wc1 = 20.1e1

$\mathbf{w_1} = (0.6)*(\tau*s+1)/(\tau*s+5e-6)$: Performances specification

W2 = [];

$\mathbf{w_3} = .85 * inv((0.5e-6.s+1)/((\frac{1}{wc1}).s+1))$: Stability specification

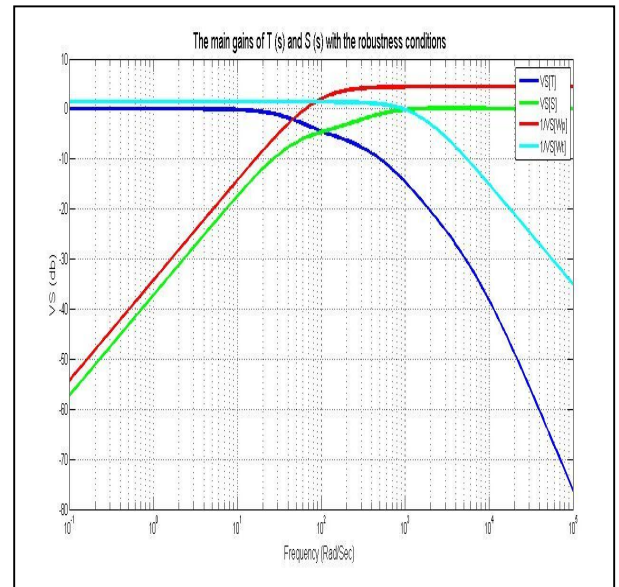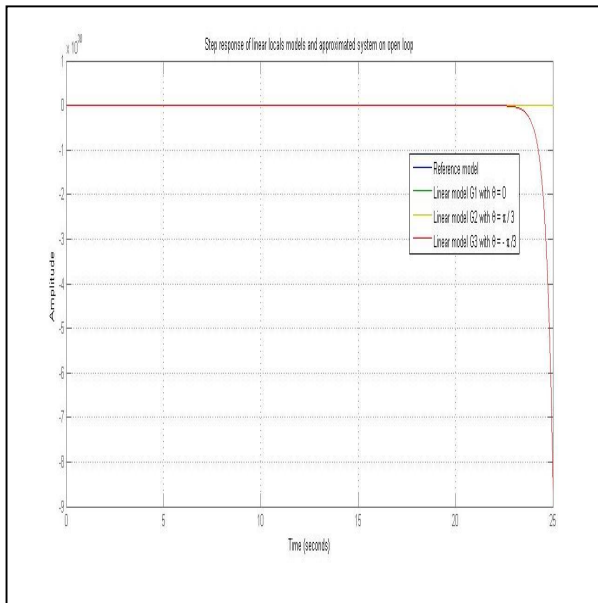The reference signal :

$$r(t) = 0.1 \cdot sin(5t) \tag{33}$$

Fig.7.step response of linear local models and approximated system

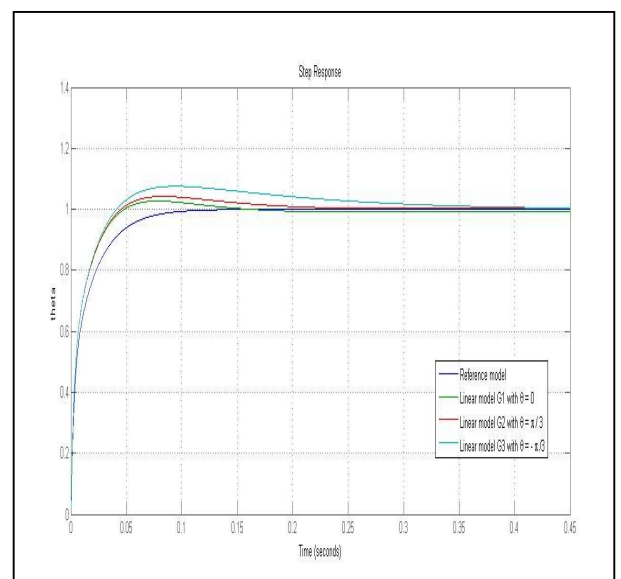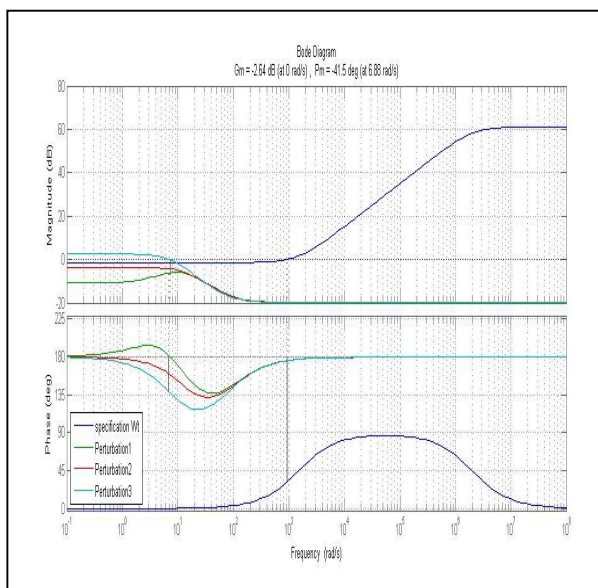Fig.8. The main gain of T and S with robustness condtion

Fig.9.Bode diagram of perturbation.

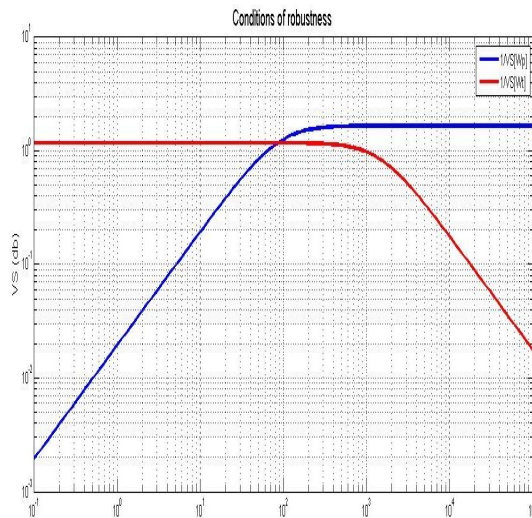Fig.10. Step response of local linear models on
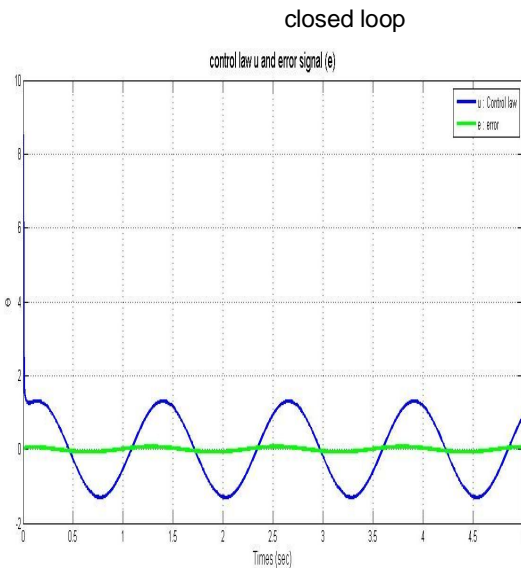
Fig.11. Condition of robustness
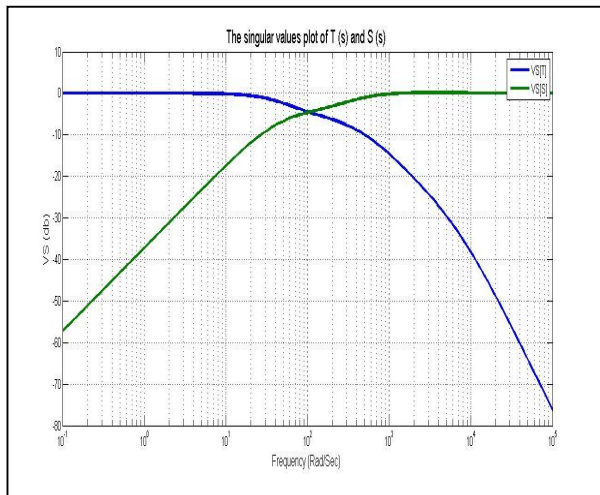

Fig.12. Control.

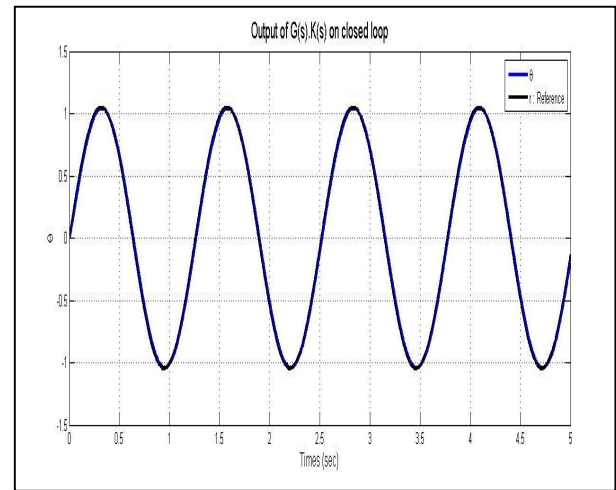Fig.13. Singular value plot of T and S

Fig.14. Output nonlinear system with Sinusoidal input.
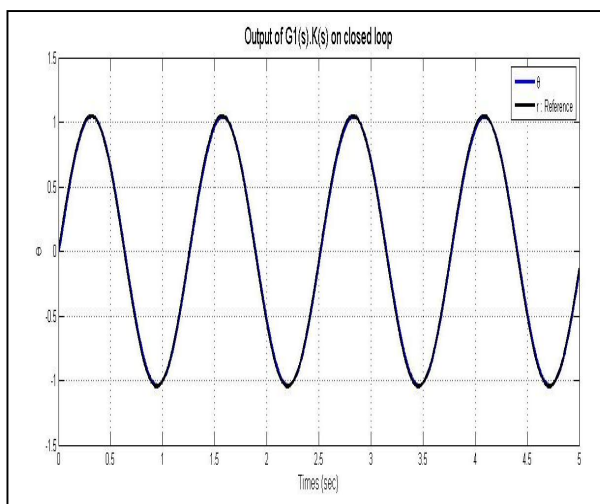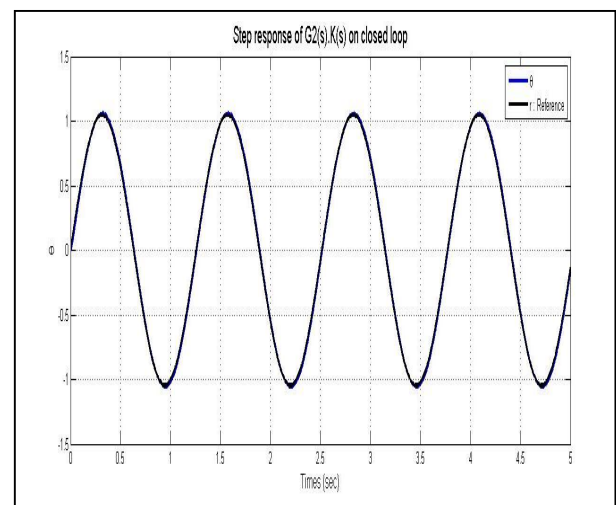

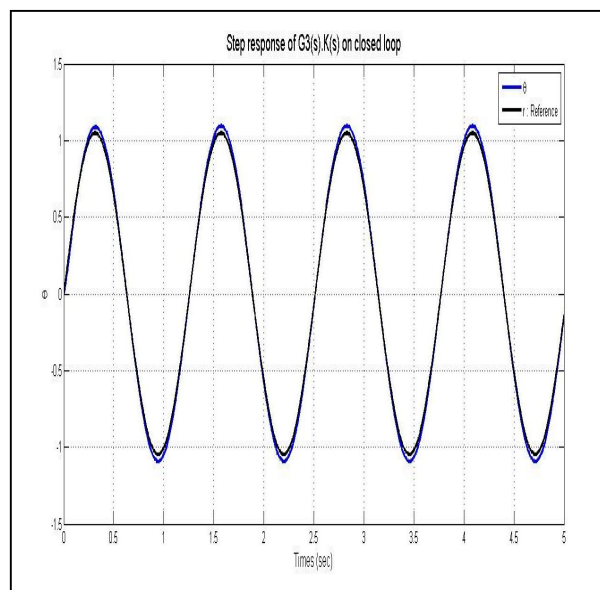. Fig.15. Output linear local model G1 and G2 with Sinusoidal input

Fig.16. Output linear local model G3 with Sinusoidal input.

We can observe with obtained results illustrated in Fig.7 at Fig.16, the controller H∞ can be powerfully control nonlinear system and all locals' linear models in the same time. We can observed too the high robustness and precision of our .controller

5. CONCLUSION

In this work, we have presented the modeling of nonlinear process (robotics wrist of RX90 Stâubli Robot). After that the local linear model near each considered operating points has been calculated.

We have described the H∞ controller with our new design control of multi-control approach. Simulation we noted that the obtained results approve the high robustness and precision of our controllers and design control approach. The results obtained allow concluding that we can control nonlinear system with robust local controllers and these controllers give good results in local linear model obtained around each operating points and nonlinear system. Finally we will study at the future work other robust control approach with optimization with algorithm inspired in biologic like (PSO, GA,..).

References

[1]. J. Balakrishnan and S. Narendra, "Improving Transient Response of Adaptive Control Systems Using Multiple Models and Switching". IEEE Trans. on Automatic Control, Vol. 39, n°9, Septembre 1994, pp. 1861-1866.

[2]. J. Balakrishnan and S. Narendra, "Adaptive Control using MultipleModels". IEEE Trans. on Automatic Control, Vol. 42, n°2, Février1997, pp. 171-187.

[3]. J. Chebassier, "Méthadologies pour la conception d'un système de comma nde par calculateur". Thèse Laboratoire d'Automatique de grenoble (INPG) , 1999.

[4]. F. Merrikh-Bayat, A. Jamshidi. Comparing the Performance of Optimal PID and Optimal Fractional-Order PID Controllers Applied to the Nonlinear Boost Converter. Dec 31 2013 math.oc arXiv:1312.7517v1, pp.6.

[5]. J-M. Duchamp, "Commutation Floue de lois de Commande applique à la Robotique". Rapport de DEA , LAMII /CESALP, 1998.

[6]. J.J.E. Slotine, "Applied Nonlinear Control". Prentice-HallInternational, 1991, ISBN: 0-13-040049.

[7]. A. Karimi and I-D. Landau, "Robust Adaptive Control of a Flexible Transmi ssion System Using Multiple Models". Laboratoire d'Automatique de Grenoble (CNRS-INPG-UJF). Design-CSD,1998.

[8]. O. Pagès, P. Mouille and B. Caron, "Two approaches of the multi- model control. Real time Implementation for a wrist of a Robot". Mechatronics 2000, 1s t IFAC Conference on mechatronic systems, Darmstadt, Allemagne, Septembre, 2000.

[9]. R Toscano., D. Martin-Calle and P. Passerieu, "Adaptation paramétrique floue d'une commande au premier ordre en fonction du point d'équilibre courant". Laboratoire d'Automatique E.N.I St ETIENNE LFA'97 -Lyon-décembre, pp.3-10.

[10].Y. Zennir, E. Guechi, L. Chetoui, "Robust multi-controller design with adaptive fuzzy switching system to robot wrist". 4th International Conference on Control Engineering & Information Technology (CEIT), At Hammamet, Tunisia, Tunisia, 2016, pp.6

[11].Y.Zennir, P. Mouille, "Multi-controllers approach applied to a wrist of a robot ".25 European Modeling and Simulation Symposium (EMSS'2013),p.6.

[12].L. Wei-Min and D. John. "H- Control of Nonlinear Systems: A Class of Controllers. TECHNICAL MEMORANDUM NO. CIT-CDS 93-008 May 13,1993, p.43.

[13].W. Min, Z. Lingbo and L. Guoping. Adaptive Robust $H\infty$ Control for Nonlinear Systems with Parametric Uncertainties and External Disturbances.

[14].X. Wang, L. Gao, S. Zhang. Robust $H\infty$ control of a class of uncertain nonlinear systems. Sinica Automatica, 1999, Vol. 25, No. 2,pp. 221-225.

[15]..K Nguang. Robust Nonlinear $H\infty$-Output Feedback Control. IEEE Transaction on Automatic Control, 1996, Vol.41, No. 7, pp. 1003-1007.

[16].G. Lu, Y. Zheng. Robust $H\infty$ control for a class of nonlinear uncertain systems. Sinica Automatica, 1999, Vol.25, No. 3, pp. 388-392

[17].M. Wu, L. Zhang, W. Gui. Robustness analysis and synithesis for a class of nonlinear systems with additive uncertainties. Control Theory and Application, 2002,Vol. 19, No. 2, pp.203-206

[18].Y. Fu, Z. Tian, S. Shi. $H\infty$ robust control of the state delayed uncertain nonlinear systems. Sinica Automatica, 2001, Vol. 27, No. 2, pp.224-228.

[19].M. Wu, L. Zhang, G. P. Liu. Robust control of a class of nonlinear systems with $l\infty$-bounded disturbances. Control Theory and Application, Vol. 19, No. 3: 369-2375, 2002 .