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Propagation of Classical Waves in One-Dimensional Disordered Media: I Formalism

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Résumé :
La propagation des ondes classiques dans les milieux aléatoires est étudiée en présence des corrélations à courtes distances dans le désordre. Un analogue classique du modèle de Kronig-Penney est proposé en considérant une chaîne de sous-systèmes identiques, chacun étant constitué d'une masse reliée à socle rigide par un ressort. Les masses sont reliées entre elles par un fil soumis à une tension uniforme. La nature des modes est examinée par le formalisme des matrices de transfert. Les propriétés de propagation du système sont étudiées d'une manière statistique mettant en valeur un grand nombre de grandeurs physiques telles que le coefficient de transmission, la longueur de localisation et les exposants critiques. En particulier il est mis en évidence que la présence de corrélation dans le désordre permet de restaurer un grand nombre de modes de propagation du type onde de Bloch en contradiction avec les conclusions obtenues par d'autres modèles de la littérature.
Mots-clés: *Désordre, système unidimensionnel, Corrélation, localisation*

Abstract:
The propagation of classical waves in one-dimensional random media is examined in presence of short-range correlation in disorder. A classical analogous of the Kronig-Penney model is proposed by means a chain of repeated sub-systems, each of them constituted by a mass connected to a rigid foundation by a spring. The masses are related to each other by a string submitted to uniform tension. The nature of the modes is investigated by using different transfer matrix formalisms. The transmission properties of the system are numerically studied using a statistical procedure yielding various physical magnitudes such the transmission coefficient, the localization length and critical exponents. In particular, it is shown that the presence of correlation in disorder restores a large number of extended Bloch-like modes in contradiction with the general conclusion of the localization phenomenon in one-dimensional systems with correlated disorder.
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Mots clés: *désordre, localisation, propagation, dimer*
Keys words : *disorder, localization, propagation,dimer*

1- INTRODUCTION

According to the universal conclusion of the scaling theory [1], it is well understood that in one-dimensional (1D) disordered systems, all the elementary excitations are localized in the Anderson sense [2]. Such localization effects are known to have a significant relevance in other wave propagation phenomena in random media [3]. Indeed, classical waves may offer easier and more direct realization for the observation of the Anderson localization in 1D disordered systems [4].

Then, it is clear that the disorder precludes the presence of long-range propagation. However, recent theoretical approaches have successfully examined different ways in delocalizing the states indicating that disorder can be creative [5-6]. A challenging scenario has been put forwards to suppress localization allowing the propagation of waves: namely correlation in disorder. Originally introduced by Dunlap et al.[7], the random dimer model (RDM) has been applied to various domains: conducting polymers [8-9], semiconductor disordered superlattices [10,11] pointing out the existence of truly extended states supported by experimental evidences [10]. The key idea is that the RDM within a short length correlation restores the tunnel effect and then the necessary condition for delocalizing the particle. However all this matter holds only for the quantum case since the competition between destructive interference and tunnel effect is the major cause leading to the localization or delocalization of the states.

To our knowledge, very few has been done in this context in the classical sense. This has prompted us to examine the effect of the correlated disorder via the dimer effect on the propagation of classical waves. The 1D classical system illustrating the analogy between electron-wave and classical-wave may constructed as follows: the wave medium at each regular lattice point is constituted by a large string having negligible mass submitted to a uniform tension; a mass is linked to a ground foundation by a spring forming a unit cell. In this situation, the wave field consists of transverse amplitudes along the string. Here the dimer is introduced by assuming a concentration of two successive identical cells at random through a host lattice of identical cells. Typically it corresponds to a random binary alloy with dimer. It is expected that such system restores the existence of extended modes. The paper is organized as follow: in the second part, we describe the formalism leading to the physical quantities of interesting by using the technique of transfer matrices such the coefficient transmission, the localization length and the critical exponents at the transition. In the third part we present and discuss the results computed numerically within a statistical treatment. Finally, the conclusion closes the paper.

2- THEORETICAL MODEL

A semi-infinite tight string with homogeneous density is submitted to a uniform tension T_0. The string is formed by a large number of subsystems at each lattice discrete points $x_n=nd$, d being the lattice spacing. Each subsystem is contructed by a mass M_n connected to a grounding rigid foundation by a spring having a linear stiffness K_n. We are interested by the propagation of transverse wave in the vertical plane. The transverse displacement y at the longitudinal coordinate x is solution of the equation of motion :

$$\frac{\partial_2 y(x)}{\partial x_2}+k\,y(x)=\sum_{n=-\infty}^{+\infty}\lambda_n\delta(x-x_n)y(x) \qquad (1)$$

with:

$$k=\frac{\omega}{v_\varphi} \text{ and } v_\varphi=\sqrt{\frac{T_0}{\rho}} \qquad (2)$$

k and v_φ denoting the wave vector and the wave velocity through the whole system respectively. Here ω is the fundamental frequency to be determined. The term λ_n related to each delta peak corres-ponds to vibration mode given by [30]:

$$\lambda_n=\frac{1}{T_0}\left(K_n-M_n\omega^2\right)=\frac{K_n}{T_0}\left(1-\frac{\omega^2}{\Omega_n^2}\right) \qquad (3)$$

where:

$$\Omega_n^2=\frac{K_n}{M_n} \qquad (4)$$

Physically Ω_n defines the free frequency of the n^{th} subsystem while λ_n has the meaning of an effective delta peak strength. λ_n^{-1} defines a characteristic length translating the bearing of the associated string. Randomness may be introduced in different ways: disorder in mass and /or stiffness. Here, we restrict ourselves to the first one without lost of generality. Thus , the masses are statistically independent variables given by a common probability distribution.

In the region

$$nd\le x\le(n+1)d,$$

the solution of Eq.(1) is a superposition of forward and back-ward scattering waves:

$$y(x)=A_n\exp(ikx)+B_n\exp(-ikx) \qquad (6)$$

where A_n and B_n denote the amplitude coefficients in the n-th region. Introducing the reflection the transmission amplitudes r_N and t_N of the system, $y(x)$ satisfies the limit conditions:

$$y(x)=\begin{cases}\exp(ikx)+r_N\exp(-ikx) & x<0\\ t_N\exp(ikx) & x>L\end{cases} \qquad (7)$$

where $L=Nd$ is the system size.

The amplitudes A_n and B_n through the initial and final amplitudes can be linearly expressed using boundary conditions giving the total transfer matrix $M(N)$ of the system,

$$\begin{pmatrix} t_N \\ 0 \end{pmatrix} = M(L,0)\begin{pmatrix} 1 \\ r_N \end{pmatrix} \qquad (8)$$

$$M(L,0) = \frac{1}{2ik_0}\begin{pmatrix} -ik_0 & -1 \\ ik_0 & 1 \end{pmatrix} S(L,0)\begin{pmatrix} 1 & 1 \\ ik_0 & -ik_0 \end{pmatrix} \qquad (9)$$

where $S(L,0)$ refers as the total diffusion matrix. This allows one to determine the transmission coefficient which is the fundamental physical quantity of interest:

$$T = \left|M_{22}\right|^{-2} = \frac{4}{\left(S_{11}+S_{22}\right)^2 + \left(\frac{S_{21}}{k} - k\,S_{12}\right)^2} \qquad (10)$$

From an analytical point of view, the wave pro pagation equation may be achieved within the Poincarré map representation, which in turns ena- bles one to relate the displacements at successive lattice points.

Defining $y_n \equiv y(x = n \cdot d)$, Eq.(1) may be exac- tly transformed [13] into a simple recursive site description :

$$\begin{pmatrix} y_{n+1} \\ y_n \end{pmatrix} = R_n(\omega)\begin{pmatrix} y_n \\ y_{n-1} \end{pmatrix} = \begin{pmatrix} 2\,a_n(\omega) & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} y_n \\ y_{n-1} \end{pmatrix} \qquad (11)$$

where:

$$a_n(\omega) = \cos(kd) + \frac{\lambda_n}{2k}\sin(kd) \qquad (12)$$

is the standard KP formula yielding the fre- quency spectrum [6]. Within the basis of the on-site wave functions, the transmission coefficient at the end of the chain, i.e. $n = N$, reads as the ratio:

$$T_N(\omega) = \frac{\left|y_N\right|^2}{\left|y_0\right|^2} \qquad (13)$$

Once the transmission coefficient is known, the nature of the propagating modes may be characterized by the normalized Lyapunov exponent given by the ratio:

$$\frac{L}{\xi} = -\frac{1}{2}\log T \qquad (14)$$

where ξ denotes the localization length.

3. CONCLUSION

The propagation of classical waves in random media has been studied by using an analogous with the well known electronic disordered Kronig-Penney model to observe the phenomenological aspects of the Anderson localization. At this stage we have been able to obtain a close formulae describing the propagation properties of the system. Towards this end we are developing a numerical simulation for the coefficient transmission and all other related physical magnitudest

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