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COMPUTER AIDED DESIGN OF INDUCTION MACHINE USING FINITE ELEMENT COUPLED TO CIRCUIT EQUATIONS

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Résumé- *L'électromagnétisme informatisé exige une grande gamme de méthodes numériques selon l'application, la topologie de la géométrie, la fréquence et la qualité de la solution. Différents efforts ont étés faits pour développer un logiciel universel capable de traiter n'importe quel problème, en intégrant facilement les modules spécifiques dans une certaine interface. Suivant cette philosophie et parce que maintenant les techniques de calcul informatique permettent la résolution de divers problèmes concernés par la classification de modèle, l'identification de système, la prévision, la commande, le robotique, et la vision d'ordinateur, cet article propose une nouvelle approche pour étudier la machine à induction par CAO en utilisant la méthode des éléments finis couplée aux équations du circuit.*

Abstract- *Computational electromagnetism requires a wide range of numerical methods depending on the application, the topology of the geometry, the frequency and the quality of the solution. Different efforts have been done to develop an universal software package able to deal with any problem, by integrating easily specific modules in the some interface. Following this philosophy and because now soft computing techniques allows to solve various problems concerned with pattern classification, system identification, prediction, control, robotics, and computer vision., this paper proposes a new approach to study the induction machine by CAD using finite element method coupled to the circuit equations.*

Key words: *CAD, Induction machine, Finite element, Circuit equations.*

1. Introduction

The goal of Computer-Aided Design is to enlist the computer to aid an interface designer in constructing an interactive interface to be eventually used by an end-user. This point of view is by analogy to Computer-Aided Design and Computer-Aided Manufacturing [1]-[3].

CAD traditionally refers to tools to visualize, describe, edit and test manufactured artefacts, which are now an indispensable part of all manufacturing and production processes. Computer-Aided Design systems are successful in manufacturing because they replace the specifications of numerical parameters, a task difficult for humans to perform reliably, with the visualization and editing of visual representations of virtual objects, which is much more natural for humans to perform. Computer-Aided Design systems can be successful in user interface design because they replace specification of programming language constructs, which are difficult for humans to perform reliably, with the visualization and editing of visual representations of virtual interfaces, which are more natural for humans to perform.

While manufacturing CAD systems are oriented towards describing the size and shape and perhaps associated attributes such as materials or cost of products, user interface CAD must not only describe the appearance and form of the user interface, but also the behavior of the interface. If, for a given interface, each interface element can have its own independent behavior, then a user interface often can be defined simply by combining and connecting pre-defined behavioral elements. But if there are interactions between the elements, if there is state to be maintained, or particular actions to be taken, then some other method for describing the behavior must be found.

The induction machine is the most rugged and most widely used machine in the industry. The main disadvantage in the line start of induction motors is the starting performance. During starting, the torque is low and the starting current is 5-8 times higher than the rated current. The low starting torque is particularly disadvantageous where the motor is directly connected to the load all the time, like the motor of a compressor. The high starting current subsides as the motor speed increases and reaches the steady-state operating point. If the load connected to the motor requires a high starting torque, the motor will accelerate slowly. This will cause a large current flow for a longer time, thereby creating a heating problem. Other than the heating problem, a large inrush of starting current deteriorates the power quality with unwanted voltage dips in the supply voltage. The design of the rotor of an induction machine is critical in meeting the performance specifications since the shape of the torque-speed curve is largely determined by the rotor design [4].

In this paper, the usual approach is for the interface designer to write code in a traditional programming language, like Matlab. This model permitting the simulation of squirrel-cage induction motor associated with nonlinear external circuits. In this simulation the electric circuit equation is directly coupled with the magnetic one and the solution of the resulting non linear time-dependent equation is obtained using step-by-step numerical integration and the Newton-Raphson iterative procedure.

2. Electromagnetic field calculation

From Maxwell's equation, we start the development of the magnetic field model:

$$\nabla\times E=-\frac{\partial B}{\partial t} \quad (1)$$

$$\nabla\times H=J_e+J+\frac{\partial D}{\partial t} \quad (2)$$

$$\nabla . D=\rho_e \quad (3)$$

$$\nabla . B=0 \quad (4)$$

In static and quasi-static (low frequency, less than 1.0 kHz) field problems, the displacement term in (2) is negligible. Therefore, (2) can be rewritten as follows:

$$\nabla\times H=J_e+J \quad (5)$$

The magnetic field intensity H is related to the flux density B through the permeability μ as follows:

$$B=\mu H \quad (6)$$

In addition to the above equations a key variable, namely the magnetic vector potential A, can be defined as follows:

$$B=\nabla\times A \quad (7)$$

Implementing the finite element discretization, using first or second order triangular elements, we can obtain the numerical solution. As a result of the finite element discretization, one obtains the following matrix equations for both the electromagnetic and mechanical problems:

$$[S].[A]=[J_e] \quad (8)$$

and

$$[K].[U]=[F] \quad (9)$$

If we solve (8) by using the Newton-Raphson method, we obtain the magnetic vector potential A.

The electromagnetic field calculation process inside of induction motor is realized using finite elements method [5]-[20] in the frequency domain. The boundary value problem, for electromagnetic field in the induction machines, is given by the following equations:

$$\frac{\partial}{\partial x}\left(\nu_{eff}.\frac{\partial A}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu_{eff}.\frac{\partial A}{\partial y}\right)-j\omega r\sigma A=-J_s \quad (10)$$

$$(x,y)\in\Omega\subseteq R^2$$

$$A(x,y)=0,\ (x,y)\in\Gamma_d \quad (11)$$

For the magnetic calculation process own software package named Supervisor Matlab 2D [21]-[24], based on the first order triangular elements, was used.

So, the above boundary value problem is discredited by finite element, and nonlinear system of equations obtained is solved using Newton-Raphson iterative technique.

In this calculation process is used effective ***B(H)*** curve represented by Fig. 1, as a representation of nonlinear influence. This approach is possible only using one of several different methods for complex approximation

approximation is reason for some other problems, like slow convergence and the convergence failure in the case of very high level of saturation.

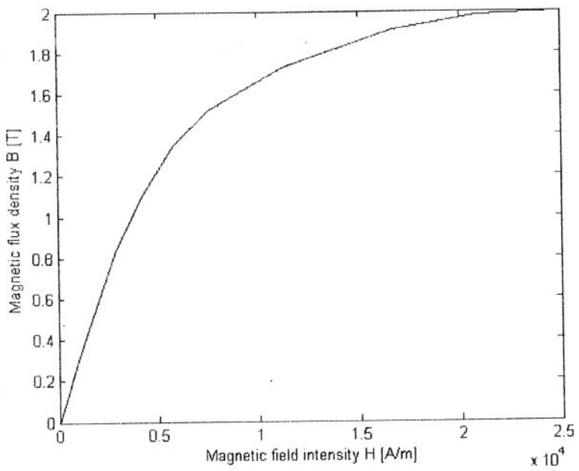

Fig. 1: Curve B(H)

The chosen model is a squirrel cage induction machine as shown in Fig. 2, with open stator slots and with nominal values of electric parameters as follows:

-Nominal voltage: 380 [V]

-Nominal power : 7.5 [kW]

-Phase number: 3

-Nominal frequency: 50 [Hz]

-Nominal power factor: 0,9

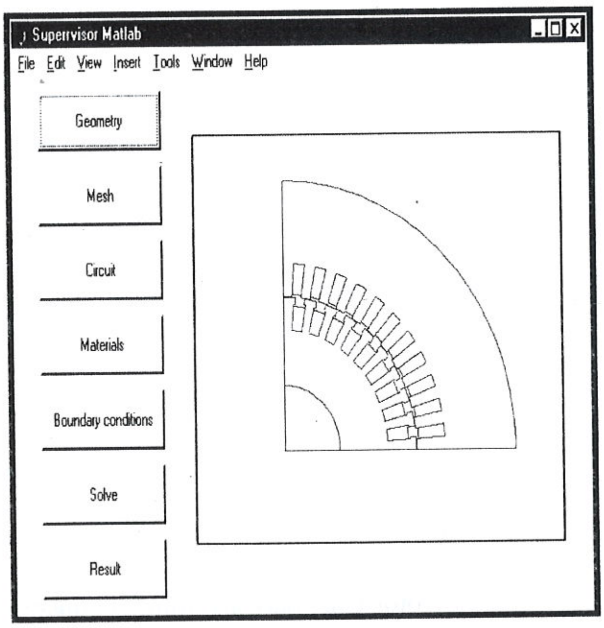

Fig. 2: Motor geometry

After building the geometry, the automatic mesh generation is affected. The finite element mesh is almost regular, because the above process is based on Deulanay triangulation and the node positions optimization based on the penalty functions method.

Rotor slip varied from 0(%) to 100(%) in finite number of values in the calculation process, results of electromagnetic field calculation are shown on the Fig 3 , it can be seen that equipotential lines change the appearance with respect to rotor slip , this phenomena is result of squirrel cage influence.

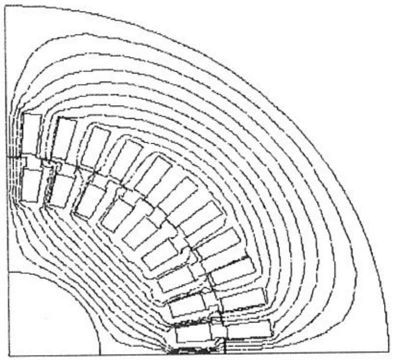

Fig. 3: Equipotential lines with s=0(%)

In modeling electromagnetic systems, the solution of magnetic and electric circuit equations can be done considering coupling between the two circuits.

Such a coupling can be indirect, in this case the parameters of one circuit are determined from its circuit equation and used to solve the other circuit equation through an iterative procedure. In the case where one or both of the two magnetic and electric circuits is nonlinear, a direct coupling of two circuit equations is necessary.

In electric machines, if the magnetic armature reaction is strong the saturation and reluctance effects in the magnetic circuit result modified internal voltages. Moreover, if the electric circuit contains nonlinearities due to contact resistances or semiconductor components, the resulting voltages will be very different from those predicted if nonlinearities in magnetic and electric circuits were absent. In such a case a direct coupling of both circuit equations must be used in modeling the machine operation.

Two-dimensional (2-D) magnetic calculations are often used in modeling electric machines with a reasonable approximation. The direct coupling of (2-D) magnetic and electric circuit equations gives good accuracy with a price of important stroge capacity and computation time.

The magnetic vector potential from of Maxwell's equations yields to the solution of (12):

$$curl\,\upsilon\, curl\,\mathbf{A}+\sigma\frac{\partial \mathbf{A}}{\partial t}+\sigma\,\mathbf{grad}\;V=0 \qquad (12)$$

Ohm's law is written as:

$$\mathbf{J}=-\sigma\left(\frac{\partial \mathbf{A}}{\partial t}+\mathbf{grad}\;V\right) \qquad (13)$$

One method that takes the term representing the eddy currents in a solid conductor, consist of solving (12) by means of complex variables, as a matter of fact we have:

$$\sigma\frac{\partial \mathbf{A}}{\partial t}=j\omega\,\sigma\,\mathbf{A} \qquad (14)$$

As a consequence of the non linearity of the magnetic materials of the iron sheets, the magnetic reluctivity is variable with respect to the field. To solve this problem, an iterative Newton-Raphson method has been used.

The presence of sinusoidal sources, with non linear materials leads to a time variation of the stored magnetic energy. In order to compute the magnetic energy corresponding to a given maximum value of the induction, an equivalent ***B(H)*** curve has been used as shown in Fig. 1.

In the case of induction motor, the rotor currents have the pulsation ($g\omega$) where g represents the slip of the rotor. In order to be able to model the induced currents in the cage, the pulsation ($g\omega$) will be considered in the rotor.

This hypothesis, however, leads to a wrong representation of the space harmonics, the pulsation of which is then:

$$(1-n(1-g))\,\omega \qquad (15)$$

n: being the rank of the harmonic.

All these hypothesis yields to solve the two following equations:

$$curl\,\upsilon eq\, curl\,\mathbf{A}+j\sigma g\omega\mathbf{A}+\sigma\,\mathbf{grad}\;V=0 \qquad (16)$$

$$\mathbf{J}=-j\sigma g\omega\,\mathbf{A}-\sigma\,\mathbf{grad}V \qquad (17)$$

To represent a voltage fed induction motor, circuit equations must be coupled with field equations, furthermore, because the rotor cage is a polyphase, tridimentional circuit, a specific coupling method has been developed. Thus, there is obtained a tool in which the resistances and the reactance of the end rings and the end windings may be introduced. These quantities are computed analytically and complete the finite element analysis of the induction machine.

2.1 Rotor equations

The finite element formulation of (16) and (17) leads to the following system of equations in the case of solid conductors.

$$([S]+jg\omega[L])[A]-[C][\Delta V]=0 \qquad (18)$$

$$-j\omega g[R][C]^t[A]+[\Delta V]=[R][D].J \qquad (19)$$

With:

$$C_{ij}=L\iint_\Omega \sigma\,\alpha_i\,d\Omega$$

$$S_{ij}=L\iint_\Omega \upsilon\,\mathbf{grad}\;\alpha_i\;\mathbf{grad}\;\alpha_j\,d\Omega$$

$$L_{ij}=L\iint_\Omega \sigma\,\alpha_i\,\alpha_j\,d\Omega$$

$$R_{kk}=R_k=\frac{L}{\int_\Omega \sigma\,ds}$$

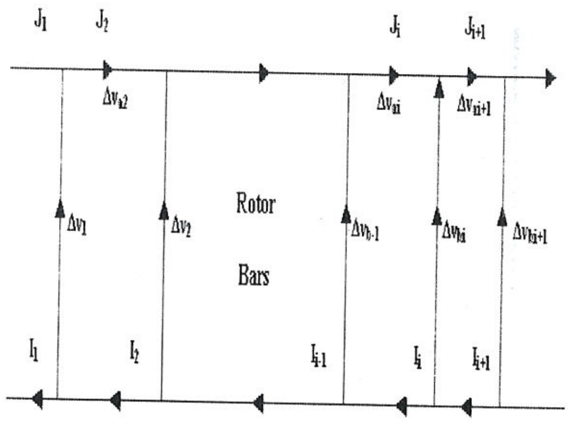

Fig. 4: The connection of bars and end-rings

But the rotor cage bars are solid conductors connected together by means of the end rings, every portion of the ring located between two bars may be considered as an external impedance. The rotor cage is then described by the polyphase circuit of Fig. 4

Let r_a and x_a be respectively the resistance and the reactance of a portion of the ring, with reference to Fig. 4 we obtain the following equations:

$$\Delta V_{bi-1}=2\,\Delta V_{ai}+\Delta V_{bi} \quad (20)$$

$$I_i=-J_{i+1}+J_i \quad (21)$$

$$\Delta V_{ai}=(r_a+jgx_a)J_i \quad (22)$$

If M is an (nXn) matrix defined as in equation (23):

$$[M]=\begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & \dots & 0 \\ \dots & & & & \\ \dots & & & & 0 \\ 0 & \dots & & 1 & -1 \\ -1 & 0 & \dots & 0 & 1 \end{bmatrix} \quad (23)$$

The two formulation (20) and (21) gives the two following matrix equations:

$$[I]=[M][J] \quad (24)$$

$$[\Delta V_a]=\frac{-1}{2}[M]^t\,[\Delta V_b] \quad (25)$$

When substituting (22) with (24) and (25) one yields to:

$$(ra+jg\,xa)[I]=-[T][\Delta V] \quad (26)$$

Where $[T]=\frac{1}{2}[M].[M]^t$ is a symmetric band matrix. Finally, (19) becomes, in the case of a rotor cage:

$$j\omega g[R][C]^t[A]+(1+[R][T]/(r_a+jgx_a))[\Delta V]=0 \quad (27)$$

The currents in the bars may be obtained by (26), the resistance and the reactance of the end-rings are computed using analytical methods.

2.2 Stator equations

Stator coils are made of thin conductors in which the skin effect my be considered as negligible, in this particular case, it is possible to find a formulation that represents a voltage fed coil. Let us consider an Ns identical turn, coil connected to an external impedance Z_{ext} and supplied by a sinusoidal voltage source as shown in Fig. 5, Ohm's law writes as follows:

$$E=Z_{ext}.I+\sum_{k=1}^{NS}(\Delta V_{1k}-\Delta V_{2k}) \quad (28)$$

When applying (13) on the Ns conductors, Ns equations are obtained leading to the following formulation:

$$-j\omega[C]^t[A]+[R]^{-1}[\Delta V]=[D]\,I \quad (29)$$

A linear combination of (28) and (29) yields finally to:

$$j\omega[D]^t\,[C']^t\,[A]+(Z_{ext}+\sum_{k=1}^{NS}R_k)\,I=E \quad (30)$$

With: $C_{ij}'=Ns\,\frac{L}{Sk}\iint_S \alpha_i\,.d\Omega$

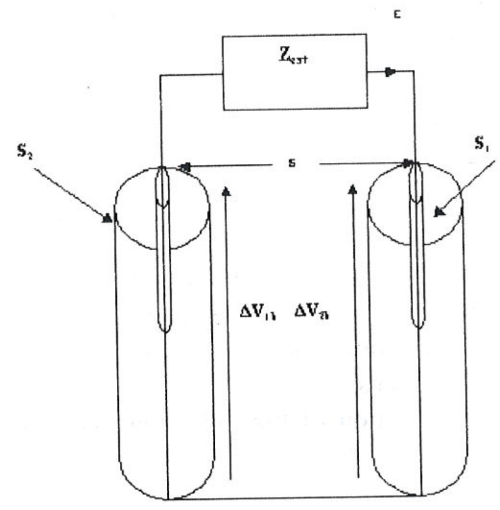

Fig. 5: Description of a coil

Since the conductors are thin, the potential difference across each conductor simply writes with respect to I, the field on (14) then becomes:

$$[S][A]-[C][D].I=0 \quad (31)$$

The voltage balanced iterative process is most important part of stator current-speed and torque-speed characteristics calculation. This method is based on the well-known voltage equation of the stator phase circuit:

$$\underline{U}_1=\underline{I}_1.(R_1+j.X_1)-\underline{E}_i \quad (32)$$

In the value of the stator 3D- effect correction reactance is included end-winding reactance and stator skewed slots reactance. The well-known analytical approach was used for the numerical calculation of the stator end-winding reactance and stator skewed slots reactance. These two 3D corrections is possible to calculate using numerical approach, but this method needs mach longer calculation time. The induced phase voltage in stator coil is calculated as follows:

$$\underline{E}_i = -j.2.\omega_1 \, N.L_e.\underline{A}_{av} \quad (33)$$

The average value of magnetic vector potential is defined as:

$$\underline{A}_{av} = \frac{1}{S_1}.\iint_{(S_1)} \underline{A}.ds \quad (34)$$

Where S_1 is the total area which is covered with one side of the coils in one phase winding. The voltage difference between the calculated stator voltage and the prescribed terminal voltage (U_t) can be calculated after the stator voltage calculation as follows:

$$voltage\ error = \frac{|U_t - U_1|}{U_t} \quad (35)$$

If the value of the above voltage error is not acceptable, it is necessary to process with the current correction process. This process contains two steps:

First, calculation of the induction machine phase total impedance:

$$\underline{Z}_t = R_1 + j.X_1 - \frac{\underline{E}_i}{\underline{I}_1} \quad (36)$$

Second, calculation of the new value of stator current:

$$\underline{I}_1^{(new)} = \frac{U_t}{|\underline{Z}_t|} \quad (37)$$

Numerical calculation of induction motor torque and speed characteristics is based on the equations from the fundamental induction machine theory, as follows:

$$T_m = \frac{P_m}{\Omega} = \frac{P_{em}}{\Omega_s} \quad (38)$$

$$P_r = s.P_{em} \Rightarrow P_{em} = \frac{P_r}{s} \quad (39)$$

$$T_m = \frac{P_r}{s.\Omega_s} \quad (40)$$

Method for numerical calculation of the torque-speed characteristic numerical calculation is given in equation (40).

3. Results

All the vector quantities (flux density, magnetic field....) can be represented in the form of vectors on a region as shown in Fig 6. The vectors are computed in each finite element, in the integration point where the quantity has a maximum value.

Fig 7 indicate the flux density components Bx, By in the induction machine.

The flux presented in Fig. 8 is assumed to vary sinusoidal with time at the frequency f or angular frequency ω and with the variation of the air-gap.

The computation results shows that the value of self-inductance and mutual inductance in the stator circuit are constant. There is a small periodical change in the self-inductance in the rotor circuit, which is concerned with the relative space position of rotor circuit in the stator teeth. The mutual space position of rotor circuit and the stator circuit presents periodical change, which is concerned with the relative space position between rotor circuit and the stator winding. As for random two rotor circuits, the mutual-inductance value is small and negative only when they are adjacent and is zero in the other cases. The Fig.9 shows the changing of the stator-rotor multi inductance.

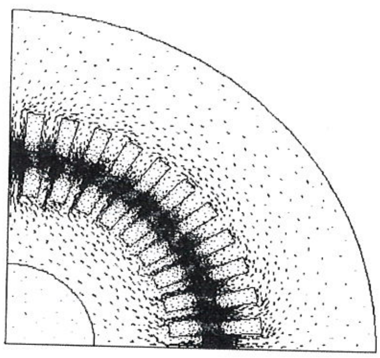

Fig. 6: Magnetic flux density vectors

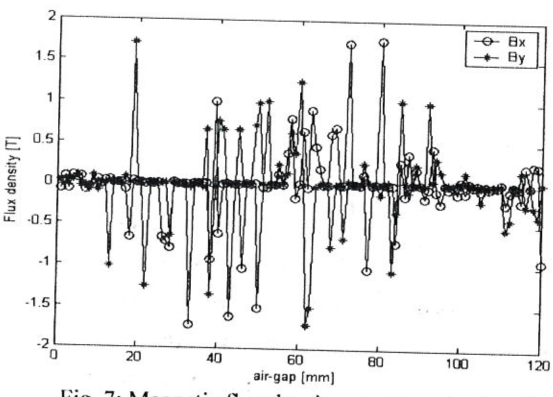

Fig. 7: Magnetic flux density components Bx, By

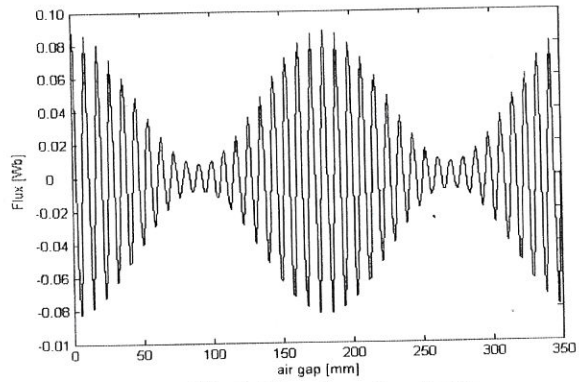

Fig. 8: Flux along the air-gap

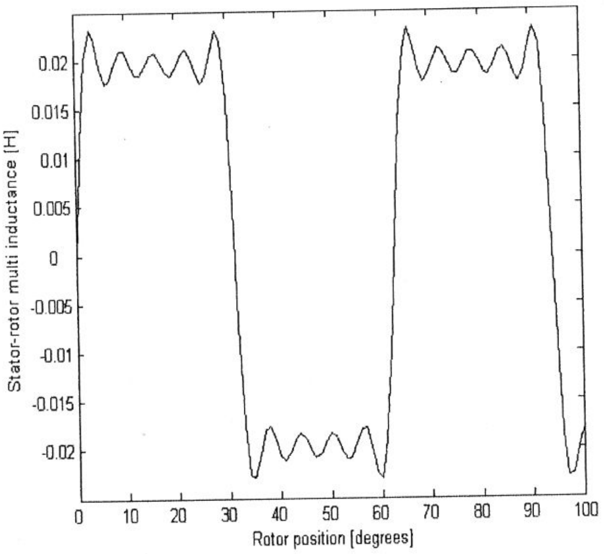

Fig 9: The stator-rotor multi inductance

4. Conclusion

Like Computer-Aided Design systems for products and manufacturing applications, Computer-Aided Design systems for user interfaces can make the design process easier, more interactive and less-error prone. The success of CAD systems for product design is predicated on the fact that the system allows the user to interact with virtual examples of the artifact under construction. This takes advantage of people's natural tendency to teach and to learn by example. But user interfaces have dynamic and interactive behavior, and to define such behavior we need more mechanism then just specification of appearance of interfaces. By recording actions of a simulated end-user and generalizing them, programming by example provides the technology for bringing the success of the CAD applications to user interface design.

In this work, we use the finite element method to predict the dynamic behavior of electric machines that is gaining in popularity because of the availability of high performance microcomputers and the development of new features such as the integration of the rotor's motion and the machine's supply circuitry in the finite element solution.

Modern CAD package use two important features to achieve this computation: the moving air-gap method for modeling the motion of the rotor and the integration of the power supply and control circuitry into the finite element solution. A novel design method has been presented here to starting performance of the software Matlab to solve the Maxwell's equations by using the finite element coupled to the circuit equations.

The simulation results presented here nicely validate the electromagnetic theory behind the proposed mythology in CAD of electric machines.

5. Nomenclature

Table I: Parameters used in the problem formulation

E : The electric field intensity vector
B : The magnetic flux density (induction) vector
H : The magnetic field intensity vector
J_e :The excitation (external) current density vector
J : The induced current density vector
D : The electric flux density (displacement) vector

ρ_e : The (external) charge density
S: Electromagnetic stiffness matrix
K: Mechanical stiffness matrix
F: Force vector
σ : Electric conductivity
A : Magnetic vector potential
Ω : Domain of numerical calculation (machine cross section)
Γ_d : Boundary of domain Ω (Condition of Dirichlet)
J_s : Source current density
ν_{eff} : Effective magnetic reluctivity
ω_r : Slip angular frequency
Z_{ext} : Impedance of the end windings
$\underline{U}_1$: Stator phase voltage as a result of the calculation
$\underline{I}_1$: Stator phase current
R_1 : Stator phase resistance
X_1 : Stator 3D effect correction reactance
$\underline{E}_i$: Stator induced phase voltage
ω_1 : Source line angular frequency
N : Number of series turns per phase
L_e : Effective core length
$\underline{A}_{av}$: Average value of magnetic vector potential
T_m : Mechanical torque of the induction machine
P_m : Useful mechanical power
P_{em} : Electromagnetic power transferred across the air-gap
P_r : Rotor bar power loss
Ω_s : Synchronous angular speed $\Omega=(1-s)\Omega_s$

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