



# **NUMERICAL MODELING OF CRACK PROPAGATION PARAMETERS BY THE SFEM METHOD FOR A TWO-DIMENSIONAL MODEL**

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# **ABSTRACT**

Fracture mechanics is a broad field that allows studying the behavior of a structure with different defects. The presence of defects can be in the form of internal cracks or superficial cracks. Fracture mechanics analysis correlates with loading parameters. The purpose of this work is to numerically model the straight and inclined initial crack by the angle  $(\alpha)$ concerning an elastic material, the SFEM method was used to make the modelling, a FORTRAN program makes it possible to create a structured parametric mesh, which keeps the same number of nodes and elements during the different stages of crack propagation. Different crack propagation scenarios have been studied. In addition, several values of the angle  $(\alpha)$  were used to orient the crack. In addition, the model used has four-node CPE4 elements. The stress state characterization parameters at the crack front (KI and KII) were evaluated based on the structural length  $(X)$ . The analytical approach was used to compare the different results obtained by the SFEM method.

**Keywords:** Numerical simulation; SFEM; CPE4; FIC; Crack propagation.

# **1. INTRODUCTION**

Fracture mechanics is also an important line of research, and very demanding from an analytical and practical point of view. In the field of mechanics, the numerical approach based on the finite element method. Its purpose is to characterize the stress field via the stress intensity factors SIF and the contour integral (J) in the vicinity of the crack front. Indeed, several works have been proposed in scientific research to model and analyze crack problems, among these works we cite the studies of Gajjar and Pathak [1] used the extended finite

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element method to study the influence of the graded property of plasticity and the limit of the thermal conditions on the fracture parameter and the integral of the contour J. Furthermore,

the evolution of SIF was modeled by Bentahar and Benzaama [2] using the mesh stretching method (SFEM) by software of the Abaqus numerical simulation of a crack propagation problem bi-dimensional and Bentahar et al [3] studied the variation of the safe stress intensity factor along the crack by the (SFEM) method of a 2D crack propagation problem.

On the other hand, Zhao et al [4] used standard and non-standard compact samples to experimentally study the crack growth behavior of aluminum alloy 7075-T651 in ambient air. On the other hand, another study presented by Alshoaibi and Fageehi [5] based on the developed finite element code is written by the Visual Fortran computer language used to model fatigue crack growth (FCG) in structures 2D.

Zhang et al [6] presented direct calculations of 3D fracture parameters, by FEM versioning method (P-FEM) and contour integral method to compare stress intensity factors (SIF) and stress (T) for straight plane cracks. The motion extrapolation method was used by Alshoaibi (2015) [7] to simulate the crack propagation of a 2D linear elastic model. On the other hand, he also characterized the singularity of the crack tip and the stress intensity factors around the crack tip.

Furthermore, a combined comparative study between two experimental and numerical techniques has been presented by Bouaziz et al. [8] to investigate the fracture properties of additively fabricated polymer parts using digital image correlation (DIC) measurements.

#### **2. ASPECTS OF FRACTURE MECHANICS**

#### **2.1. Law of fatigue crack propagation (Paris and Erdogan, 1963)**

The paris law (also known as the Paris-Erdogan equation (1963) [9]) is a crack growth equation that gives the growth rate of a fatigue crack. The Paris equation is:

$$
\frac{d_a}{d_N} = C(\Delta K)^m \tag{1}
$$

Where:

 $\alpha$  is the crack length;

(*N*) is the number of loading cycles and the variation;

 $(\Delta K)$  of the stress intensity factor;

*C* and *m* are the material constants.

#### **2.2. Maximum circumferential stress criterion (MCSC)**

The maximum circumferential stress criterion (MCSC) was introduced by Erdogan and Sih (1963) [10] for elastic materials. It specifies that the crack propagates in the direction for which the circumferential constraint is maximum.

$$
\tan(\frac{\theta}{2}) = \frac{1}{4}(\frac{K_L}{K_H}) \pm \frac{1}{4}\sqrt{(\frac{K_L}{K_H})^2 + 8}
$$
 (2)

**Fig. 1.** Field of stresses in the vicinity of the front of the crack

Tada et al [11] gave the general 2D stress field equation near the crack front to define the stress intensity factor.

$$
\sigma_{i,j}^{I,H}(r,\theta) = \frac{K_{I,H}}{\sqrt{2\pi r}} f_{ij}(\theta)
$$
\n(3)

 $\sigma_{i,j}^{I,II}$ is the stress field associated with mode I.

 $K_I$ ,  $\text{II}$  is the SIF in mode I and II, The stress field near the crack point with polar coordinates (r, θ) near the crack tip are shown in Figure 1.

$$
\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \left( 1 - \sin\frac{\theta}{2} \sin\frac{3\theta}{2} \right)
$$
\n
$$
\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \left( 1 + \sin\frac{\theta}{2} \sin\frac{3\theta}{2} \right)
$$
\n
$$
\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2} \cos\frac{\theta}{2} \cos\frac{3\theta}{2}
$$
\n11\n12\n13\n14

**Fig.2.** Illustration of cracking modes

- Mode I: opening
- Mode II: plane shear
- Mode III: anti-plane shear

In the continuous medium, cracking is an irreversible separation phenomenon. Three main

modes of cracking can be distinguished (Fig. 2).

#### **2.3 Stress intensity factor**

The stress intensity factor K is an essential parameter in fracture mechanics, which makes it possible to know the state of stress and deformation at any Saverio crack point [12]. (Ewalds and Wanhill, [13]) gave the stress intensity factor by the following relationship:

$$
K_{I} = F \sigma \sqrt{a\pi} \tag{5}
$$

Where, *F* is the geometric correction factor of the model used.

$$
F = 1.12 - 0.23(a/w) + 10.6(a/w)^{2} - 21.7(a/w)^{3} + 30.4(a/w)^{4}
$$
 (6)

Where the stress intensity factor  $K_{II}$  is calculated by the relation.

$$
K_I \sin \theta + K_{II} (3\cos \theta - 1) = 0 \tag{7}
$$

#### **3. NUMERICAL MODEL**

The model of our study has the following dimensions, the width  $X=7$ mm and  $L=8$ mm the parametric mesh was used to make the modeling, the creation of this mesh was made by the FORTRAN program, and the finite element code ABAQUS was applied for this analysis. The structure considered has a crack of length $(a)$ , the model is subjected to a uniform tensile stress  $\sigma$ =120Mpa. Bilinear 4-node CPE4 elements were used as shewn the figure 3. The mechanical properties of the structure are  $E = 1 \cdot 10^7$ Pa and  $v = 0.25$ .



**Fig. 3.** SFEM model and boundary conditions

## **4. MODELING MODEL**

We propose to model crack propagation with different orientation angles as shown in figure 4.



## **5. RESULTS AND DISCUSSIONS**





**Fig. 5.** Evolution of SIF of the different crack propagation; a) first propagation  $\alpha = -14.20^{\circ}$ , b) second propagation  $\alpha$ = -0.620°, c) The third propagation  $\alpha$ = 0.820° d) fourth propagation  $\alpha$ = 1.500° and e) fifth propagation  $\alpha$  = 2.100°

Figure 5 shows the evolution of FIC as a function of (X) concerning the different crack propagation angles, it can be seen that the results obtained change regularly with respect to the crack length. As well as the increase in crack length causes an increase in FIC, concerning the first crack propagation KI varies between 30Mpa to 40Mpa in the interval of (X) from 3.60mm to 3.85mm see Fig 5(a). Figure 5(b) shows the evolution of FIC as a function of  $(X)$ in the case of the second crack propagation concerning the interval of  $(X)$  which varies between 3.9 mm and 4.4 mm. Further, the variation of KI in the third crack propagation is varied between the value 70Mpa and 80Mpa mm in the intarval of (X) 4.4 mm to 4.9 mm as illustrated in Fig. 5(c). Figure 5(d-e) illustrates the two intervals of 4.9 mm to 5.4 mm and 5.4 mm to 5.9 and the results obtained from KI vary between 100 Mpa and 120 Mpa and 140 Mpa and 200 Mpa, respectively the variation of KII results in all steps of the crack propagation almost is zero.The comparison between the SFEM method and the analytical method gives comparable results.

#### **6. CONCLUSION**

We conclude that the studied model is a well-known problem in fracture mechanics.

Numerical simulation by the stretched finite element method SFEM was used to characterize the different crack parameters of a two-dimensional model.

The values of KI are always larger compared to KII.

The characterization of the crack parameters is very necessary to know the stress state at the level of the crack front.

Five crack propagations were modeled by the SFEM method.

There is proportionality between the results obtained and the results of the analytical method.

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