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MECHANICAL BEHAVIOR OF EXPONENTIAL FUNCTIONALLY GRADED SANDWICH PLATES SUBJECTED TO AXIAL LOADING AND RESTED ON ELASTIC FOUNDATION

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ABSTRACT

Mechanical buckling of functionally graded (FGM) sandwich plates is analyzed in this research using new higher-order shear deformation plate theory which captures the shear deformation influences needless of any shear correction factor. New functionally graded sandwich structure based on variable exponential function is presented in this analysis. Material properties of functionally graded face layers are assumed to vary continuously through-the-thickness according to an exponential function in terms of the volume fractions of the constituents (E-FGM), while the core layer is made of ceramic. Equilibrium and stability equations of E-FGM sandwich plate with simply supported boundary conditions are derived using the generalized higher-order shear deformation plate theory and the Hamilton's variational principle, and solved by using the Navier's solutions. Several numerical results indicate the influence of the plate aspect ratio, the relative thickness, the gradient index and the sandwich scheme on the critical buckling load of FGM sandwich plates are investigated.

Keywords: Mechanical buckling, E-FGM sandwich plate, higher-order shear deformation theory, simply supported boundary conditions.

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1. INTRODUCTION

Developing new biomaterials for medical applications is one of the most challenging tasks in materials science today. When we look at biomaterials, we typically observe design principles that are not normally applied in traditional materials processing. A distinctive feature of biomaterials is the formation of layered structures. Furthermore, the complex functions of various tissues involve continuous changes from one structure or composition to another. For example, the exquisite design of bones, ranging from a dense, rigid external structure (cortical bone) to a porous internal structure (cancellous bone), suggests that functional grading through biological adaptation has been exploited. This structure optimizes the material's response to external loads. Therefore, the optimized structure of artificial implants should have similar gradients.

The design of Functionally graded materials (FGMs) has the potential to mimic the biological functions of these heterogeneous tissues and advance our understanding of how biomaterials integrate in vivo [1,2]. Consequently, properties are also spatially gradually varied to meet specific non-homogeneous service needs without abrupt macro-interfaces. FGM is an emerging material for orthopaedic prostheses because functional gradients can be tailored to reproduce the local properties of the original bone, which helps minimize stress shielding effects while reducing shear between the implant and surrounding bone tissue stress, two key prerequisites for achieving orthopaedic repair. Transplants have a longer lifespan.

Functionally Graded Materials (FGM) are a novel class of composite materials. Whereas traditional composites are homogeneous in composition, FGMs possess a gradual spatial compositional variation of the composite material in terms of volume fraction and microstructure [1]. These new materials were proposed to reduce the local stress concentrations induced by abrupt transitions in material properties across the interface between discrete materials [3]. Typically, FGMs are made of a ceramic and a metal for the purpose of thermal protection against large temperature gradients. The ceramic material has excellent characteristics in heat resistance due to its low thermal conductivity. On the other hand, the ductile metal constituent prevents fracture due to its greater toughness. FG structures can be seen in nature. For example, the bone, human skin and the bamboo tree are all different forms of FGM. With the developments in manufacturing methods, (FGMs are taken into account in the sandwich structure industries. In general, the sudden change in the material properties of sandwich structure from one layer to another can result in stress concentrations which often lead to delamination. To overcome this problem, The FG sandwich structure is proposed because of the gradual variation of material properties at the interfaces between the face layers and the core.

Several researchers used power-law FG sandwich plates [4]. Daikh and Zenkour [5] proposed another FG sandwich plates based on sigmoid function. In this paper, we present an analytical analysis on buckling of new FG sandwich plates based on variable exponential function E-FGM. The equilibrium and stability equations are obtained based on new higher-order shear deformation plate theory (HSDT).

2. FGM SANDWICH PLATES

Consider a rectangular E-FGM sandwich plate composed of three layers as. The vertical positions of the bottom, the two interfaces, and the top are denoted by $h_0 = -h/2$, h_1, h_2 and $h_3 = h/2$, respectively.

The volume fraction of the sandwich plate can be expressed as [6]:

$$V^{(1)}(z) = \left(\frac{z-h_0}{h_1-h_0}\right)^k, \quad h_0 \le z \le h_1$$

$$V^{(2)}(z) = 1, \quad h_1 \le z \le h_2$$

$$V^{(3)}(z) = \left(\frac{z-h_3}{h_2-h_3}\right)^k, \quad h_2 \le z \le h_3$$

Where k denotes volume fraction index. By using the rule of mixture, the effective material properties $P^{(n)}$ of layer n (n=1, 2, 3) can be expressed as:

For Power-law FG sandwich plate:

$$P^{(n)}(z) = P_m + (P_c - P_m)V^{(n)}(z) \tag{1}$$

For exponential FG sandwich plate:

$$P^{(n)}(z) = P_m + e^{ln(P_c/P_m).V^{(n)}(z)} \tag{2}$$

3. MATHEMATICAL FORMULATIONS

Based on the HSDT, the displacement components may be expressed as follows :

$$u(x,y,z) = u_0 - z\frac{\partial w_0}{\partial x} + f(z)\varphi_1 \;, \quad v(x,y,z) = v_0 - z\frac{\partial w_0}{\partial y} + f(z)\varphi_2 \;, \quad w(x,y,z) = w_0 \tag{3}$$

Where (u_0, v_0, w_0) and (φ_1, φ_2) denote the displacements and rotations of transverse normals on the plane $z = 0$, respectively. The displacement field of Third-order shear deformation plate theory of Reddy [7] is obtained by setting $f(z) = z\left(1 - \frac{4z^2}{3h^2}\right)$, while the sinusoidal shear deformation plate theory of Touratier [8] is obtained by setting $f(z) = \frac{h}{\pi}\sin\left(\frac{z}{h}\right)$. The displacement field of the classical thin plate theory CPT is obtained by setting $f(z) = 0$, whereas the displacement of the first-order shear deformation plate theory FSDT is obtained by setting $f(z) = z$. the proposed HSDT is given as:

$$f(z) = z\left(1 - \frac{3z^2}{2h^2} + \frac{2z^4}{5h^4}\right) \tag{4}$$

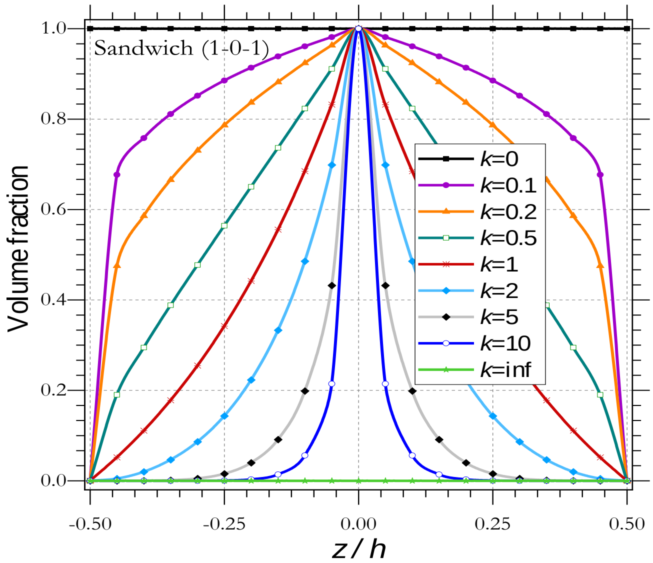

Fig. 1. Variation of volume fraction through the E-FGM sandwich plate thickness.

By using Hamilton's principle, the governing equations of motion of plate subjected to an uniform in-plane compressive loads $\bar{N}_{xx}^{0}$ and $\bar{N}_{yy}^{0}$ can be derived as:

$$\frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0 \ , \qquad \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_{yy}}{\partial y}=0 \tag{5}$$

$$\frac{\partial^2 M_{xx}}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_{yy}}{\partial y^2}+\bar{N}_{xx}^{0}\frac{\partial^2 w_0}{\partial x^2}+\bar{N}_{yy}^{0}\frac{\partial^2 w_0}{\partial y^2}-k_w w+k_s\left(\frac{\partial^2 w_0}{\partial x^2}+\frac{\partial^2 w_0}{\partial y^2}\right)=0 \tag{6}$$

$$\frac{\partial P_{xx}}{\partial x}+\frac{\partial P_{xy}}{\partial y}-R_{xz}=0 \ , \qquad \frac{\partial P_{xy}}{\partial x}+\frac{\partial P_{yy}}{\partial y}-R_{yz}=0 \tag{7}$$

k_w and k_s are coefficients of Winkler and Pasternak foundation. Here N, M, P and R are total in-plane force resultants, total moment resultants, additional stress couples and the transverse shear stress resultants respectively. The following approximate solution is seen to satisfy both the differential equation and the boundary conditions

$$\{u_0, \varphi_1\}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\{U_{mn}, X_{mn}\}\cos(\lambda x)\sin(\mu y) \tag{8}$$

$$\{v_0, \varphi_2\}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\{V_{mn}, Y_{mn}\}\sin(\lambda x)\cos(\mu y) \tag{9}$$

$$w_0=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}\sin(\lambda x)\sin(\mu y) \tag{10}$$

where $\lambda=m\pi/a$, $\mu=n\pi/b$ and U_{mn}^{1}, X_{mn}^{1}, V_{mn}^{1}, Y_{mn}^{1} and W_{mn}^{1} are arbitrary parameters. Finally we obtains

$$[L]\{\Lambda\}=0 \tag{11}$$

where $\{\Lambda\}=\{U, V, W, X, Y\}^T$ denotes the columns and $[L]$ is the stiffness matrix.

4. RESULTS

Consider a simply supported rectangular E-FGM sandwich plate made of a mixture of metal and ceramic subjected to in-plane load in two directions $\left(\bar{N}_{xx}^0 = \gamma_1 N_{cr},\ \bar{N}_{yy}^0 = \gamma_2 N_{cr}, (\gamma_1 = \gamma_2 = -1)\right)$. The combination of materials consists of Titanium alloy and Zirconia (Ti-Al6-4V/ZrO_2). Young's modulus for Titanium alloy are $E_m = 70$ Gpa, and for Zirconia are $E_c = 380$ Gpa. Poisson's ratio is chosen as constant $(\upsilon_c = 0.3)$. We note that several kinds of sandwich plates are used:

The (1–0–1) FGM sandwich plate: $h_1 = 0$, $h_2 = 0$

The (1–1–1) FGM sandwich plate: $h_1 = -h/6$, $h_2 = h/6$.

The (1–2–1) FGM sandwich plate: $h_1 = -h/4$, $h_2 = h/4$.

The (2–1–2) FGM sandwich plate: $h_1 = -h/10$, $h_2 = h/10$.

The (2–2–1) FGM sandwich plate: $h_1 = -h/10$, $h_2 = 3h/10$.

The (2–1–1) FGM sandwich plate: $h_1 = 0$, $h_2 = h/4$.

For convenience, the following non-dimensionalizations are used in presenting the numerical results in tabular form:

$$\bar{N} = \frac{N_{cr} a^2}{E_m h^3}, \qquad K_w = \frac{k_w a^4}{D_m}, \qquad K_s = \frac{k_s a^2}{D_m}, \qquad D_m = \frac{E_m h^3}{12(1-\upsilon^2)}$$

Table 1. Nondimensional critical buckling load of P-FGM sandwich plates.

Theory	1 0 1	1 1 1	1 2 1	2 1 2	2 2 1	2 1 1
Present	2.58391	3.23253	3.75317	2.92032	3.47479	3.09713
TSDT [7]	2.58357	3.23237	3.75328	2.92003	3.47472	3.09697
SSDT [7]	2.58423	3.23270	3.75314	2.92060	3.47490	3.09731
FSDT [7]	2.57118	3.21946	3.74182	2.90690	3.46286	3.08510
CPT [7]	2.66624	3.34075	3.89203	3.01366	3.59831	3.20195

Table 2. Nondimensional critical buckling load of e-fgm sandwich plate resting on elastic foundations.

Kw, Ks	a/h	k	1 0 1	1 1 1	1 2 1	2 1 2	2 2 1	2 1 1
0, 0	5	0	8.01053	8.01053	8.01053	8.01053	8.01053	8.01053
		0,5	3.65596	4.43065	5.02370	4.06557	4.70828	4.26937
		1	2.62225	3.42662	4.12891	3.02674	3.74973	3.25509
		2	1.99244	2.70730	3.44526	2.32765	3.04479	2.55819
		5	1.66251	2.22285	2.93298	1.90551	2.55272	2.12555
	20	0	9.67638	9.67638	9.67638	9.67638	9.67638	9.67638
		0,5	4.11897	5.00562	5.73281	4.57533	5.35301	4.83267
		1	2.90481	3.79145	4.62595	3.33562	4.18149	3.61311
		2	2.19434	2.94813	3.80316	2.52788	3.34470	2.80094
		5	1.85871	2.39425	3.19993	2.05502	2.77488	2.31264
0, 100	5	0	17.16803	17.16803	17.16803	17.16803	17.16803	17.16803
		0,5	12.81347	13.58816	14.18121	13.22308	13.86579	13.42687
		1	11.77976	12.58413	13.28642	12.18425	12.90724	12.41260

		2	11.14995	11.86481	12.60277	11.48516	12.20230	11.71570
		5	10.82002	11.38036	12.09049	11.06302	11.71023	11.28306
	20	**0**	18.83389	18.83389	18.83389	18.83389	18.83389	18.83389
		0,5	13.27648	14.16313	14.89031	13.73284	14.51052	13.99018
		1	12.06232	12.94896	13.78346	12.49313	13.33900	12.77062
		2	11.35185	12.10564	12.96067	11.68539	12.50221	11.95844
		5	11.01622	11.55176	12.35744	11.21253	11.93239	11.47015
100, 0	**5**	**0**	8.47445	8.47445	8.47445	8.47445	8.47445	8.47445
		0,5	4.11989	4.89458	5.48763	4.52949	5.17220	4.73329
		1	3.08618	3.89054	4.59284	3.49066	4.21366	3.71901
		2	2.45636	3.17123	3.90918	2.79158	3.50871	3.02211
		5	2.12643	2.68678	3.39691	2.36943	3.01665	2.58947
	20	**0**	10.14030	10.14030	10.14030	10.14030	10.14030	10.14030
		0,5	4.58289	5.46955	6.19673	5.03926	5.81693	5.29660
		1	3.36873	4.25538	5.08987	3.79954	4.64541	4.07703
		2	2.65827	3.41206	4.26708	2.99181	3.80862	3.26486
		5	2.32264	2.85818	3.66386	2.51894	3.23881	2.77657
100, 100	**5**	**0**	17.63196	17.63196	17.63196	17.63196	17.63196	17.63196
		0,5	13.27740	14.05209	14.64514	13.68700	14.32971	13.89080
		1	12.24368	13.04805	13.75035	12.64817	13.37117	12.87652
		2	11.613878	12.32874	13.06669	11.94909	12.66622	12.1796
		5	11.28394	11.84428	12.55442	11.52694	12.17416	11.74698
	20	**0**	19.29781	19.29781	19.29781	19.29781	19.29781	19.29781
		0,5	13.74040	14.62706	15.35424	14.19677	14.97444	14.45411
		1	12.52624	13.41289	14.24738	12.95705	13.80292	13.23454
		2	11.81578	12.56957	13.42459	12.14932	12.96613	12.42237
		5	11.48015	12.01569	12.82137	11.67645	12.39632	11.93408

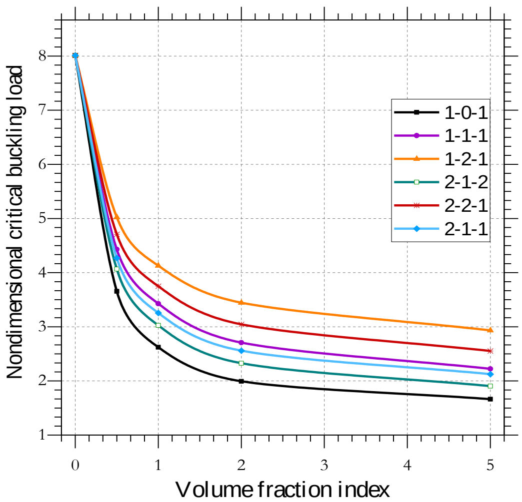

Fig. 2. Nondimensional critical buckling load of square E-FGM sandwich plate versus the inhomogeneity parameter (*a*/*h*=5, *a*/*b*=1).

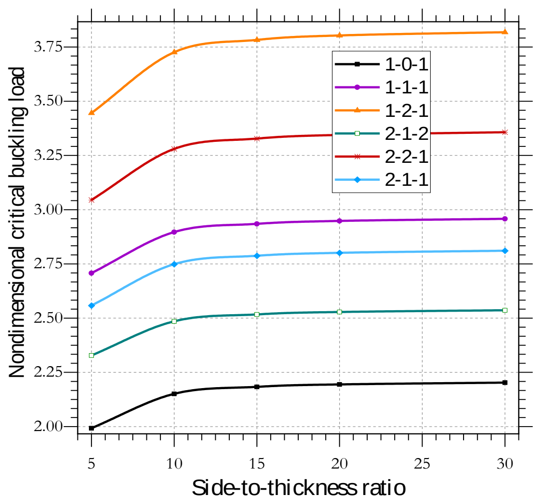

Fig. 3. Nondimensional critical buckling load of square E-FGM sandwich plate versus the side-to-thickness ratio (*a*/*b*=1, *k*=2).

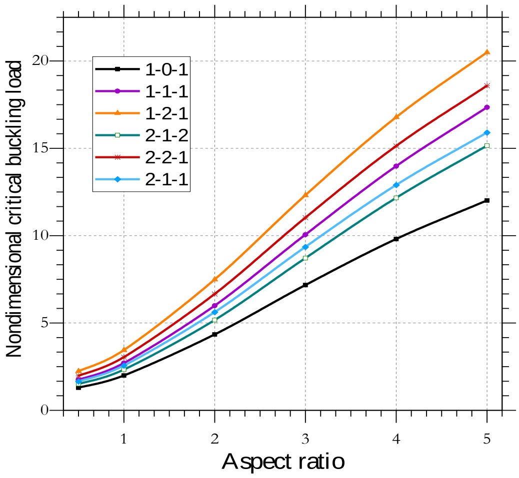

Fig. 4. Nondimensional critical buckling load of square E-FGM sandwich plate versus the aspect ratio (a/h=5, k=2).

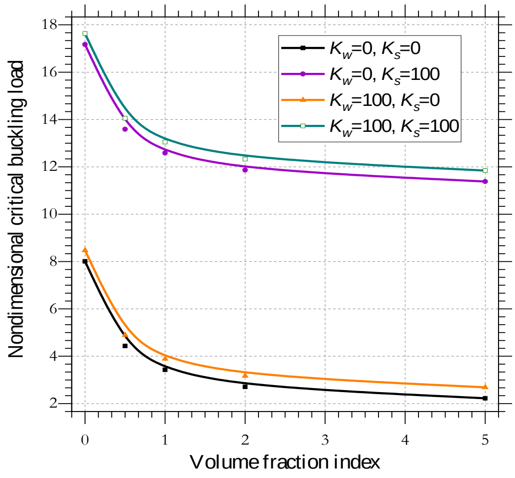

Fig. 5. Nondimensional critical buckling load of E-FGM sandwich plate (1-1-1) on elastic foundation versus the inhomogeneity parameter (a/h=5, a/b=1).

The table 1 illustrates the impact of the sandwich schemes on power-law functionally graded sandwich plate (P-FGM) is presented by using different plates theories. It is clear that the present theory is in a good agreement with those generated by Zenkour (2005), and particularly the third-order shear deformation theory.

The effect of inhomogeneity parameter, side-to-thickness and elastic foundations parameters on the nondimensional critical buckling load of exponential functionally graded plate using different sandwich schemes are presented in table 2. Figure 1 shows the effect of inhomeneity parameter k on the nondimensional critical bukling load. It can be seen that with the increase of the parameter k, nondimensional critical buckling load decreases. The case of k=0, mean that all the schemes of sandwiches have the same composition (céramique), this can explain the same results in this case.

In Fig. 2, the variations in nondimensional critical buckling load of FGM sandwich plates for different sandwichs schemes versus the side-to-thickness ratio a/h. It can be observed that with the increase of the side-to-thickness ratio, nondimensional critical buckling load decreases gradually wherever the sandwich scheme is Figure 3 presents the nondimensional critical buckling load versus the aspect ratio b/h. From this figure, we can see that the critical bucling load increases with the increasing of the aspect ratio a/b. The effect of the parameter k and the elastic foundation on the nondimensional critical buckling load is shown in the figure 5.

4. CONCLUSION

In this paper, nondimensional critical buckling load of exponential functionally graded sandwich plates have been analyzed. Different types of E-FGM sandwich plates are presented. Material properties of E-FGM layers are assumed to vary continuously through-the-thickness according to an exponential function in terms of the volume fractions of the constituents. The equilibrium and stability equations of E-FGM sandwich plates have been derived based on the higher-order shear deformation theory. As a result, the characteristics of nondimensional buckling load for E-FGM sandwich plates are significantly influenced by volume fraction distributions, and the geometric parameters. The inhomogeneity parameter *k* has considerable effect on the nondimensional critical buckling load of E-FGM sandwich plate. The nondimensional critical buckling load of FGM sandwich plates is high in thin and long plates.

6. REFERENCES

[1] Miyamoto Y, Kaysser WA, Rabin B H, Kawasaki A, Ford R G., Functionally Graded Materials: Design, Processing and Applications, Kluwer Academic, Boston, (1990).

[2] Roy S., Functionally graded coatings on biomaterials: a critical review. Functionally graded coatings on biomaterials: a critical review. Materials today, (2020). (18), 100375. https://doi.org/10.1016/j.mtchem.2020.100375

[3] Finot M, Suresh S., Bull C., Sampath S., Curvature changes during thermal cycling of a compositionally graded Ni/$A1_2O_3$ multi-layered material, Mat. Sci. Eng. A–Struct., (1996), **205**, 59–71.

[4] Zenkour A M., A comprehensive analysis of functionally graded sandwich plates: Part 2—Buckling and free vibration. International Journal of Solids and Structures, (2010), 42, 5243–5258.

[5] Daikh A. A. and Zenkour A M., Effect of porosity on the bending analysis of vaious functionally graded sandwich plates, Mater. Res. Express, (2019), 6 065703.

[6] Zenkour A M, Sobhy M., Thermal buckling of various types of FGM sandwich plates, Composites Structure, (2010), **93**, 93–102.

[7] Reddy J N., A general non-linear third order theory of plates with moderate thickness, International Journal of Non-linear Mechanics, (1990), **25** (6), 677–686.

[8] Touratier M., An efficient standard plate theory, International Journal of Engineering Science, (1991), **29**(8), 901–916.