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# ADAPTIVE FIR FILTER DESIGN APPLIED ON DIGITAL IMAGE COMPRESSION IN WAVELETS DECOMPOSITION

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**Abstract:**
*In this paper, data image compression with loss, wavelet and adaptive filtering are considered. Performances of the subband adaptive digital filter are discussed. We propose an adaptive low pass filter in the wavelets decomposition tree. The idea is to decompose a digital image using an adaptive FIR filter in each low frequency subband and compare it with an invariant filter of Daubechies. The adaptive filter is conceived by the ADFFLS (Adaptive Digital Filter with Fast Least Square) method. We employ a framework that includes the main comparison parameters. It is clearly shown here that it provides better performances than invariant filters in most applications. The filter performances on the different stages of the data image compression chain are then valued and compared with an invariant and biorthogonal filter of Daubechies. This is done studying of the main parameters. This study resides in the meticulous choice of comparison parameters and the different stages of comparison. Results show that the PSNR, the correlation between original and reconstructed images and the compression rate are better with the adaptive filter for different lengths, quantifiers, and quantification levels.*

**Key words:**
*Adaptive filtering, wavelet, subband coding, source coding, image compression.*

**Résumé:**
*Dans cet article, la compression avec pertes des images numériques, les sous-bandes et ondelettes et le filtrage adaptatif sont considérés. Nous proposons un filtre adaptatif passé bas dans l'arbre de décomposition en sous-bandes. L'idée consiste à décomposer une image numérique, utilisant un filtre RIF adaptatif passe bas à tous les niveaux de décomposition et le comparer à un filtre invariant de Daubechies. Le filtre adaptatif est conçu par la méthode des moindres carrés rapides (ADFFLS). Les performances du filtre au niveau de chaque étage de la chaîne de compression sont évaluées et comparées à celles d'un filtre biorthogonal de Daubechies, moyennant certains paramètres. L'essentiel de ce travail réside dans le choix des parameters, les différentes étapes de comparaison au niveau de la chaîne de codage de source et enfin, le type d'application. Les résultats que nous avons obtenu avec le filtre adaptatif en matière de PSNR, de corrélation entre les images originale et reconstruite et de taux de compression, sont meilleurs quelque soit la longueur du filter, pour différents quantificateurs et niveaux niveaux de quantification.*

## 1. Introduction:

The study of image compression method has been an active area of research. For transformation coding, the *Karhunen-Loève* Transformation (KLT) is the optimal linear transformation in the sense that it minimizes the mean squared reconstruction error. Many adaptive methods have been used in lossy composed of a wavelet decomposition tree, different quantifiers and the *Huffman* encoding. These methods go from conventional adaptive filtering up to complex combinations, where some or all the elements of the chain are time varying [1-2-3-4]. This paper proposes an approach to adaptation transform coding in which the low pass filters in the wavelet decomposition tree are time varying. This paper is organised as follows: Section 2 reviews the techniques of adaptive digital filtering. Section 3 presents the application of the conceived adaptive filter in the lossy source coding chain and the mean comparison parameters. Performances of this method for compressing images, results and discussion are investigated in section 4. Finally, section 5 concludes the paper.

## 2. Adaptive digital filtering:

Features in adaptive filtering are time varying according to certain predefined criteria. The principle of an adaptive digital filter is shown in figure 1. The error signal *e(n)* is used to obtain the best coefficients of the programmable filter, according to some predefined criteria. Two categories appear according to the choice of reference *y(n)*: If the input signal is taken as reference then, the filter is called of prediction, otherwise, if *y(n)* is different of the input signal then, the adaptive filter realizes a modelling of the system that has produced this reference. The main parameters common to the different algorithms for the coefficients update are [2-5]:

- The number of coefficients;
- The number of iterations;
- The choice between prediction and modelling;
- The addition of a noise signal to the input signal or to the reference signal;
- The value of the adaptation step or that of the length of the observation temporal window.

The most important option in adaptive filtering is the algorithm for updating the coefficients. The gradient algorithm is the most used in 1-D signal. The Fast Least Square algorithms continue to develop and may present some considerable advantages in processing speed and in adaptation precision [6-5].

### 2.1. ADFLMS algorithm

In this conception, there is an appearance of two filter structures: The transverse and the lattice structures. Many versions of the gradient algorithm exist in the literature [2-5]. In the basis version of the transverse structure, the system equations are the following [2-5]:

$$e(n+1) = y(n+1) - H^t(n)x(n+1) \quad (1)$$

$$H(n+1) = H(n) + \delta x(n+1)e(n+1) \quad (2)$$

### 2.2. ADFFLS algorithm

In this conception, as in ADFLMS (see section 2.1), appear the transverse and the lattice structures. In addition, two more parameters intervene: the weighting factor *W* and the initial energy of the prediction error $E_0$. In transverse structure, the coefficients update is obtained using the following equation [5-6]:

$$H(n+1) = H(n) + R_L^{-1}(n+1)x(n+1)e(n+1) \quad (3)$$

Some simulation examples in the modelling of the lattice structure show clearly the performances improvement obtained with the least square algorithm with regard to the gradient algorithm. Reference [6] presents in details, the principles, structures and algorithms of the adaptive filtering, as well as some related applications.

## 3. Applications:

In this paper, the work has been applied to a lossy source coding chain (see Fig.3), consisting of a two levels structure wavelets decomposition (see Fig.2). Three types of scalar quantifiers and a *Huffman* encoding have been used [1]. The ADFFLS algorithm (see Section 2.2) has been chosen for the design of the adaptive FIR filter. This latter has been applied in every low frequency subband of the decomposition tree (see Fig.2). Performances of the adaptive filter ($A_d$) are compared with those of the invariant filter of

*Daubechies* (bior) [8-9]. Simulations are made using the popular (256x256) *Lena* image (see Fig.4).

**3.1. Comparison parameters:**

In order to evaluate the performances of the designed adaptive filter, this latter has been compared to a *Daubechies* wavelet filter using the following parameters [8-9]:

**3.1.1. Covariance:**

The covariance between two images $X$ and $Y$ is given by the following equation:

$$\mathrm{cov}(X,Y)=\frac{\sum_{i=0}^{N-1}\sum_{j=0}^{M-1}X_c(i,j)Y_c(i,j)}{NM} \quad (4)$$

**3.1.2. Covariance matrix:**

The covariance matrix ''$M_C$'' is calculated for the four sub-images after decomposition (see Fig. 2). Two interesting parameters are derived from this matrix: the subband coding gain ''$G_s$'' and the concentration of the energy ''$C_E$'' in the low frequency subband ''LL''. The covariance matrices of *Lena* obtained after two decompositions levels using adaptive ($A_d$) and invariant (bior) filtering of length $L=16$ are given in expressions (5) and (6)

$$Mc_{(A_d)}=\begin{array}{c|cccc|} & LL & LH & HL & HH \\ LL & 12196 & 86 & 20 & 58 \\ LH & -86 & 244 & 1 & 12 \\ HL & 20 & 1 & 161 & 9 \\ HH & 58 & 12 & 9 & 68 \end{array} \quad (5)$$

$$Mc_{(bior)}=\begin{array}{c|cccc|} & LL & LH & HL & HH \\ LL & 15889 & 2 & 113 & -9 \\ LH & 2 & 84 & 3 & 2 \\ HL & 113 & 3 & 467 & 3 \\ HH & -9 & 2 & 3 & 3 \end{array} \quad (6)$$

$$G_S=\frac{1}{N_{SB}}\frac{\sum_{i=1}^{N_{SB}}\mathrm{cov}(i,i)}{\sqrt[N_{SB}]{\sum_{i=1}^{N_{SB}}\mathrm{cov}(i,i)}} \quad (7)$$

$$C_E(i)=\frac{\mathrm{cov}(i,i)}{\sum_{j=1}^{N_{SB}}\mathrm{cov}(j,j)}100 \quad (8)$$

**3.1.3. Correlation:**

The normalised and centred correlation is defined as:

$$C(x,y)=\frac{E(xy)-E(x)E(y)}{E(x^2)E(y^2)} \quad (9)$$

In practice, the correlation is estimated using the following relation:

$$C(X,Y)=\frac{1}{T}\frac{\sum(X-\bar{X})(Y-\bar{Y})}{\sqrt{\sum(X-\bar{X})^2\sum(Y-\bar{Y})}} \quad (10)$$

**3.1.4. PSNR:**

The maximum signal to noise ratio is given by the following equation:

$$PSNR=10\log\frac{255^2}{QME^2} \quad (11)$$

**3.1.5. Quantization and encoding:**

For the entire obtained subband, scalar quantifiers have been chosen for their easiness. They are either uniform or adapted to the coefficients distribution (using *Lloyd-Max* for example) [10] for these:

- The three scalar quantifiers have been studied separately: Uniform, *Lloyd-max* and DPCM;
- The PSNR of the low frequency subband LL for different quantification levels has been studied;
- The compression rate $R$ and a *Huffman* encoding have been studied [11].

**4. Results and discussion:**

The obtained results are summarised in Figs. 5, 6 and Tables 1, 2. The covariance matrices of *Lena* obtained after two decomposition levels, using adaptive filtering ($A_d$) and invariant filtering (bior) of length $L=16$, are given in expressions (5) and (6) respectively. The correlation between the original image and the *LL* subband, as well as the PSNR are better with an adaptive filter ($A_d$) and this whatever its length is (see Fig.5) (see Tab.1). The reconstructed image is obtained after two wavelets decomposition levels, a scalar quantification and a *Huffman* coding (see Fig. 6). The *LL* subband has been quantified differently for every reconstructed image (uniform, *Lloyd-max* and DPCM). Each of the subbands *LH, HL* and *HH* has been through a uniform scalar quantification (see Fig. 2). Quantification levels of 22, 10, 19 and 8 are affected respectively to subbands *LL, LH, HL* and *HH.* The same work has been realised for quantification levels of 22, 9, 7 and 3 (see Fig.6). These results show the importance of the choice of the quantifier, mainly for the low frequency subband and the quantification levels. Table 2 shows the bit allocation performances of the adaptive filter ($A_d$), comparing it with a biorthogonal filter of *Daubechies* ($bior_{2-2}$) of length $L=6$. In this comparison, the *Lloyd-Max* and the DPCM algorithms work the same way and offer the same performances in the two cases.

## 5. Conclusions:

A comparative survey to adaptive compression is proposed in this paper, based on an adaptive digital FIR filter. The results presented herein have shown that the adaptive filtering can outperform the globally optimal linear transform. The same image was coded using both the invariant filter of Daubechies and the adaptive filter. For our approach, the correlation, the PSNR, the compression ratio are better and the image quality was improved. A better compromise for the main comparison parameters is obtained for a *Lloyd-Max* quantifier in the low frequency subband LL Finally, the obtained results show indeed the interest of the use of the adaptive filter. In fact, the correlation and the PSNR are better whatever its length is. The different quantifiers offer a better compression ratio and a better compromise for the correlation and the PSNR, in addition the compression ratio is obtained for a *Lloyd-max* quantifier in the low frequency subband *LL.*

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| *Symbol* | *Designation* |
|---|---|
| $\delta$ | *Adaptation step* |
| $A_d$ | *Adaptive* |
| ADFFLS | *Adaptive Digital Filter with Fast Least Square* |
| ADFLMS | *Adaptive Digital Filter with the Least Mean Square algorithm* |
| bpp | *Bits per pixel* |
| Bior | *Biorthogonal* |
| $C_E$ | *Concentration of Energy* |
| C(X,Y) | *Correlation* |
| DPCM | *Differential Pulse Coded Modulation* |
| e(n) | *Error signal* |
| $E_0$ | *Prediction error* |
| E | *Expectation* |
| FIR | *Filter Impulse Response* |
| $G_s$ | *Subband coding gain* |
| HL | *High-Low* |
| HH | *High-High* |
| H(n) | *Vector of the $L^{th}$ filter coefficients* |
| $H^t(n)$ | *Transposed vector of H(n)* |
| LH | *Low-High* |
| LL | *Low-Low* |
| L | *Filter Length* |
| M | *Number of columns* |
| $M_C$ | *Covariance matrix* |
| N | *Number of lines* |
| PSNR | *Pic signal noise ratio* |
| QME | *Quadratic means error* |
| $R_L^{-1}(n)$ | *Estimated auto-correlation matrix of the input signal* |
| W | *Weighting factor* |
| USQ | *Uniform Scalar Quantization* |
| x(n) | *Vector of the $L^{th}$ most recent input data* |
| X | *Original image* |
| $X_c$ | *Image X centred with regard to the mean* |
| $\overline{X}$ | *Mean value of the original image* |
| y(n) | *Reference signal* |
| Y | *Reconstructed image* |
| $Y_c$ | *Image Y centred with regard to the mean* |
| $\overline{Y}$ | *Mean value of the reconstructed image* |

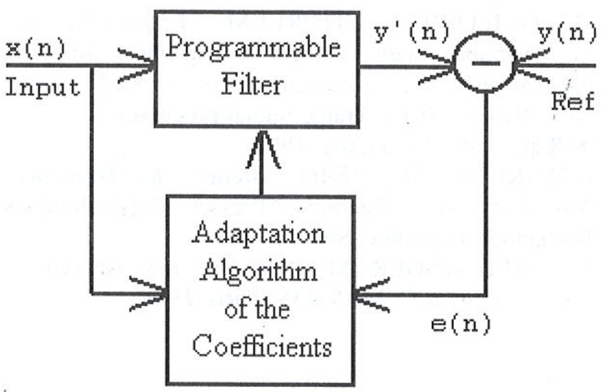


Figure 1. Structure of adaptive filter

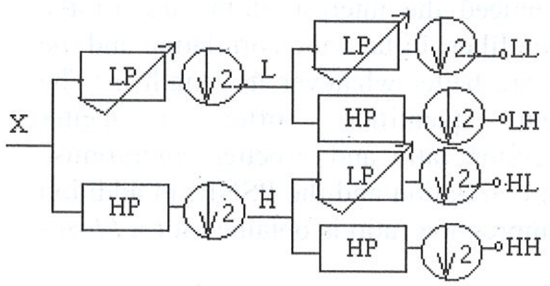


Figure 2. Tree structure for 4-band separable subband filter.

Table 2. "Lena" image bitrate "bpp" comparison between the adaptive ($A_d$) and the Daubechies (bior) filters, with length ''L=6'', different quantizations and quantification levels.

| ***Quantification Levels*** | ***(LL-LH-HL-HH) 22-9-7-3*** | | ***(LL-LH-HL-HH) 22-10-19-8*** | |
|---|---|---|---|---|
| **Filter** | $A_d$ | $Bior_{2.2}$ | $A_d$ | $Bior_{2.2}$ |
| **USQ** | 0.3173 | 2.0749 | 0.3757 | 1.8258 |
| **DPCM** | 0.3060 | 2.0281 | 0.3644 | 1.7790 |
| **Lloyd-Max** | 0.3197 | 2.1230 | 0.3782 | 1.8740 |

*a)*

*b)*

*c)*

*d)*

*e)*

*f)*

Figure 6. Reconstructed "Lena" image obtained with the following parameters: a) C=0.9953, PSNR=33.8820, bpp=2.0749 ; b) C=0.9948, PSNR=33.4857, bpp=2.0281 ; c) C=0.9955, PSNR=34.1115, bpp=2.1230 ; d) C=0.9864, PSNR=29.2545, bpp=1.8258 ; e) C=0.9859, PSNR=29.1151, bpp=1.7790 ; f) C=0.9866, PSNR=29.3322, bpp=1.8740