



The weak mixing angle effect on the cross sections of the $e^- e^+ \rightarrow \nu_l \bar{\nu}_l$ ($l = \mu, \tau$) interactions

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Abstract

In this study, the cross section $\sigma_{\mu,\tau}$ of the interaction $e^- e^+ \rightarrow \nu_i \bar{\nu}_i$; ($i = \mu, \tau$) was calculated .The effect of the weak mixing angle $\sin^2 \theta_w$ on the cross section $\sigma_{\mu,\tau}$ of the same reaction in the energy field $89 GeV \leq \sqrt{s} \leq 94 GeV$ was also studied. The effect of the center mass energy \sqrt{s} on the cross section $\sigma_{\mu,\tau}$ was also studied for two boundary values of the weak mixing angle $\sin^2 \theta_w$: 0.23221 , 0.2233. The study showed that the effect of the weak mixing angle on the sectional difference $\Delta\sigma_{\mu,\tau}(nb)$ gets in the energy field $\sqrt{s} = (91 \pm 1) GeV$, and takes a maximum value almost equal to the mass of the Z- boson .

Keywords: *the weak mixing angle, creation of (neutrino-antineutrino) pairs, annihilation of (electron-positron) pairs .*

Introduction:



In 1980, after a rigorous test of the standard model and the theory of uniformity of electromagnetic interactions with the weak interactions, the relationship $\sin^2 \theta_w = 1 - m_w^2 / m_z^2$ which defines the weak mixing angle became under scientific deliberation. Based on the importance of the weak mixing angle in the study of solar neutrino oscillation, the Borxino experiment (In recent years) , has examined the available data on the value of the weak mixing angle to confirm their current value and to confirm predictions of their value at energies below (1 MeV) [1-4].The ATLAS experiment also measured this angle by colliding the proton-proton at energy $\sqrt{s} = 8 TeV$ in the Large Hadronic Collider (LHC) and the value of this angle was equal to $\sin^2 \theta_w = 0.23140 \pm 0.00021$ [5].The weak mixing angle was precisely measured in LEP and SLC by electron collision with positron [6,7] and more accurately in Tevatron [8-13] and in LHC [14]. The combined mixing angle in LEP and SLC was equal to $\sin^2 \theta_w = 0.23149 \pm 0.00016$, and in Tifatron equals to $\sin^2 \theta_w = 0.23148 \pm 0.00033$ [13], while its value in the LHC was published by CMS [14] and was equal to $\sin^2 \theta_w = 0.23101 \pm 0.00053$. Hadron Collider recently provided another value for the weak mixing angle: $\sin^2 \theta_w = 0.23149 \pm 0.0007$ [15,16].In this paper, we calculated the cross-section of the reaction $e^- e^+ \rightarrow \nu_l \bar{\nu}_l$ ($l = \mu, \tau$) for energy values located in the vicinity mass of Z-boson. We focused on the effect of the weak mixing angle on it.

Goal and importance of research:

The weak mixing angle is a basic structural parameter in the standard model of particle physics (SM). Numerous experiments were concerned with determining their numerical value and the results were mostly different. This discrepancy created scientific excitement and multiple questions, and many research papers attempted to explain this discrepancy. This interest led us to study the effect of the weak mixing angle on the cross sections of interactions $e^-e^+ \rightarrow \nu_i\bar{\nu}_i$ ($i = \mu, \tau$) used in the calculation of the energy spectrum carried by the neutrino pairs ($\nu_i\bar{\nu}_i$) resulting from the electron-positron (e^-e^+) annihilation, in the hot stellar medium. The aim of this research is to study the differential cross-sections of the reactions: $e^-e^+ \rightarrow \nu_\mu\bar{\nu}_\mu$, $e^-e^+ \rightarrow \nu_\tau\bar{\nu}_\tau$, and to know the effect of the weak mixing angle $\sin^2\theta_w$ accurately using the center mass system for the energies which almost equal to the mass of the Z- boson. This simulates the hot stellar medium in which neutrino pairs ($\nu_i\bar{\nu}_i$) are formed by the annihilation of electron-positron (e^-e^+) pairs, as mentioned above. The importance is that neutrinos can carry energy from the hot stellar medium to the outer medium away from it and contribute to its cooling and thus can be considered a warning bell and a good cosmic message that helps predict the near death of stars and their end of life.

Research methods and materials:

First: Calculate the effective differential cross-sections of the reaction:

$$e^-e^+ \rightarrow \nu_l\bar{\nu}_l \quad (l = \mu, \tau) \quad (1)$$

We begin from the Feynman diagrams (Figure 1) corresponding to the relationship (1)

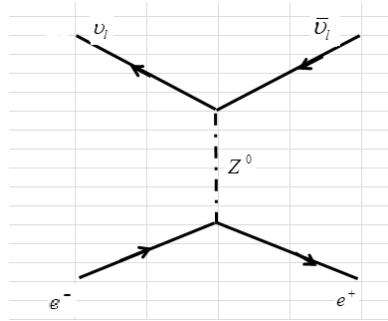


Figure (1): shows the Feynman diagram of the interaction $e^- e^+ \rightarrow \nu_i \bar{\nu}_i$ ($i = \mu, \tau$)

The amplitude for the process $e^- e^+ \rightarrow \nu_l \bar{\nu}_l$ ($l = \mu, \tau$) is shown

$$\begin{aligned}
 iM &= \frac{g^2}{2} \frac{D_{\mu\nu}^{(z)}(q)}{2 \cos^2 \theta_w} \bar{u}(k) \gamma^\mu \frac{1-\gamma^5}{2} v(k') \times \\
 &\times \bar{v}(p') \gamma^\nu \left(-\frac{1}{2} + 2 \sin^2 \theta_w + \frac{\gamma^5}{2} \right) u(p) \quad (2)
 \end{aligned}$$

Where $q = p + p' = k + k'$ expresses the four - momentum, while the symbols k', k, p', p is, respectively, the four momentum of the electron, positron, neutrino, anti-neutrino. While $D_{\mu\nu}^{(z)}(q)$ is a propagator for Z-boson according to Feynman's laws and it is write as:

$$D_{\mu\nu}^{(z)}(q) = \frac{i(-g_{\mu\nu} + q_\mu q_\nu / m_z^2)}{q^2 - m_z^2 + i\Gamma_z m_z} \quad (3)$$



Here,

Γ_z, m_z are the widths, masses of z-boson respectively.

The differential cross-sections for unpolarized incoming beams in C.M. system for the process $e^- e^+ \rightarrow \nu_i \bar{\nu}_i$ ($i = \mu, \tau$) is

$$\frac{d\sigma_{\mu,\tau}}{d\Omega} = \frac{\alpha^2 s}{\sin^4 2\theta_w} \frac{\sin^4 \theta_w \sin^4(\theta/2) + (1/2 - \sin^2 \theta_w)^2 \cos^4(\theta/2)}{(s - m_z^2)^2 + \Gamma_z^2 m_z^2} \quad (4)$$

Where

$$g = e/\sin \theta_w, \quad \alpha = e^2/4\pi, \quad d\Omega = 2\pi \sin \theta d\theta$$

Are adopted. The total cross-section can be calculated by integrity the relationship (4), then we find

$$\sigma_{\mu\tau} = \frac{\pi}{6} \frac{s \alpha^2}{(s - m_z^2)^2 + \Gamma_z^2 m_z^2} \left[\left(\frac{\sin^2 \theta_w}{\sin^2 2\theta_w} \right)^2 + \frac{(1/2 - \sin^2 \theta_w)^2}{\sin^4 2\theta_w} \right] \quad (5)$$

To illustrate the effect of the weak mixing angle on the cross section of the interaction $e^- e^+ \rightarrow \nu_{\mu,\tau} \bar{\nu}_{\mu,\tau}$ we will use the following table (1) [6].

Table (1)

Accelerator or detector	Value of $\sin^2 \theta_w$
LEP + SLD	0.23153 ± 0.00016
LEP + SLD : A_{FB}^{Qb}	0.23221 ± 0.00029
SLD: A_t	0.23098 ± 0.00026
CDF $ee + \mu\mu$ $9.4 fb^{-1}$	0.23221 ± 0.00046
DO $ee + \mu\mu$ $9.7 fb^{-1}$	0.23095 ± 0.00040
ATLAS $ee + \mu\mu$ $4.8 fb^{-1}$	0.23080 ± 0.000120

LHCb $\mu\mu$ $3 fb^{-1}$	0.23142 ± 0.00108
CMS $\mu\mu$ $18.8 fb^{-1}$	0.23125 ± 0.00060
CMSee $19.6 fb^{-1}$	0.23056 ± 0.00086
CMSee + $\mu\mu$	0.23101 ± 0.00053

Also we take the parameters, which appear in the formula, from PDG [17]:

$$m_z = 91.1876 GeV, \Gamma_z = 2.4952 GeV$$

$$m_w = 80.385 GeV, \Gamma_w = 2.085 GeV$$

$$\alpha = 1/137, m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau} \ll 0.0 eV$$

To study the change of cross-section values by the weak mixing angle we make the necessary calculations and arrange the results in Table(2)

Table (2)

\sqrt{s} (MeV)	89	90	91	92	93	94
$\sin^2 \theta_w$	$\sigma_1^{\mu,\tau}$	$\sigma_2^{\mu,\tau}$	$\sigma_3^{\mu,\tau}$	$\sigma_4^{\mu,\tau}$	$\sigma_5^{\mu,\tau}$	$\sigma_6^{\mu,\tau}$
0.23221	0.466	1.007	1.906	1.395	0.646	0.333
0.23142	0.470	1.012	1.924	1.408	0.652	0.336
0.23116	0.472	1.019	1.930	1.413	0.654	0.334
0.23098	0.473	1.021	1.934	1.419	0.657	0.339
0.22310	0.520	1.123	2.127	1.557	0.721	0.372

The dependence of the total unpolarized cross-sections on the weak mixing angle for six different values of \sqrt{s} is shown in Figures(2)and (3):

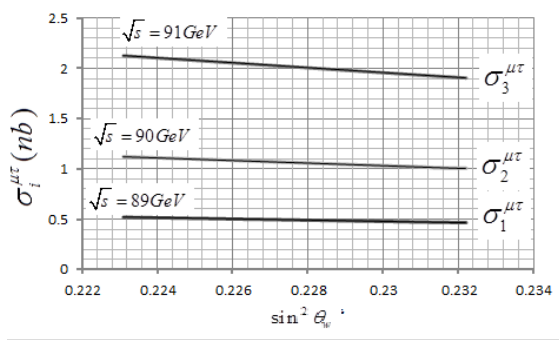


Figure (2): shows $\sigma_i^{\mu\tau} = f(\sin^2 \theta_w)$; $i = 1, 2, 3$ for $\sqrt{s} = 89, 90, 91 GeV$

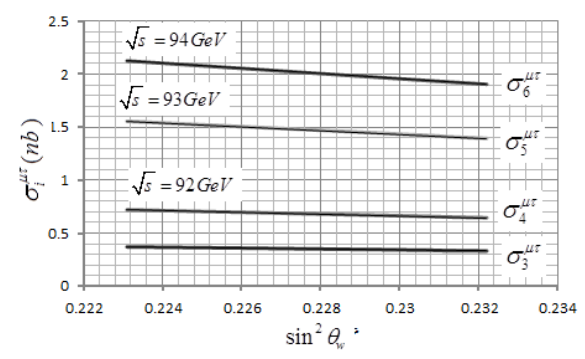


Figure (3): shows $\sigma_i^{\mu\tau} = f(\sin^2 \theta_w)$; $i = 3, 4, 5, 6$ for $\sqrt{s} = 91, 92, 93, 94 GeV$

Figure (2) contains the first three cross sections for $\sqrt{s} = 89, 90, 91 GeV$, while Figure (3) contains the last four cross sections for $\sqrt{s} = 91, 92, 93, 94 GeV$.

Now focusing our attention on the two cross-sections corresponding to the two boundary values of the weak

mixing angles:0.23221; 0.2231 in the energy field (89–94)GeV , then we get the Table (3)

Table (3)

$\sin^2 \theta_w = 0.23221$						
$\sqrt{s} (GeV)$	89	90	91	92	93	94
$\sigma_1^{\mu\tau} (nb)$	0.46	1.00	1.90	1.39	0.64	0.33
	6	7	6	5	6	3
$\sin^2 \theta_w = 0.2231$						
$\sqrt{s} (GeV)$	89	90	91	92	93	94
$\sigma_6^{\mu\tau} (nb)$	0.5	1.12	2.12	1.55	0.72	0.37
	2	3	7	7	1	2

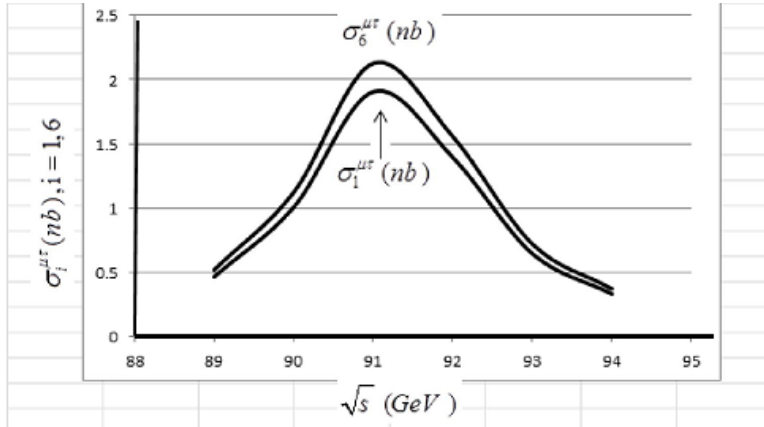


Figure (4): $\sigma_1^{\mu\tau} (nb) = f(\sqrt{s})$, and $\sigma_6^{\mu\tau} (nb) = f(\sqrt{s})$ for $\sin^2 \theta_w = 0.2231$; $\sin^2 \theta_w = 0.23221$.

We observe from Figure (4) that the cross section corresponding to the maximal mixing angle is putting under the cross section corresponding to the minimum mixing

angle. We define $\Delta \sigma_{61}^{\mu\tau}(nb)$ by the relation:
 $\Delta \sigma_{61}^{\mu\tau}(nb) = \sigma_6^{\mu\tau} - \sigma_1^{\mu\tau}$ and the curve $\Delta \sigma_{61}^{\mu\tau}(nb) = f(\sqrt{s})$ for the two weak mixing angles $\sin^2 \theta_w = 0.23221$, $\sin^2 \theta_w = 0.2231$ is plotted in Figure (5):

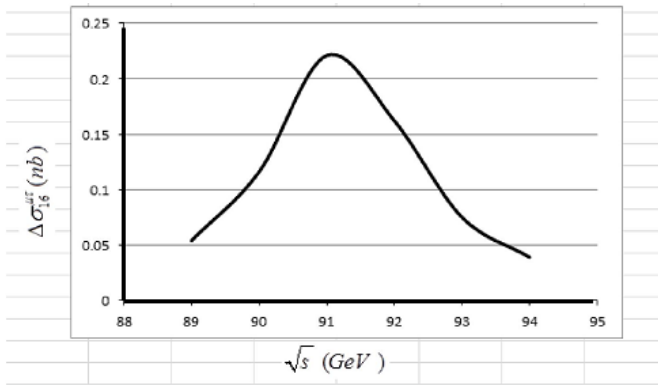


Figure (5): $\Delta \sigma_{61}^{\mu\tau}(nb) = f(\sqrt{s})$

For comparison, we put the last two curves in one curve, look Figure (6)

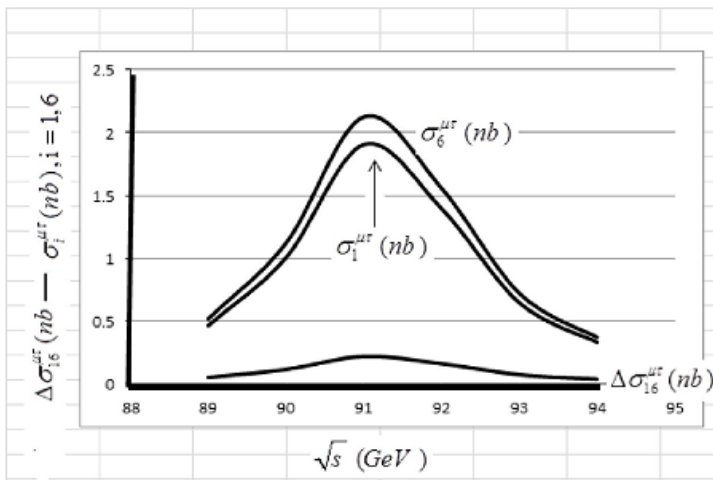


Figure (6) $\sigma_1^{\mu\tau}(nb) = f(\sqrt{s})$, $\sigma_6^{\mu\tau}(nb) = f(\sqrt{s})$, $\Delta\sigma_{16}^{\mu\tau}(nb) = f(\sqrt{s})$

Figure (6) shows the effect of the weak mixing angle on the cross section of the studied interaction in the energy field $89\text{GeV} \leq \sqrt{s} \leq 94\text{GeV}$

Results and discussion:

1. The value of the cross section increases with decreasing the value of the weak mixing angle and vice versa, as show Figures (2) and (3).
2. Figure (4) shows that the values of the cross-section of the reaction $(e^-e^+ \rightarrow \nu_{\mu,\tau}\bar{\nu}_{\mu,\tau})$ corresponding to the maximum mixing angle ($\sin^2\theta_w = 0.23221$) are smaller than the values of the same cross-section corresponding to the values of the minimum mixing angle ($\sin^2\theta_w = 0.2231$). This means that for a large cross section we need a small mixing angle and vice versa.
3. Figure (6) shows that the maximum influence of the mixing angle on the cross-section is obtained at energy $\sqrt{s} = (91 \pm 1) \text{ GeV}$ and the peak is approximately equal to $\sqrt{s} = 91 \text{ GeV}$, and outside this area the effect is approximately equivalent and not equal to zero.

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