Transmission Line Matrix (TLM) modeling of 1D phononic crystals

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***Abstract*:**

In this paper an acoustic Transmission Line Matrix (TLM) method for the calculation of phononic band gaps is presented. It is applied to the study of 1D propagation of longitudinal waves in a MEMS Silicon thin structure. The system is modeled as a series of acoustic transmission lines with different characteristic impedances representing the two phononic materials with different Young's modulus and mass density. The phononic band gap predicted by this model is verified against FEM results. Further application of the model to the design of 1D phononic crystals shows that increasing the holes length l_x of phononic crystal shifts the band gap towards lower frequencies. The broadening effect of band gaps was related to the filling factor. This broadening was found to reach it maximum when the lattice constant of the two phononic crystals are the same.
Keywords: Phononic crystal, TLM, Acoustic waves, modeling.

Résumé:
Dans cet article, une méthode acoustique de la Matrice de Line de Transmission (TLM) pour le calcul des largeurs de bande phononiques interdites est présenté. Il est appliqué à l'étude de la propagation des ondes longitudinales dans une structure mince 1D de MEMS au Silicium. Le système est modélisé sous la forme d'une série de lignes de transmission acoustique avec des impédances caractéristiques différentes, représentant les deux matériaux phononiques avec différents module de Young et masse volumique. La bande interdite phononique prévue par ce modèle est vérifiée par rapport aux résultats FÉM. En plus l'application du modèle de la conception des cristaux phononiques 1D montre que l'augmentation de la longueur des trous lx du cristal phononique décale la largeur de bande vers les basses fréquences. L'effet d'élargissement de la bande interdite est lié au facteur de remplissage. Cet élargissement atteint un maximum lorsque la constante du réseau des deux cristaux phononiques sont les mêmes.
***Mots-clés**: Cristaux phononiques, TLM, Ondes acoustiques, Modélisation.*

Nomenclature

Symbol	Designation	SI units
lx	Hole length along the x-axis	µm
ly	Hole length along the y-axis	µm

α_x	Filling ratio along the x-axis	
α_y	Filling ratio along the y-axis	
g	Capacitance transducer width	µm
ρ	Mass density	$kg.m^{-3}$
E	Young's modulus	MPa
$\sigma_x(x,t)$	Stress along x	N/m
$v_x(x,t)$	Velocity of the section at x	m/s
Θ	Voltage or current	V or A
G'	Conductance per unit length	$(\Omega m)^{-1}$
R'	Resistance per unit length	Ωm^{-1}
L'	Inductance per unit length	Hm^{-1}
C'	Capacitance per unit length	Fm^{-1}
Z_{ac}	Acoustic impedance	Ω
V_l^+	Wave front propagation from left side	
V_l^-	Wave front propagation from right side	
Σ_x	Pressure at point x	Pa
k	Wave vector	m^{-1}

1. Introduction

Since the pioneering work of Sigalas and Economou in 1993 on band structure of elastic waves in 2D systems [1], the propagation of elastic or acoustic waves in periodic structures has received considerable attention [2–6]. One of the main focus areas is to obtain frequency ranges, known as phononic band gaps or stop bands, in which the propagation of elastic or acoustic waves is forbidden. These phononic band gaps in periodic structures are commonly generated by two methods that are Bragg scattering and local resonances [7,8]. Recently, a new band gap generation method based on inertial amplification was introduced [9,10]. This latter method overcomes the drawbacks of the previous two methods at low frequencies. However, the existence of inertial amplification induced gaps has been only shown using lumped parameter (mass–spring) lattices [11]. Such finite periodic band gap structures can be used in mechanical filtering, demultiplexing of acoustic waves, phononic crystal plates and high-frequency radiofrequency devices, sonic insulators, acoustic and elastic waveguides and even energy harvesting [12,13,14]. Thus, maximizing isolation bandwidth of these systems and obtaining good isolation characteristics inside their bandwidth has important practical implications.

The amount of vibration transmission can be determined by analyzing the depth profile of the gap (stop band) in a frequency response function (FRF) plot [8–11]. Original phenomena, such as focusing or sound tunneling through a heterostructure, have already been observed in 3D phononic crystals [2,3]. Both these phenomena are related to the anisotropy or to the large sound velocity dispersion for frequencies outside, but close to the band gap. Getting a deeper understanding of these effects requires the prior knowledge of how the periodicity affects the wave transport.

2. Phononic crystals

Phononic crystals are artificial structures made of inclusions of a material A, periodically embedded in a host matrix of material B [15], Figure 1. Classical elastic/acoustic waves propagate in these materials where the density and/or elastic constants of the structure change periodically. This changes the speed of waves at interfaces between materials, which, in turn, leads to the formation of phononic band gap. Phononic crystals are in general materials with one-(1D), two-, or three-dimensional periodicity in their elastic properties. This spatial modulation with a proper choice of acoustic impedances and geometric features can lead to the appearance of forbidden frequencies (phononic band gaps) at which propagation of acoustic waves/phonons is not allowed. It is now well established, both theoretically and experimentally, that frequency band gaps over which the propagation of elastic waves is forbidden, exist in these systems, even if the mismatch of the acoustic impedance for the materials A and B is small.

Analogously to the photonic crystals, Kushwaha [16] proposed the concept of "phononic crystals" as an artificial periodic elastic/acoustic structure that exhibits the so-called phononic bandgap. The application of phononic crystal is mainly determined by the regulation performance of its bandgap. The regulation of bandgap in traditional research works is mainly fulfilled by changing the geometry, filling fraction and orientation of scatterers in a phononic crystal [17]. With the introduction of MEMS micromachining technologies, operating frequencies of phononic crystals have been scaled to a range suitable for radio-frequency and sensing applications [18, 19].

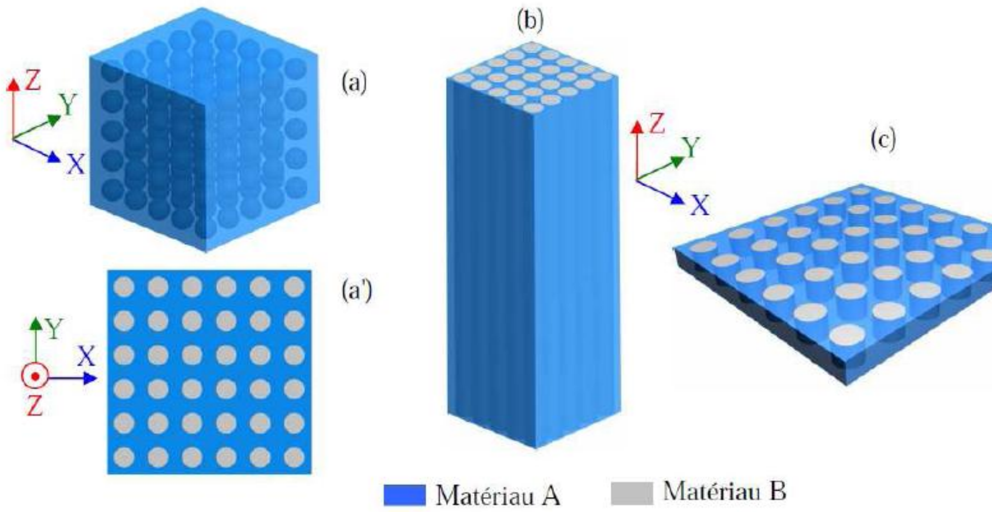

Fig.1 Phononic crystals obtained by inclusions of material B in a material A matrix:
(a-a') For bulk acoustic waves, (b) For surface acoustic waves, (c) For Lamb waves [15].

In this paper, we consider a MEMS structure in the form of 1D phononic crystal (SPnC) of rectangular-like periodic grooves made on the (001) surface of crystalline silicon, figure 2. The dimensions used in this example are ly = lx = 40 µm, l_{ROD} = 820 µm, $\alpha_x = \alpha_y = 0.5$ are the filling ratios along the x-axis and y-axis respectively, g = 1 µm, V = 20 V, v_{ac} = 1 V. The rod thickness is 60 µm. The mechanical properties (mass density ρ and Young's modulus E), which are needed to realize a phononic crystal, are achieved through rectangular perforations in the rod. The propagation of longitudinal acoustic waves along such a constant section structure is governed by the following equations of motion along x axis:

$$\frac{\partial \sigma_x(x,t)}{\partial x} = \rho \frac{\partial v_x(x,t)}{\partial t} \quad \text{and} \quad \frac{\partial v_x(x,t)}{\partial x} = \frac{1}{E}\frac{\partial \sigma_x(x,t)}{\partial t} \qquad (1)$$

where $\sigma_x(x,t)$ is the stress along x and $v_x(x,t)$ the velocity of the section at x. These equations lead to the wave equations:

$$\frac{\partial^2 \sigma}{\partial x^2} = \frac{\rho}{E}\frac{\partial^2 \sigma}{\partial t^2} \quad \text{or} \quad \frac{\partial^2 v}{\partial x^2} = \frac{\rho}{E}\frac{\partial^2 v}{\partial t^2} \qquad (2)$$

The first capacitor is polarized with two voltages: A DC voltage to maintain a permanent energy (force) level and a low AC voltage to create a frequency variable electrostatic force, which generated an electrostatic pressure. The second capacitor is a simple electrostatic sensor.

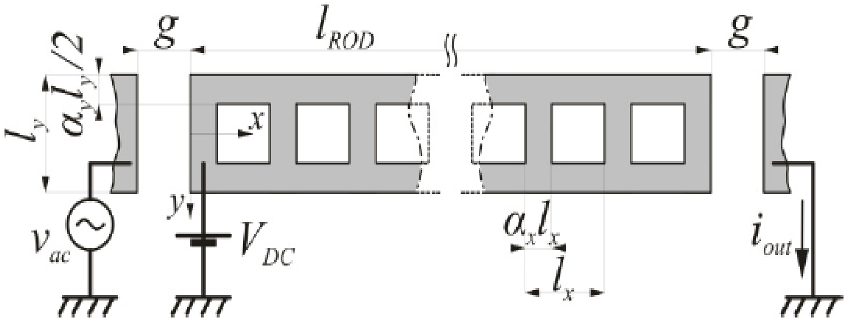

Fig.2. Schematic structure of the modeled rod with its driving and sensing terminations [20].

3. TLM Model

We seek to define the relationship between the frequency and the wave vector in relation to other parameters specific to the propagation medium. This relationship is called dispersion relation. This latter is linear in homogeneous media and depends directly on the wave velocity in the medium. Several numerical methods have been employed to investigate this dispersion in phononic crystals. Plane wave expansion (PWE) [21], finite difference time domain (FDTD) [22] and the layer multiple scattering method (LMSM) [23, 24] are among the well known methods. These methods are difficult to set and often unnecessary for simple cases where acoustic waves propagate in one direction. For this raison, we propose to use Transmission Line Matrix (TLM) method. It was originally developed in 1971 by B.P.Johns and R.Beurle [25] for modeling the propagation of electromagnetic waves in time domain. It was subsequently applied in several fields such as optical and acoustical propagation [26]. It is based on replacing the physical problem by a network of transmission lines. For a 1D problem, the electrical equivalent network is shown in figure 3. The application of Kirchhoff's laws to this equivalent circuit leads to the telegraph equation:

$$\frac{\partial^2\Theta}{\partial x^2} = G'R'\Theta + (G'L' + C'R')\frac{\partial\Theta}{\partial t} + C'L'\frac{\partial^2\Theta}{\partial t^2} \qquad (3)$$

Where Θ may be voltage or current. G', R', L' and C' are the conductance, resistance, inductance and capacitance per unit length, respectively.

To limit the model to wave propagation, the resistance and loss of the transmission line are neglected to leave a simplified equation:

$$\frac{\partial^2\Theta}{\partial x^2} = C'L'\frac{\partial^2\Theta}{\partial t^2} \qquad (4)$$

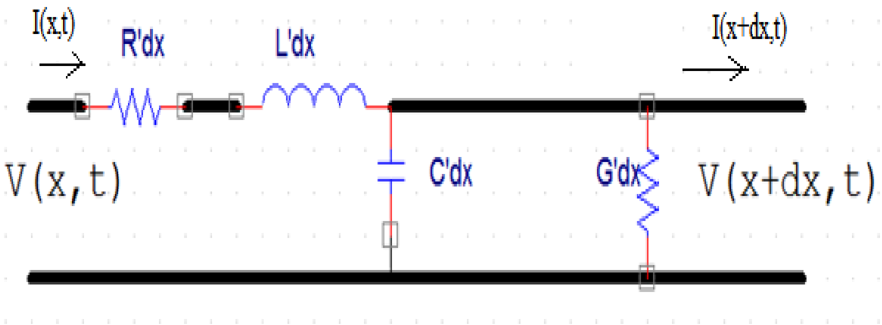

Fig.3 Electrical model of a transmission line.

This equation is similar to equation (2) as long as the stress and the velocity are mapped to the voltage and current along the line, respectively. To preserve consistency with the standard definitions in acoustics (were the hydrostatic pressure is used instead of the stress) the acoustic impedance is defined as $Z_{ac} = -\frac{\sigma}{v}$. In the analogy, mass density and the inverse of Young's modulus take the place of the inductance and capacitance per unit length of the line respectively. The characteristic acoustic impedance of the equivalent acoustic line is $Z_0 = \sqrt{\rho E}$. In the frequency domain, the equation solution is given by:

$$V_x(x) = V_x^+ e^{-ikx} + V_x^- e^{ikx} \tag{5}$$

$$\Sigma_x(x) = Z_0\left(-V_x^+ e^{-ikx} + V_x^- e^{ikx}\right) \tag{6}$$

Where V_x is the velocity and Σ_x is the pressure at point x. $k = \omega\sqrt{\rho/E}$ is the wave vector. The wave front V_l^+ propagates from left side along the positive direction of the x-axis. V_l^- propagates in the opposite direction. The transmission matrix that links the velocity waves from left and right in filled parts is:

$$\begin{pmatrix} V_l^+ \\ V_l^- \end{pmatrix} = \begin{bmatrix} e^{ikl_x\alpha_x} & 0 \\ 0 & e^{-ikl_x\alpha_x} \end{bmatrix} \begin{bmatrix} V_r^+ \\ V_r^- \end{bmatrix} = M_f \begin{bmatrix} V_r^+ \\ V_r^- \end{bmatrix} \tag{7}$$

A similar matrix M_p can be written for holes in the rod.

The matrix modeling the interface between the filled and perforated sections has the expression:

$$\begin{pmatrix} V_l^+ \\ V_l^- \end{pmatrix} = \begin{bmatrix} \frac{1+\alpha_y}{2\alpha_y} & \frac{\alpha_y-1}{2\alpha_y} \\ \frac{\alpha_y-1}{2\alpha_y} & \frac{1+\alpha_y}{2\alpha_y} \end{bmatrix} \begin{bmatrix} V_r^+ \\ V_r^- \end{bmatrix} = M_{fp} \begin{bmatrix} V_r^+ \\ V_r^- \end{bmatrix} \tag{8}$$

A similar matrix M_{pf} can be written to model the interface between the perforated and filled sections.

Finally, the transmittance of the phononic crystal for n^{th} holes is:

$$T = M_f\left[M_{fp} M_{pf} M_p M_f\right]^n \tag{9}$$

Equations (5) to (9) form the basic routine that the TLM method will be based on.

4. Results and discussion

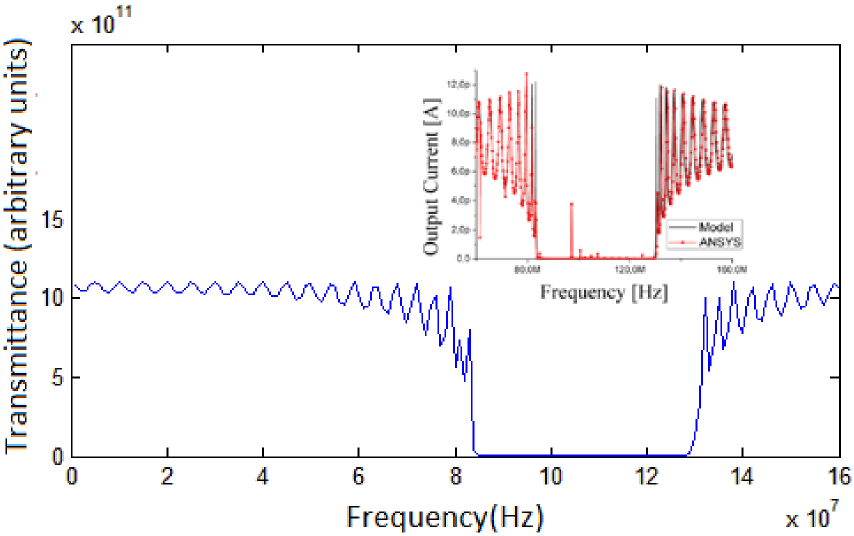

Fig.4. Frequency response of the 1D structure.
(The insert is the courant output for the same structure from reference [20]).

The generation of defect modes is mainly due to the function of impurity. Apart from the filling fraction of impurity, the change in the position, geometry and orientation of impurity can also generate the defect modes. To investigate the effect of impurity geometry and orientation on the defect modes, we consider the rod have a Young's modulus E=170MPa and a masse density $\rho = 2.33.10^3 kg.m^{-3}$ with 20 holes. Fig.4 shows the transmittance spectrum observed at the end of the rod. A band gap is clearly observed and corresponds to that found in reference [20]. The TLM model correctly predicts the position of the first frequency gap in good agreement with FEM simulations [20]. This position is found to occur at f=83MHz in good agreement with FEM results shown in the insert of Fig.4.

Fig.5 shows the effect of holes size on the frequency response. Notice that the band gap is shifted towards lower frequencies when le length lx is increased from 5µm to 10µm. The width of the gap band remains constant. Such results show that band gap width is related to materials physical properties and not to the structural geometry.

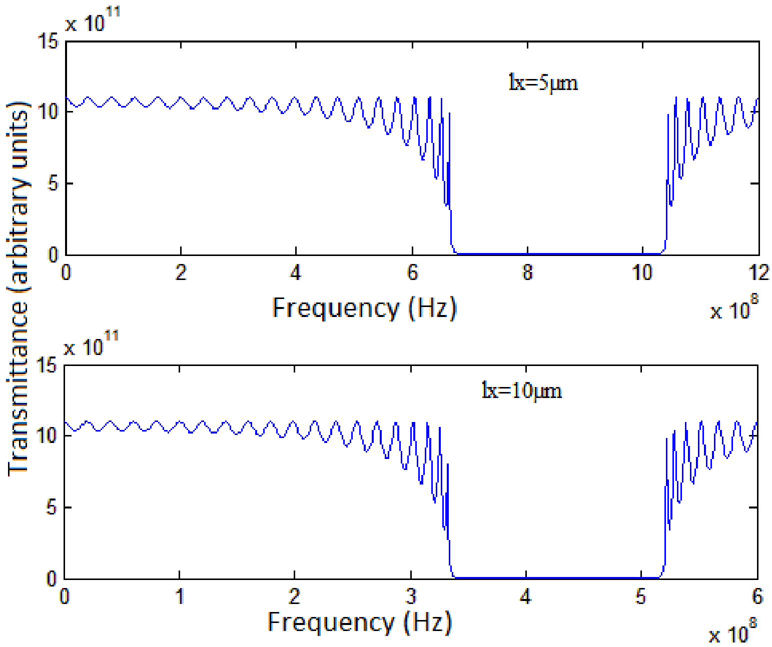

Fig.5 Effect of holes length on frequency response.

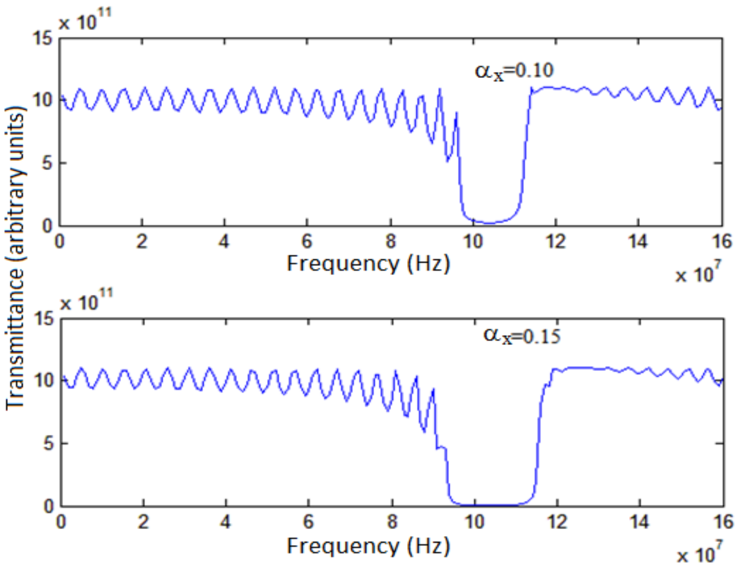

Fig.6 Effect of filling ratio on frequency response.

Fig.6 shows the effect of filling ratio along the x-axis α_x upon band gap width and band gap position. It is clear that this filling ratio has a big effect on band gap width and change only slightly the band gap position. The study shows that band gap width increase with α_x values up to 0.5 then start to decrease afterwards as reported on Fig.7. Such results show the importance of filling factor in the design of phononic crystals to be used for isolating resonators and filters.

5. Conclusion

The band structures of 1D phononic crystals with different lattice constant were investigated using the TLM method. The results show that the method is powerful in the simulation of these acoustic structures and may be used in the design of acoustic components for phononic applications. Increasing the lattice constant of one phononic crystal (holes length l_x in our case) was found to shift the band gap towards lower frequencies. The broadening effect of band gaps was related to the filling factor. This broadening was found to reach it maximum when the lattice constant of the two phononic crystals are the same. These preliminary results show the crucial role that TLM modeling can play in further enriching and developing phononic crystals for specific applications.

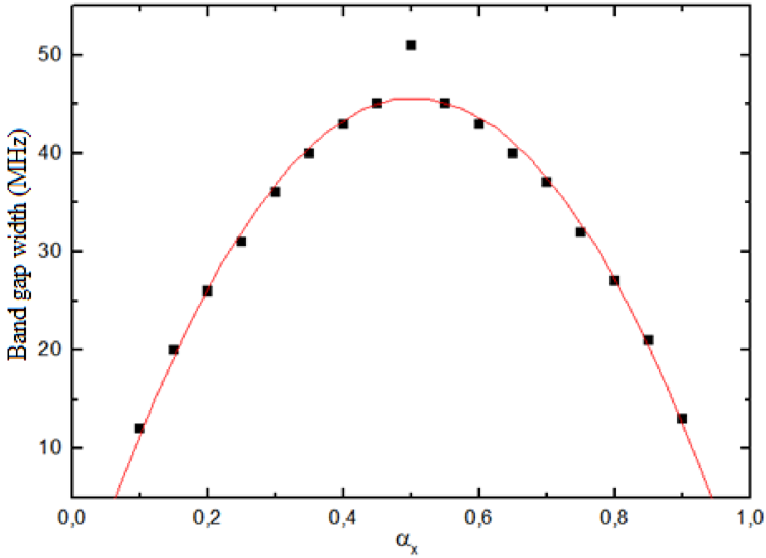

Fig.7 Change in band gap width with filling factor.

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