CFD simulation of heat transfer in a two dimensional vertical conical cylinder partially annular space

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Abstract *— In This work, numerical study of two-dimensional steady flow analysis has been made on natural convection in a differentially heated vertical conical cylinder partially annular space. The heat transfer is assumed to take place by natural convection. Where the inner and outer surfaces of annulus are maintained at uniform wall temperature. The CFD FLUENT12.0 code is used to solve the governing equations of mass, momentum, energy quantities using constant properties and Boussinesq approximation for density variation. In addition, the numerical results of the heat transfer are discussed for various values of physical parameters of the fluid and geometric parameters of the annulus on the heat transfer. A good agreement of Nusselt number has been found between the present predictions and reference from the literature data.*

Résumé - Dans ce travail, une étude numérique bidimensionnelle stationnaire, a été réalisé sur la convection naturelle dans un cylindre vertical conique partiellement annulaire différentiellement chauffée. Le transfert de chaleur est supposé avoir lieu par convection naturelle. Où les parois interne et externe de l'espace annulaire sont maintenues à température uniforme. Le code CFD FLUENT12.0 est utilisé pour résoudre les équations régissant de quantités de masse, de quantité de mouvement et d'énergie, à l'aide des propriétés constantes et approximation de Boussinesq pour la variation de densité. En outre, les résultats numériques de transfert de chaleur sont traités pour différentes valeurs de paramètres physiques de fluide et des paramètres géométriques de l'espace annulaire sur le transfert de chaleur. Un bon accord du nombre de Nusselt a été trouvé entre les prédictions actuelles et de référence à partir des données de la littérature.

***Keywords**:* **Two dimensional; Natural convection; Heat transfer; Annular space; Conical cylinder; numerical simulation, Axisymmetric flow.**

NOMENCLATURE

Symbol	Designation	SI unity
Ar	External aspect ratio=H/L	
g	Gravity Acceleration	(ms^{-2})
h	Height of hot cylinder	(m)
H	Height of the conical cylinder	(m)
k	Thermal conductivity	$(W.m^{-1}.K^{-1})$
L	Annulus gab width $L=r_o-r_i$	(m)
Nu	Nusselt number (See eq. (6) and (7)	
p	Pressure	$(N.m^{-2})$
P	Pressure dimensionless	
Pr	Prandtl number=$\mu c_p/k$	
r	Dimensional radial coordinate	(m)
R	Dimensionless radial coordinate	
Ra	Rayleigh number=$g\beta\Delta TL^3/\nu\alpha$	
T	Dimensional temperature	(K)
ΔT	Temperature difference $\Delta T=T_i-T_o$	(K)
u,v	Radial and axial Dimensional velocity respectively	(m/s)
U,V	Radial and axial Dimensionless velocity respectively	
z	Dimensional axial coordinate	(m)
Z	Dimensionless axial coordinate	

Greeks symbol

Symbol	Designation	SI unity
β	Thermal expansion coefficient	(K^{-1})
θ	Dimensionless temperature	
ρ	Mass density	($kg.m^{-3}$)
χ	Internal aspect ratio=h/L	

Indices symbol

i	inner
o	outer

1. INTRODUCTION

The natural convection heat transfer in annulus space is an important research topic due to its wide application in engineering problems, such applications are found in energy conversion, storage and transmission systems, Examples of using annulus geometry include electrical cooling, solar energy collector, nuclear reactor design. A significant number of experimental and theoretical works have been carried out in the past decades in an attempt to understand heat transfer flow in a cavity. A comprehensive review of natural convection in annular cavities has been documented in the literature. Among the very first investigations. A problem that has been widely studied owing to its many practical applications is natural convection in vertical annuli with isothermal hot inner and cold outer vertical surfaces received much attention. The studies conducted by Fujii and Verhara [1] in a vertical cylindrical cavity, by Thomas and De Vahl Davis [2] in a cavity annular vertical showed that the average Nusselt number varies weakly with the Prandtl number. This aspect also meets in the weak value of the Prandtl number as shown by De Vahl Davis and Thomas [3]. Farouk et al [4] and Glakpe et al [5], conducted extensive reviews of the technical literature. In a study on natural convection in an annular cavity Kumar and Kalam [6], provided some correlations for cavities to weak and average Aspect ratio 0.3 and 10, for the radius ratio between 1 and 15 and the Rayleigh numbers ranging from 10 to 10^6. Prasad and kulacki [7] are the first to have studied experimentally the effect of the curvature on the natural convection in annular cavity. The radius ratio effect of the annular cavity was investigated at Rayleigh numbers ranging between 8 10^6 and 3 10^{10}. The case of a vertical annulus with isoflux inner vertical surface and isothermal outer vertical surface was considered in Keyhani et al [8] and Khan and Kumar [9]. The coupling of conduction and natural convection in a vertical annulus was studied in a long time ago by Fishbaugher and Kim [10], and recently by Choukairy et al [11]. Few research works have been reported for the case of conical Yih [12] considered a vertical cone embedded in a porous medium to study the boundary layer for uniform lateral mass flux effect on natural convection of non-Newtonian power-law fluids. Cheng [13] used an integral numerical approach to investigate heat and mass transfer by natural convection from truncated cones in porous media. Murthy and Singh [14] studied the thermal dispersion effects over a cone with the help of similarity solution. Pop and Tsung-Yen [15] showed that the thermal transfers for a cone to the waved wall are lower to those for a cone in smooth wall. Chamkha et al [16] has studied simultaneous heat and mass transfer by natural convection about a vertical wedge and a cone embedded in a porous medium. Pornchaloempong et al [17,18] carried out thermal analysis on different shapes including cones and geometries with elliptic cross sections. Chamkha [19] considered the steady, laminar, free convection flow along a vertical cone and a wedge immersed in an electrically conducting fluid saturated porous medium in the presence of a transverse magnetic field. The effect of thermal radiation on the non-Darcy natural convection flow over a vertical cone and wedge embedded in a porous medium with variable viscosity and wall mass flux was analyzed by Al- Harbi [20]. These studies were restricted to conduction heat transfer only. The present paper covers the natural convection flow in a differentially heated vertical conical cylinder partially annular space. In this context, we analyze the effect of the Rayleigh number and aspect ratio of the annulus as well as the cavity geometry on the heat transfer.

2. MATHEMATICAL FORMULATION

2.1. Physical domain

The physical problem, analyzed in the present work, is the flow generated by natural convection in a vertical conical cylinder partially annular; where the annulus is filled with air. The domain of analysis is bounded by cold concentric conical cavity and hot cylinder with isothermal walls of the inner and outer axial length h, H respectively. The top and bottom walls of the outer cone are considered adiabatic, as shown in Figure 1. The bottom radii of the inner cone and the bottom radii of the outer cone is r_i and r_o the wall temperature are T_i and T_o, respectively. For the inner and outer wall, where $T_i > T_o$, where as the top and bottom walls of the outer cone are kept insulated. The buoyancy induced flow is assumed to be laminar, and the fluid studied is incompressible with constant properties except for the density. The variation of the density with the temperature is calculated using the Boussinesq approximation. Then, the mathematical model used is based on the assumption of a two-dimensional flow.

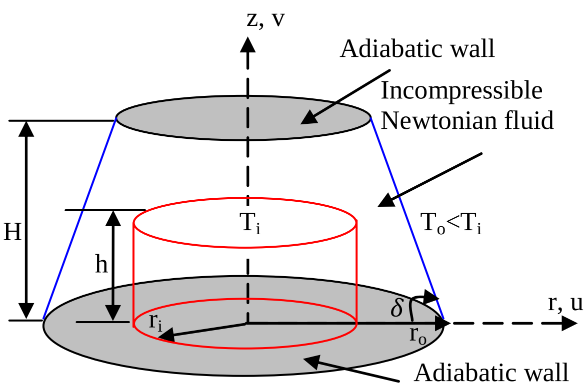


Fig. 1.Physical Model.

2.2. Governing equations

The governing dimensionless equations in cylindrical coordinates for the present study take the following forms.

Continuity,

$$\frac{1}{R}\frac{\partial}{\partial R}(RU)+\frac{\partial V}{\partial Z}=0 \qquad (1)$$

R momentum,

$$U\frac{\partial U}{\partial R}+V\frac{\partial U}{\partial Z}=-\frac{\partial P}{\partial R}+Pr\left[\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}(RU)\right)+\frac{\partial^2 U}{\partial Z^2}\right] \qquad (2)$$

Z momentum with the Boussinesq approximation for the buoyancy term,

$$U\frac{\partial V}{\partial R}+V\frac{\partial V}{\partial Z}=-\frac{\partial P}{\partial Z}+Pr\left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial V}{\partial R}\right)+\frac{\partial^2 V}{\partial Z^2}\right]+PrRa(\theta-0.5) \qquad (3)$$

Energy

$$U\frac{\partial \theta}{\partial R}+V\frac{\partial \theta}{\partial Z}=\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \theta}{\partial R}\right)+\frac{\partial^2 \theta}{\partial Z^2} \qquad (4)$$

where the dimensionless parameters are defined as follows:

$$R=\frac{r}{L};\ Z=\frac{z}{L};Ar=\frac{H}{L};\ \chi=\frac{\mathrm{h}}{L}\ \theta=\frac{T-T_o}{T_i-T_o} \qquad (5)$$

$$Pr=\frac{\nu}{\alpha};\ Ra=\frac{\beta g\Delta T L^3}{\alpha\nu}$$

2.3. Boundary conditions

The dimensionless boundary conditions for the two velocity components are all set to zero at all the solid walls. The temperature boundary conditions are as follows.

At $0\le Z\le H/L$ and $R_o\le R\le R_o$-Ar cotgδ: U= V= 0 and θ=0 for the isothermal Cold tilted wall

At $0\le Z\le h/L$ and $R_i\le R\le R_i$: U= V= 0 and θ=1 for the isothermal hot vertical wall

At Z= h/L and $0\le R\le R_i$: U= V= 0 and θ=1 for the isothermal hot horizontal wall

At Z =H/L and Z =0: U= V= 0 and $\frac{\partial\theta}{\partial z}$ =0 for the adiabatic walls

3. NUMERICAL METHOD

The FLUENT 12.0, CFD code was used to solve the governing equations using finite the volume method by segregated implicit solver with first order formulation. The segregated solver solves conservation governing equations independently. The first order upwind differencing scheme was used for momentum and energy equations. The discretization scheme used for pressure was body force weighted to take the density variations in consideration.

The pressure–velocity coupling is ensured using the SIMPLE algorithm. The geometrical model is created and meshed using GAMBIT with a simple quadrilateral cell. The cavity was filled with a conical cylinder partially annular grid with a very fine spacing near the walls, as needed for accurately resolving the steep gradients in the thin buoyancy-driven boundary layer. All the variables were calculated right up to the walls without using any wall function. On the wall surface, the boundary values for the velocity components were set to zero. The dimensionless temperature of the cold and the hot walls are set to 0 and 1 respectively.

3.1. Nusselt Number

The energy transported across the inner cylinder of the annulus is expressed in terms of local and mean Nusselt numbers. The local Nusselt number for the inner cylinder of annulus can be obtained from temperature gradients by the following relationships:

$$Nu_1 = \left.\frac{\partial\theta}{\partial R}\right|_{R=R_i} \text{ and} Nu_2 = \left.\frac{\partial\theta}{\partial Z}\right|_{Z=\frac{h}{L}} \qquad (6)$$

The mean Nusselt number is defined by

$$\overline{Nu} = \frac{1}{\chi}\int_0^{\chi} Nu_1 dZ + \frac{1}{\mathrm{R_i}}\int_0^{\mathrm{R_i}} Nu_2 dR \qquad (7)$$

4. RESULTS AND DISCUSSION

Numerical simulations have been performed to elucidate the effect of Rayleigh number and aspect ratio of the annulus. The results are summarized as follows:

4.1. Effects of Rayleigh Number

Understanding the fluid dynamics in the entire annulus is possible by estimating the temperature distribution in the annulus with respect to different parameters being Ar=1, K=2, and δ=70°. Figure 2 shows for Rayleigh number $10^3 \leq Ra \leq 10^4$ that there is a logarithmic temperature distribution in the entire region of the annulus suggesting that heat from the hot wall to the cold wall is transferred by conduction for Rayleigh number Ra=5 10^4. The temperature gradient in the core becomes slightly flatter and non-linear suggesting that the dominance of conduction is reduced due to a small amount of convection setting up in the core part. Figure 2 shows for Rayleigh number Ra=10^5 a steep vertical temperature gradient near the wall. This confirms the existence of the cellular patterns formed inside the annulus which increases the rate of heat transfer. This is the transition regime in which the boundary layers formed at the hot and cold wall reach the central part of the annulus core and merge together. Hence, there is a rotary motion of the fluid, causing mixing and enhancement in heat transfer. However for Rayleigh number Ra=10^6 the core was found to be nearly isothermal with the temperature gradient nearly equal to zero.

Vertical velocity profiles are presented in Figure 3 for different Rayleigh numbers ranging from Ra=10^3 to Ra=10^6. Profiles are plotted along the horizontal direction. Other geometric parameters were chosen as Ar=1, K=2 and δ=70°. As expected, the velocity is very weak everywhere in the annulus when Ra≤10^4. Under this condition, fluid motion is not effective confirming that the conductive regime is predominant.This indicates that the fluid is practically stagnant in the annulus at 0≤R≤0.5 and 1.3≤R≤1.5. But, when the Rayleigh number is in the range 5 10^4≤ Ra≤10^5, the fluid driven by the buoyancy force begins to accelerate from the bottom and forms the primary flow in the vertical upward direction.The velocity increases when

Rayleigh number is big enough Ra=10^6, and the fluid flow is now confined adjacent to both the hot surface ($0 \le R \le 1$) and the cold surface ($1.58 \le R \le 1.72$).

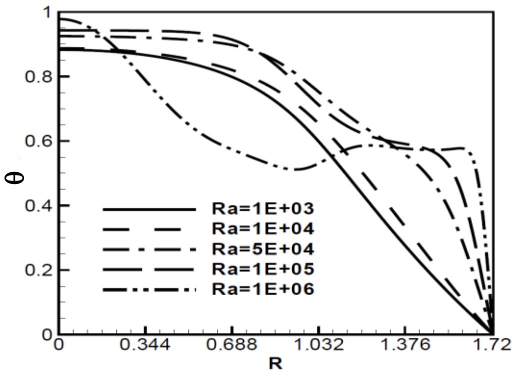


Fig.2. Temperatures profiles variation with horizontal distance for annulus Aspect ratio Ar=1, K=2, χ=0.5 and δ=70°at Z=0.75

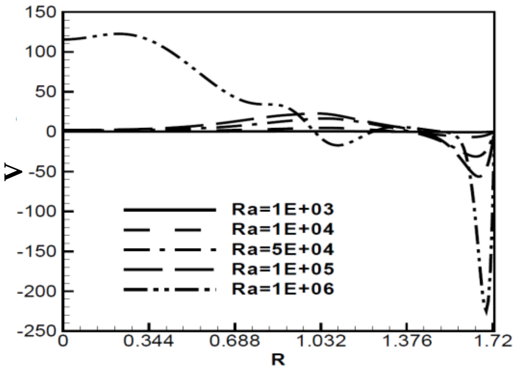
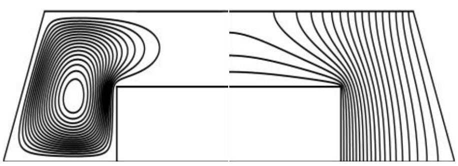
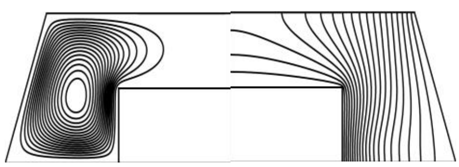


Fig. 3.Temperatures profiles variation with horizontal distance for annulus Aspect ratio Ar=1, K=2, χ=0.5 and δ=70° at Z=0.75

The Effects of Rayleigh number on streamlines (on the left) and isotherms (on the right) Figure 4 (a)-(d) present for Ar=1, K= 2, χ=0.5 and δ=70°.Visualizations are given from Ra=10^3 to Ra=10^6.As can be seen from these figures a single cell is formed for all values of the Rayleigh numbers considered, which is in clockwise rotation direction. For small values of the Rayleigh numbers such as Ra$\le 10^4$ (figure 4 a-b), isotherms are nearly vertical in most of the annulus, showing the conduction-dominated heat transfer mechanism. For higher values of the Rayleigh number, (i.e.$10^5 \le$Ra$\le 10^6$) as shown in (figure 4 c-d), due to increasing of convection mode of heat transfer, two thin boundary layers have been formed adjacent to the hot and cold walls at $0 \le R \le 1$. The isotherms get distorted and crowded near lower left and upper right of the annulus and are horizontally stratified. Most of the flow moves to the top and upper right of the annulus due to existence of partially cooled and heated wall. Flow becomes motionless at the top and floor of the annulus due to adiabatic boundaries. However, they show almost parallel distribution to the horizontal walls with increase of Rayleigh number due to increase of convection mode of heat transfer.



a) Ra=10^3



b) Ra=10^4

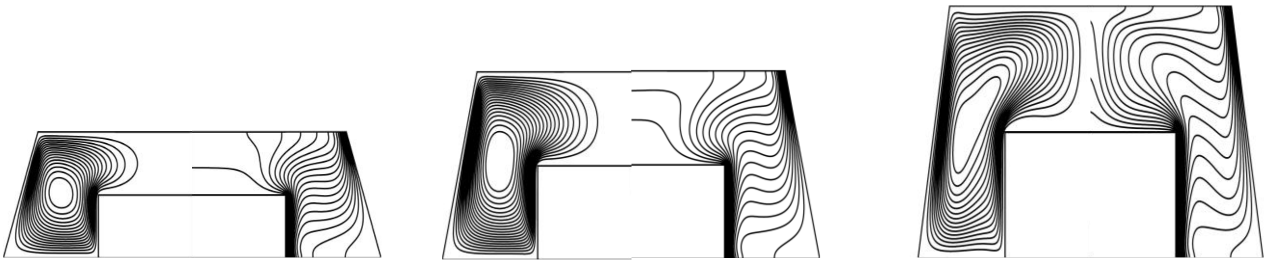
c) $Ra=10^5$ d) $Ra=10^6$

Fig.4. Streamlines and isotherms for annulus Aspect ratio Ar=1, K=2, χ=0.5 and δ=70°.

4.2. Effects of the Aspect Ratio

Figure 5 reflects the isotherms (on the right) and streamlines (on the left) variations with respect to aspect ratio Ar. The figure is obtained for four values of aspect ratio i.e. Ar = 2, 3, 4 and 5 and different values of δ=70°, 76°and 80° corresponding to K =2 and $Ra=10^5$.The isotherms reveal that the increase in the aspect ratio leads to crowding of isotherms at lower left and upper right of the annulus. There is continuous variation in the temperature gradient in hot surface at lower aspect ratio Figure 5 a indicating that the heat transfer rate continuously varies along the height of the vertical inner cone which is not the case at higher aspect ratio Figure 5 d. The fluid circulation centre increased when aspect ratio increases.

Figure 6 shows the Nusselt number variation with the Rayleigh number corresponding to different values of aspect ratio Ar=2, 3, 4 and 5 and different values of δ=70°, 76° and 80°. The other parameters being K=2. As it can be seen from the figure, the heat transfer rate increases with increase of Rayleigh number.And the heat transfer rate increases with increase in the aspect ratio.



a) Ar=1, δ=70° b) Ar=1.5, δ=76° c) Ar=2, δ=80°

Streamlines and isotherms for different
Annulus Aspect ratio at $Ra=10^5$ and K=2.

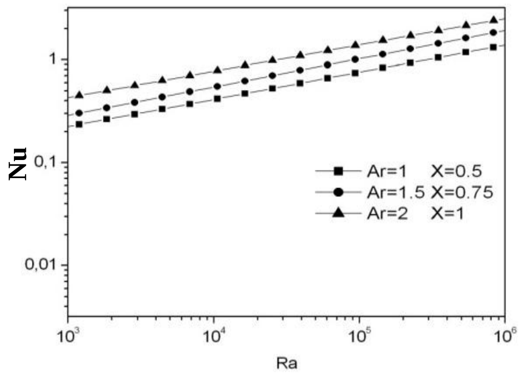


Fig. 6.Nusselt number variation with Rayleigh number and annulus aspect ratio, height ratio at K=2 and δ=70°, 76° and 80°.

5. CONCLUSION

In this paper, the results of a numerical study of natural convection in a Two-Dimensional vertical conical cylinder partially Annular space for steady-state regime with differentially heated walls are presented using main parameters of interest as Rayleigh number, Aspect ratio.

Based on the obtained results, the following findings may be summarized:

For lower values of the Rayleigh numbers we observed a domination of conduction heat transfer. When highest values of the Rayleigh numbers, due to increasing of convection mode of heat transfer.

The average Nusselt number increases with increase in Rayleigh number. The average Nusselt number increasing are an function of the Aspect ratio.

The circular cell of the fluid concentrates at the centre of the annulus for low aspect ratio, but it moves downwards as the aspect ratio increases.

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