Sciences & Technologie **B** – **N°20**, Décembre (2003), pp. 57-62.

A THREE-DIMENSIONAL INVERSE COMPUTATION FOR THE DESIGN OF TURBOMACHINERY

Received 20/02/2002 – Accepted 26/10/2003

Abstract

A practical quasi-three dimensional inverse computation model is developed for the design of turbopump under the assumption of inviscid incompressible flow. The three-dimensional flow through impellers is decomposed into a circumferentially averaged mean flow and a periodic flow according to flow periodicity. In this computation the blade geometry of impeller is designed according to specified blade bound circulation distribution and normal thickness distribution on a given meridional geometry. In the first step, we use the meridional stream function to define the flow field. The governing equation is deduced from the relation between the azimuthal component of the vorticity and the meridional velocity. In the second step, the inverse problem corresponding to the blade to blade flow confined in each stream sheet is analyzed. The blade geometry is determined according to a specified blade bound circulation distribution by iterative computation of *S2* and *S1* surfaces.

***Keywords**: Inverse computation, Design, Turbomachinery.*

Résumé

L'objectif de ce travail est de présenter une méthode quasi-tridimensionnelle de calcul inverse en fluide parfait et incompressible appliqué à une turbopompe. L'écoulement tridimensionnel est analysé en deux étapes, par un calcul axisymétrique suivi d'un calcul aube à aube. Dans cette méthode, la géométrie des aubes est déterminée en respectant la loi d'épaisseur exigée par la structure et la distribution du moment cinétique en entrée et en sortie de la roue pour une géométrie méridienne donnée. En première étape, les tourbillons simulant les aubes sont étalés d'une façon homogène et l'écoulement devient axisymétrique, la fonction de courant étant utilisée pour définir le champ. L'équation la régissant est déduite de la relation entre la composante azimutale du rotationnel et la vitesse méridienne. Dans la seconde étape, le problème inverse sur chaque nappe de courant pour le passage aube à aube est analysé. La géométrie du squelette est alors déterminée après un couplage S2-S1 tout en respectant la distribution des tourbillons liés.

***Mots clés**: Calcul inverse, Conception, Turbomachines.*

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ملخص

الهدف من هذا العمل هو تقديم طريقة هندسية عكسية تعتمد على شرط سريان مزاح لاستخراج الشكل الهندسي للأرياش وهذا باتخاذ المائع مثالي. في هذه الطريقة الهندسية نتحقق من إحترام توزيع السمك للرياش المحدد وتوزيع عزم الحركة الزاوي في مدخل ومخرج كل عجلة في المستوى الهاجري للشكل المعطى. الانسياب ثلاثي الأبعاد يدرس على مرحلتين على أساس الطريقة التقريبية لثلاثي الأبعاد S2-S1. في المرحلة الأولى الدوامات موزعة بطريقة متجانسة في الاتجاه السمتي، الانسياب يصبح متماثل محوريا. دالة السريان تكون مستعملة لتمثيل مجال الانسياب. الدالة المعرفة للقانون مستخرجة من علاقة المركب السمتي للدوران والسرعة الهاجرية. في المرحلة الثانية، الطريقة العكسية لكل طبقة سريان الانسياب بين بين الأرياش تدرس. الشكل الهندسي لهيكل الأرياش يستخرج بعد عملية إقتران بينS2-S1. وهذا باحترام توزيع الدوامات الجبرية.

***الكلمات المفتاحية**: الطريقة العكسية, تصميم هندسي، العنفات.*

Most of the blading design procedures consider the velocity distribution on both sides of the blade as the initial data. They use the hodograph plane which allows to linearize the equations in case of the potential flow [1]. But the designer loses the control of thickness distribution (airfoil presenting a fish-tail at the trailing edge or an opened leading edge). Léonard [2] recognise to use singularities such as vortices to modify the velocity distribution on the blade until to obtain the desired one. But some restrictions on the required velocity are necessary to avoid problems of thickness at the leading or trailing edge. Others use the pressure or velocity distribution in physical plane with Euler equations in case of non potential flow, but the discontinuity near leading or trailing edge still exists.

For the development of computational fluid dynamics, it became possible to analyze the complex internal flow in turbomachinery by solving the *3D* Euler equation or the Navier-Stokes equation. Compared with the direct problem, the inverse problem is much more difficult. A careful survey of published literature on fluid machinery indicates that there are a few real three-dimensional inverse models. Those can be classified as the Fourier Series expansion singularity method [3-5], the Taylor Series expansion method [6], the pseudostream function method [7] and the inverse time marching method [8].

In the present work, both thickness distribution of the blades and the desired swirl $V_\theta r$ distribution are the initial data [9,10]. This variation can be interpreted as a distribution of vortices which the blades must generate. Note that the total swirl variation is related to flow angle difference between upstream and downstream. This vortex distribution is represented by the function $\Delta(V_\theta r)f$ where f is given monotonous increasing regular function of streamwise coordinate. f depends on the nature of the machine (axial or radial). Zanetti [11] makes the same approach but he handles with pressure jump between two sides of the blade instead of swirl variation.

Only incompressible inviscid flow is analyzed in this paper. This method gives a physical realistic blade shape which respects the thickness distribution which could be preconized by structure analysis, has no problem of round-off of leading or trailing edge and provides the desired flow deviation. An improvement for the scheme is made by introducing an efficiency factor η for each stream line in order to take into account dissipation loss as suggested by Horlock [12]. This model provides possibilities for achievement of a design of a stage which guarantee both desired total pressure jump and exit swirl distribution (rotor + stator or vice versa) by modifying the swirl jump between blade rows inside the stage. A curvilinear body fitted coordinate system is adopted and the tensor formulation is used in order to handle with all kind of geometries (axial, radial or mixed flow).

1- MERIDIONAL FLOW, S2 APPROACH

In the first step, the vortex distribution is transformed into an axisymmetrical one by spreading in the azimuthal direction, this situation is equivalent to the case where the number of blades in the rotor and in the stator is assumed to be infinite, the flow becomes also axisymmetrical and can be analyzed in a meridian plane [10],[13].

The fluid is considered inviscid, the flow is supposed incompressible and permanent. The hypothesis of axisymmetric flow allows us to write : ($\partial(..)/\partial\theta=0$). In the meridian flow channel, a boundary fitted coordinate system ξ^1,ξ^3 is created with $\xi^2=\theta$. Let ζ^i denote the coordinates z,θ and r. We have:

$$g_{ij}=\frac{\partial\zeta^m}{\partial\xi^i}\frac{\partial\zeta^n}{\partial\xi^j}\left(g_{mn}\right)_\zeta \quad\text{and}\quad \sqrt{g}=\frac{D\left(\zeta^1,\zeta^2,\zeta^3\right)}{D\left(\xi^1,\xi^2,\xi^3\right)}(r)^2$$

The meridian velocity is represented by: $\mathbf{U}=V^1\mathbf{e_1}+V^3\mathbf{e_3}=W^1\mathbf{e_1}+W^3\mathbf{e_3}$; the continuity equation becomes :

$$\frac{1}{\sqrt{\tilde g}}\left[\frac{\partial\sqrt{\tilde g}U^1}{\partial\xi^1}+\frac{\partial\sqrt{\tilde g}U^3}{\partial\xi^3}\right]=0 \qquad (1)$$

where $\tilde g$ represents the determinant of the modified metric tensor due to the flow channel striction produced by the thickness of the blades. Therefore $\sqrt{g}$ represents the elementary volume of the cube : $(\mathbf{e_3}\times\mathbf{e_1})\cdot\mathbf{e_2}$, in the free space $|\mathbf{e_2}|=\sqrt{g_{22}}=r$. However in the blade row space, the thickness of the blade reduces the flow channel, if $r\delta\theta_e$ denotes the thickness of measured section following the peripheral direction, N_b the number of the blading period in the rotor or stator, the modified metric tensor must be expressed by :

$$\tilde g_{22}=\left(1-\frac{N_b\delta\theta_e}{2\pi}\right)^2 r^2$$

$\tilde g$ used to simulate the flow channel striction in Eq. 1 is evaluated with the modified $\tilde g_{22}$. Using the stream function ψ to represent the flow field by setting:

$$U^1=\frac{1}{\rho\sqrt{\tilde g}}\frac{\partial\psi}{\partial\xi^3} \qquad\text{and}\qquad U^3=-\frac{1}{\rho\sqrt{\tilde g}}\frac{\partial\psi}{\partial\xi^1}$$

The equation (1) is satisfied automatically. The governing equation for ψ is obtained by writing $\nabla\times U=\Omega^2\mathbf{e_2}$, where Ω^2 represents the peripheral component of $\nabla\times V$, it is deduced from the radial equilibrium condition, which writes:

$$\frac{\partial U_1}{\partial\xi^3}-\frac{\partial U_3}{\partial\xi^1}=\sqrt{g}\Omega^2$$

where U_1 and U_3 are the covariants components of the velocity expressing from the stream function ψ, in using the relations $U_m=g_{mn}U^n$. H the enthalpy and I the rothalpy are given by :

$$H=\frac{p}{\rho}+\frac{V^2}{2}=\frac{p_t}{\rho} \quad\text{and}\quad I=\frac{p}{\rho}+\frac{W^2}{2}-\frac{\omega^2r^2}{2}=H+\omega\left(V_\theta r\right)$$

The momentum equation is:

$$\begin{cases}\Omega\times\mathbf{W}=-\nabla I+\dfrac{\mathbf{F_b}}{\rho}+\dfrac{\mathbf{F_d}}{\rho} & \text{rotor case}\\[2mm] \Omega\times\mathbf{V}=-\nabla H+\dfrac{\mathbf{F_b}}{\rho}+\dfrac{\mathbf{F_d}}{\rho} & \text{stator case}\end{cases} \qquad (2)$$

where $\mathbf{F_b}/\rho$ represents the blades force. The loss scheme related to the plausible value of efficiency η for each streamline of the stage is added, this scheme suggests that the dissipative force $\mathbf{F_d}/\rho$ is related to the variation of $V_\theta r$ via η [12] in writing:

$$\frac{\mathbf{F_d}}{\rho}=-C_f\frac{|\mathbf{U}|^2}{\cos\beta}\frac{\mathbf{U}}{|\mathbf{U}|}$$

$\mathbf{F_d}=0$ as well as $\mathbf{F_b}=0$ are imposed in the free space. Combining the $\mathbf{e^2}$ and the $\mathbf{e^3}$ components of (2), we have:

$$\sqrt{g}\Omega^2=\ \frac{1}{W^1}\frac{\partial I}{\partial\xi^3}-\frac{F_{d_3}}{\rho}+\frac{n_1}{n_2}\frac{\partial\left(V_\theta r\right)}{\partial\xi^3}-\frac{n_3}{n_2}\frac{\partial\left(V_\theta r\right)}{\partial\xi^1} \quad \text{rotor case}$$

$$\sqrt{g}\Omega^2=\ \frac{1}{V^1}\frac{\partial H}{\partial\xi^3}-\frac{F_{d_3}}{\rho}+\frac{n_1}{n_2}\frac{\partial\left(V_\theta r\right)}{\partial\xi^3}-\frac{n_3}{n_2}\frac{\partial\left(V_\theta r\right)}{\partial\xi^1} \quad \text{stator case}$$

$$\sqrt{g}\Omega^2=\ \frac{1}{\mathrm{V}^1}\left(\frac{\partial H}{\partial\xi^3}-\frac{\left(V_\theta r\right)}{r^2}\frac{\partial\left(V_\theta r\right)}{\partial\xi^3}\right) \quad \text{free space} \qquad (3)$$

where n_i denotes the covariant components of the normal $\mathbf{n}$ of the camber surface of the blade. Use has been made that $\mathbf{V}\perp\mathbf{n}$ in the stator, $\mathbf{W}\perp\mathbf{n}$ in the rotor and $\mathbf{F_b}\parallel\mathbf{n}$. The dot product of the momentum equation with $\mathbf{V}$ in the stator and in the free space or with $\mathbf{W}$ in the rotor leads to the following relations which serve to update the nodal values of H or I:

$$\frac{\partial I}{\partial m}= (1-\eta)\omega\frac{\partial(V_\theta r)}{\partial m} \qquad \text{rotor case}$$

$$\frac{\partial H}{\partial m}= (\eta-1)\omega\frac{\partial(V_\theta r)}{\partial m} \qquad \text{stator case}$$

$$\frac{\partial H}{\partial m}= 0 \qquad \text{free space}$$

Writing $\nabla\times\mathbf{U}=\Omega^2\mathbf{e_2}$, we obtain the governing equation of ψ:

$$\frac{\partial}{\partial\xi^3}\left(\frac{g_{11}}{\rho\sqrt{\tilde{g}}}\frac{\partial\psi}{\partial\xi^3}\right)+\frac{\partial}{\partial\xi^1}\left(\frac{g_{33}}{\rho\sqrt{\tilde{g}}}\frac{\partial\psi}{\partial\xi^1}\right)-\frac{\partial}{\partial\xi^3}\left(\frac{g_{13}}{\rho\sqrt{\tilde{g}}}\frac{\partial\psi}{\partial\xi^1}\right)-\frac{\partial}{\partial\xi^1}\left(\frac{g_{31}}{\rho\sqrt{\tilde{g}}}\frac{\partial\psi}{\partial\xi^3}\right)=\sqrt{g}\Omega^2 \tag{4}$$

For the inverse problem, the distribution of $V_\theta r$ is assigned, Ω^2 is updated iteratively. Let the form of the blade camber be defined by $\theta=\xi^2\left(\xi^1,\xi^3\right)$ if the coordinate lines $\xi^3=$ const are updated to the streamlines iteratively, ξ^2 can be computed using the slip condition:

$$\xi^2=\xi^2_{le}+\int_{\xi^1_{le}}^{\xi^1}\frac{U^2}{U^1}d\xi^1 \tag{5}$$

1.1- Blade surface pressure evaluation

Usually the *S2* approach leads to the determination of the mean velocity on both faces of the blade:

$$\begin{cases} W=\left[g_{11}V^1V^1+2g_{13}V^1V^3+g_{33}V^3V^3+g_{22}\left(V_\theta r+\omega r^2\right)\right]^{1/2} \\ \quad\text{rotor case} \\ V=\left[g_{11}V^1V^1+2g_{13}V^1V^3+g_{33}V^3V^3+g_{22}\left(V_\theta r\right)^2\right]^{1/2} \\ \quad\text{stator case}\end{cases} \tag{6}$$

Let ΔU denote the difference of the absolute velocities V^+-V^- or the relative velocity W^+-W^- on the two faces of the blade, when the number of blades is finite, this difference is related to the local density of bound vortex generated by the blade. In the *S2* scheme, consider the blade section cut by a $\xi^3=$ const surface, the flux of bound vortices generated by the element $\delta\xi^1$ of the blade is determined by the flux of Ω through the elementary surface $\left(\delta S\right)_3\mathbf{e^3}=\sqrt{g}\delta\xi^1\delta\xi^2\mathbf{e^3}$, where $\delta\xi^2$ should be equal to $2\pi/N_b$. Using the Stokes relation that implies the circulation produced by ΔU is equal to the flux of the bound vortices, we get the following relation :

$$\left(\Delta U\right)_{i,k}=\frac{2\pi}{N_b}\frac{\cos\beta}{\sqrt{g_{11}}}\left|\left[V_\theta r\right]_{i-\frac{1}{2},k}^{i+\frac{1}{2},k}\right|$$

where β denotes the local angle of the blade camber line with respect to the meridian plane. The relation (6) is used to compute surface velocity on both faces of the blades, then the pressure distribution by the *S2* approach can be deduced.

2- BLADE TO BLADE FLOW, S1 APPROACH

The blade to blade flow confined in each axisymmetrical stream sheet is analyzed in order to define the final geometry for each section of the blade and to obtain the pressure distribution. At the beginning, the contour of the blade is created from the camber line obtained from the *S2* step with the assigned thickness distribution. The conformal mapping $(m,\theta)\Rightarrow(x^1,x^2)$:

$$\begin{cases} x^1=r_0\int_{m_0}^{m}\frac{dm}{r} \\ x^2=r_0\left(\theta-\theta_0\right)\end{cases}$$

transforms the blade to blade flow confined in an axisymmetrical stream sheet into 2*D* cascade flow in the (x^1,x^2) plane. The body fitted coordinate system constituted by the equipotential lines $\xi^1=$const and the streamlines $\xi^2=$ const of a *2D* flow around the cascade is created using the method of singularities [2]. In this system, the continuity equation becomes:

$$\frac{1}{\sqrt{g}}\left[\frac{\partial}{\partial\xi^1}\left(\rho\sqrt{g}U^1\right)+\frac{\partial}{\partial\xi^2}\left(\rho\sqrt{g}U^2\right)\right]=0 \tag{7}$$

where U^i represent the contravariant components of the absolute velocity $\mathbf{V}$ for the stator and relative velocity $\mathbf{W}$ for the rotor and

$$\sqrt{g}=\frac{D\left(x^1,x^2\right)}{D\left(\xi^1,\xi^2\right)}\left(\frac{r}{r_0}\right)^2\tau$$

where τ represents the local thickness of the stream sheet. Introducing the stream function ψ with :

$$\begin{cases} U^1=\frac{1}{\rho\sqrt{g}}\frac{\partial\psi}{\partial\xi^2} \\ U^2=-\frac{1}{\rho\sqrt{g}}\frac{\partial\psi}{\partial\xi^1}\end{cases} \tag{8}$$

The relation (7) is satisfied. From the momentum equation (2) applied to the S2 flow, it can be shown that the free vortex must be tangential to the axisymmetrical stream sheet. The governing equation of the blade to blade flow stream function is deduced from this condition: for the relative flow around the blades of the rotor, we have:

$$-\left[\frac{\partial}{\partial\xi^1}\left(\frac{g_{22}}{\rho\sqrt{g}}\frac{\partial\psi}{\partial\xi^1}\right)+\frac{\partial}{\partial\xi^2}\left(\frac{g_{11}}{\rho\sqrt{g}}\frac{\partial\psi}{\partial\xi^2}\right)\right] = -\frac{\partial g_{21}W^1}{\partial\xi^1}+\frac{\partial g_{12}W^2}{\partial\xi^2}+2\sqrt{g}\,\frac{\omega r}{\tau}\frac{d\log r}{dm} \qquad (9)$$

2.1- Boundary conditions for the inverse problem

Bound vorticity assigned : : We consider the difference of velocity potential as the circulation generated by the bound vortex located between P^+ and P^- the centers of the upper and lower associated elements, it should be a fraction of circulation, this condition implies:

$$\left[W^1\delta\xi^1-\omega r^2\delta\theta\right]_-^+=\Gamma\delta f$$

Penetrating flux conservation : The form of the camber line is the unknown of the problem. The boundary conditions of the flow field have to be imposed on the presumed contour of the blade. As the contour is not yet the blade surface, the slip condition could not be respected, the penetration of the fluid flow must be tolerated. The flux penetrating condition has to be defined so that the flux detected can be used to correct the form of the camber line. To have a properly field problem, it is suggested that the flux through each pair of associated boundary elements must be conservative. This condition implies :

$$[\psi]_-^+=0 \quad \text{or} \quad \left[\sqrt{g}\rho W^2\delta\xi^1\right]_-^+=0$$

So that the presence of the presumed blade contour does not produce any extra flux.

Input of Design Method
- Loading distribution
- Meridional geometry
- Axial and radial velocity distribution at inlet
- Blade thickness distribution
- Blade number and rotational speed

Initialization
- Blade shape
- Velocity distribution

Axisymmetric Computation: (S2 approach)
- Determination of velocity distribution (Eq. 2)
- Determination of vorticity distribution (Eq. 3)
- Determination of the blade shape (Eq. 5)

Blade to blade flow: (S1 approach)
- Determination of velocity distribution (Eq. 8)
- Determination of the blade shape (Eq. 10)

Figure 1: Flow diagram for determination of blade shape.

The solution of the inverse problem leads to the determination of flux penetration on the blade contour, the camber line inclination correction $\delta\vartheta$ is given by:

$$\delta\vartheta=0.5\left[\tan^{-1}\left(\frac{\sqrt{g}}{\tau}\frac{W^2}{W^1}\right)^+ +\tan^{-1}\left(\frac{\sqrt{g}}{\tau}\frac{W^2}{W^1}\right)^-\right] \qquad (10)$$

This method starts from an initial blade shape, then the blade shape is updated using the inviscid slip condition, which implies that the flow must be aligned to the blade surfaces. Equation (5) is integrated along the meridional projections of streamlines on the blade surface to find the blade shape. Once the updated blade shape is obtained, it is then possible to calculate the vorticity throughout the flow using Eq. (3). Having a new estimate for the velocity field, it is then possible to calculate a new blade shape using Eq. (5). This iterative process is repeated until changes in blade shapes between two iterations fall below a certain tolerance and can be presented in figure 1.

3- NUMERICAL TECHNIQUES

A code program written on Fortran language based on the formulation equations is prepared and used to solve the inverse problem. In this section, the numerical techniques used to solve the relevant equations described in section 1 for the meridional flow and section 2 for the blade to blade flow are presented. The computation of the terms in Eq. (4) in the meridional plane and Eq. (9) in the blade to blade plane is based on the Finite Volume approach. Structured grids are used to discretize the numerical domain, and the nodes are located at the centers of the cells, which are then identical to the control volumes. The three-dimensional grid is composed of two-dimensional ones, defined on a series of axisymmetric surfaces, distributed between the hub and tip endwalls.

The present inverse model has a good convergence. On the present inverse model, the influence of blade geometry on the mean flow is considered in the process of defining an initial blade on the S2 surface. So the defined initial S1 stream surfaces are much more closed to those of the finally designed turbopump. The iteration of S1 surfaces and the S2 surface is reduced. The convergence of the meridional streamline is obtained after 18 iterations and is equal to 10^{-4}. The present quasi-three-dimensional inverse model based on S1 stream surfaces is a simple and effective design method fit for engineering practice.

A typical computational mesh employed in these calculations comprises 105 cells in the meridional direction, 16 cells in the radial direction for the meridional plane and 24 cells in the pitchwise direction for the blade to blade plane. For one stream sheet, the CPU time on a Pentium MMX-200 is about 5 minutes.

4- RESULTS

The present inverse model is applied to design a turbopump using Finite Volume Method. Design parameters are given as follows : n= 34100 tr/mn, N_b = 6, $\boldsymbol{R}$= 0.91 m, q= 41.1 kg/s, ρ = 70.18 kg/m^3. The meridional of impeller passage and positions of blade leading edge and

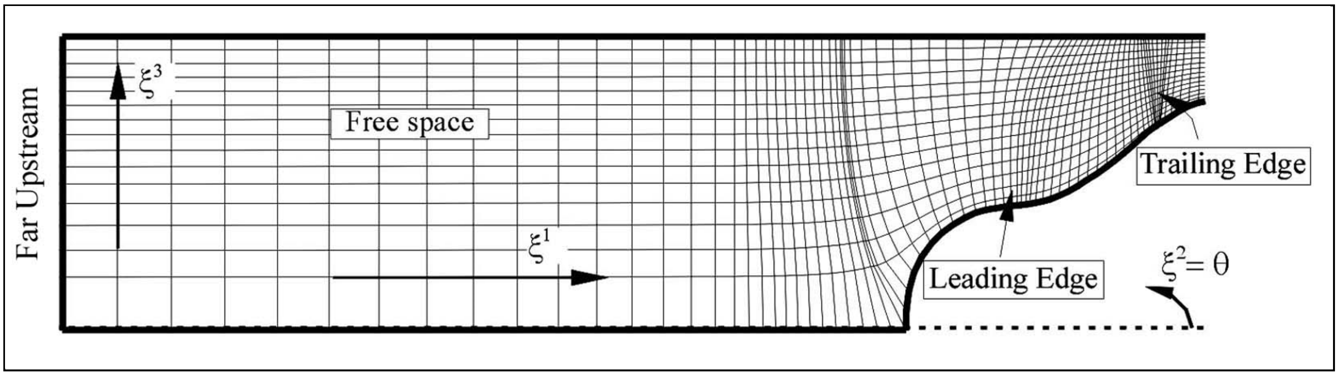

Figure 2: Geometry of flow passage and computational mesh.

trailing edge are selected to be as shown in figure 2. The network in the meridional plane obtained by the computation for the 2D incompressible potential flow [10].

Figure 3 shows the comparison of the camber lines of the impeller obtained from the S2 approach and rectified by the S1 approach. The results from the S2 and S1 computations are similar, but not identical, the need of the S1 computation to obtain the final geometry definition of the blades is confirmed. The S2 approach leads to the same geometric definition for the camber line of these blades, the S1 computation determines the modifications when the number of blades constituting the row is finite.

Figure 4 shows the pressure distribution obtained from the S2-S1 calculation for the upper and the lower sides of

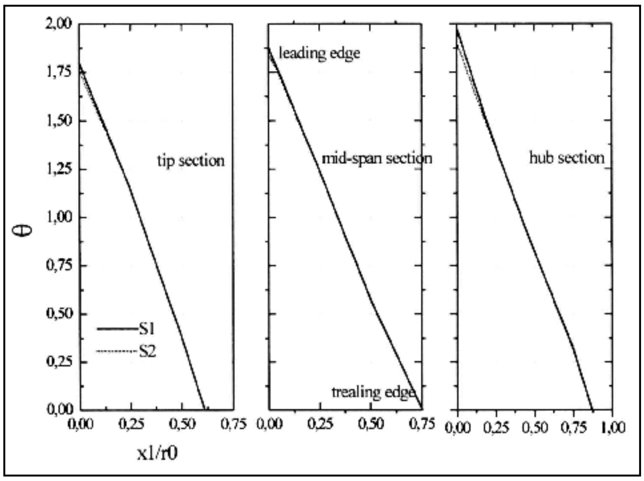

Figure 3: Camber line of the impeller obtained from S2-S1 approaches.

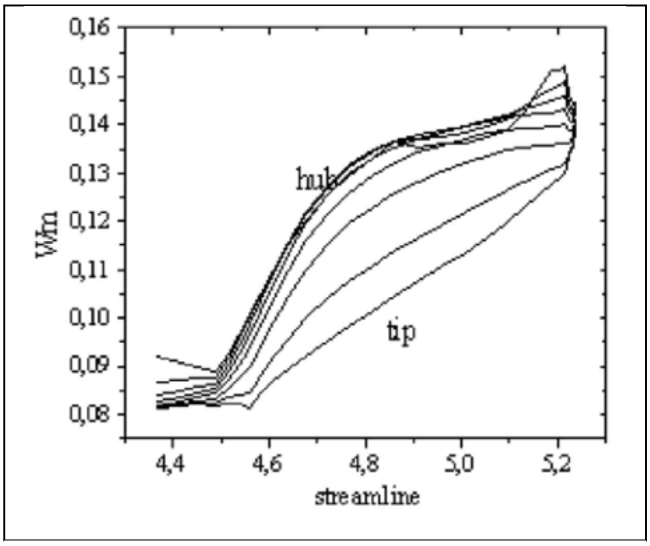

Figure 5: Meridional velocity distribution along streamlines.

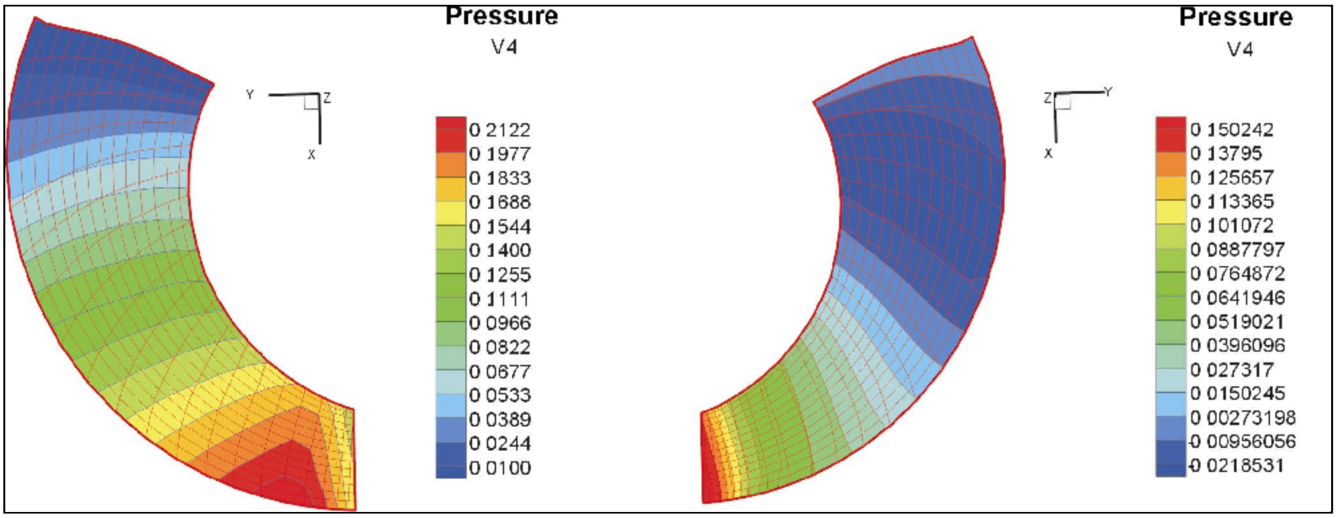

Figure 4: Pressure distribution for Upper and Lower sides of the impeller from S2-S1 approaches.

blade. For the case of the centrifugal impeller, the loading function *f* is optimized in order to avoid the cavitation and the separation of the boundary layer.

The meridional velocity distributions along averaged mean flow streamlines are shown in figure 5.

In table 1, we show the obtained values for torque coefficient $C_Q = Q / \rho\omega^2 R^4$, dissipation loss cause by viscous effects and power coefficient $C_W = P / \rho\omega^2 R^5$ for different values of turbopump efficiency.

Efficiency η	0.70	0.80	0.90	1.0
C_Q	0.02899	0.02948	0.02993	0.03040
Dissipation	0.00140	0.00092	0.00047	0.00000
C_W	0.02128	0.02432	0.02736	0.03040

Table 1: Turbopump performance.

CONCLUSION

A design procedure more suitable for engineering use is proposed in this paper. This design approach accepts the prescription of the thickness distribution of the blade and the loading distribution as the initial data. It is relatively easy to obtain the optimized design by some adjustment of the initial data. The representation of the blades by the vortex distribution enables the formulation of the well-posed inverse problem, and which leads to design the blading of a turbomachine**.**

NOMENCLATURE

V	: resultant velocity
W	: relative velocity
Ω	: curl of the velocity
ψ	: stream function
x,y,z	: cartesian coordinate system
z, θ, r	: cylindrical coordinate system
ξ^1, ξ^2, ξ^3	: body fitted curvilinear coordinates
$\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$	: covariant base vectors
$\mathbf{e^1}, \mathbf{e^2}, \mathbf{e^3}$	: contravariant base vectors
ω	: turbomachine rotational speed, radians per unit time.
U_1, U_2, U_3	: covariant components of the absolute or relative velocity
p	: static pressure
ρ	: mass density of fluid
H	: total enthalpy, p_t / ρ
I	: rothalpy, $H + \omega(V_\theta r)$
$\mathbf{F_b}$	: vector force on blade surface
g_{ij}	: metric tensor elements
g	: determinant of the metric tensor
η	: pump efficiency factor
Q	: torque
P	: pump power
q	: flow rate
R	: impeller radius
n	: vector normal to blade camber surface
N_b	: number of blades
Γ	: circulation around blade section

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