# STRUCTURAL AND THERMAL PROPERTIES OF FE<sub>2</sub>(ZR,NB) SYSTEM IN C15, C14 AND C36 LAVES PHASES: FIRST-PRINCIPLES STUDY

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# Abstract

The pseudopotential method (PP), based on Density Functional Theory (DFT) using the Generalized Gradient Approximation (GGA) was applied to investigate the  $Fe_2(Zr, Nb)$  system within the three Laves phases structures: Cubic C15, Hexagonal C14 and C36. The effects of Nb concentration on structural, thermal and stability of the system were studied. The lattice parameters and bulk modulus of the three phases were predicted and showed a good agreement with the available experimental data. The rigidity of the Fe<sub>2</sub>Nb was higher than the Fe<sub>2</sub>Zr one for C15, C14 and C36. The energetic phase diagram of the systems was also established by determining the heat formation of the different phases. The obtained results showed that the Laves phases have close formation energies which suggests that the C15, C14 and C36 phases can co-exist at low-temperature. Finally, the temperature effect on the structural parameters, thermal expansions, heat capacities and Debye temperatures are determined from the non-equilibrium Gibbs functions and discussed accordingly.

Keywords : Debye's Quasi Harmonic Model, First-principles calculations, Laves phases, PP method.

#### Résumé

La méthode du pseudopotentiel (PP), sur la base de théorie fonctionnelle de densité (DFT) en utilisant le Generalized Gradient Approximation (GGA) a été utilisée pour étudier le système Fe2 (Zr, Nb) dans les trois Laves phases structures: Cubic C15, Hexagonal C14 et C36. Les effets de la concentration sur la structure de Nb, thermique et la stabilité du système ont été étudiés. Les paramètres de maille et module de masse des trois phases ont été prévues et ont montré un bon accord avec les données expérimentales disponibles. La rigidité de la Fe2Nb est plus élevé que celui Fe2Zr pour C15, C14 et C36. Le diagramme de phase dynamique des systèmes a également été établi en déterminant la formation thermique des différentes phases. Les résultats obtenus ont montré que les phases de Laves ont près énergies de formation qui suggère que les phases C15, C14 et C36 peuvent coexister à basse température. Enfin, l'effet de la température sur les paramètres structurels, des dilatations thermiques, les capacités de chaleur et des températures de Debye sont déterminées à partir des fonctions de Gibbs non-équilibre et discuté en conséquence.

Mots clés: Modèle quasi harmonique de Debye, premiers principes calculs, Laves phases, méthode PP.

#### ملخص

# الكلمات المفتاحية:

وقد تم تطبيق الطريقة (PP) pseudopotential ، على أساس الكثافة النظرية الوظيفية (تجهيز الدوائر) باستخدام المعمم التدرج التقريب (GGA) للتحقيق في نظام لل Fe2 (عنصر الزركون، ملحوظة) ضمن ثلاثة Laves مراحل الهياكل: مكعب C15، سداسي C14 و C36. تم دراسة تأثير تركيز ملحوظة على الهيكلية، والحرارية واستقرار النظام .وتوقع المعلمات شعرية ومعامل الحجم من المراحل الثلاث وأظهر اتفاق جيد مع البيانات التجريبية المتاحة .وكانت صلابة من Fe2Nb أعلى من واحد Fe2Zl، C15، C14 و C36. تأسست الرسم البياني مرحلة حيوية من الأنظمة أيضا تحديد تشكيل الحرارة للمراحل المختلفة .أظهرت النتائج المتحصل عليها أن مراحل لعوالي الماقات تشكيل وثيقة مما يوحي بأن مراحل 513، 2014 و C15، و214 مالي من واحد Fe2Zl، 2014، 2015 و C15 تأسست الرسم البياني مرحلة حيوية من الأنظمة أيضا تحديد تشكيل الحرارة للمراحل المختلفة .أظهرت النتائج المتحصل عليها أن مراحل Laves لها الطاقات تشكيل وثيقة مما يوحي بأن مراحل 513، 2014 و C15، وراحلة من المنحقضة .وأخيرا، يتم تحديد تأثير درجة الحرارة على المعلمات الحرارية، التوسعات الحرارية، والقدرات الحرارة ورجات حرارة ديباي من وطائف جيبس عدم التوازن ومناقشتها وفقا لذلك.

كلمات البحث: ديباي في شبه متناسق النموذجي، أول مبادئ الحسابات، Lavesمر احل، طريقة. PP

#### I. INTRODUCTION

Laves phases have Been considered for many important attractive applications, and such as superconducting materials, giant magneto-strictive materials and hydrogen storage materials, because they have excellent physical and chemical properties [1-4]. Recently, Laves phases have also been investigated for high-temperature structural applications due to their high melting point, high strength and reasonably good oxidation resistance at elevated temperatures [4]. Although several calculation methods have been successively applied to predict the stable ground state structure of various Laves phases, these calculations are always limited to only one structure type at the stoichiometric composition, and very often restricted to 0 K [1]. However in real systems the stability of Laves phase type is a function of temperature and composition and also different Laves phases types may coexist in a single system [1]. In order to avoid these limitations, the Pseudo-Potential (PP) method is applied in the present work to investigate the stability and structural properties of Fe2(Zr,Nb) system within the three Laves phases structures. In addition, the quasi-harmonic Debye model [5], which is added within the framework of the PP method, is used to reproduce the temperature effects. Thus, the aim of this work is threefold: (i) testing the validity of the quasi-harmonic Debye model using the total energy and molar volume set obtained with the (PP) method and making it suitable for a large number of solids, (ii) highlighting the stability of Fe<sub>2</sub>(Zr,Nb) Laves system, and (iii) giving structural and vibrational properties of this phases since there is a lack in experimental data.

#### **II. COMPUTATIONAL METHODS**

#### Ab initio calculations

All calculations in this study were done using the density functional theory (DFT) using pseudopotential method as implemented in the pseudo-potential plane wave self-consistent field package Quantum Espresso [6]. The many-body problem of interacting electrons and nuclei was mapped to a series of one-electron equations, the so-called Kohn-Sham (KS) equations [7-8]. The generalized gradient approximation (GGA) of Perdew et al. (PBE) to the local density approximation was taken to include the exchangecorrelation energy [9] and ultrasoft pseudopotentials of Vanderbilt were used [10]. A well converged value of the cut-off energy and the k-point mesh over the Brillouin zone were considered. All structures were allowed to fully relax using Broyden-Fletcher-Goldforb-Shanno (BFGS) scheme [11] until the total energy has converged to less than  $10^{-5}$ eV/atom. The maximum force has converged to lower than 0.004 *eV*/Å.

# Thermal properties within the Quasi-harmonic Debye model

In order to take into account the temperature effects, the quasi-harmonic Debye model has been applied [5]. Accordingly, the non-equilibrium Gibbs function  $G^*$  (*V*, *T*, *P*) takes the following formulation:

$$G^*(V;T,p) = E(V) + pV + A_{vib}(T,\theta(V))$$
(1)

where  $\theta(V)$  is the Debye temperature, and the vibrational term  $A_{vib}$  can be expressed as:

$$A_{vib}(T, \theta(V)) = nkT \left[\frac{9}{8}\frac{\theta}{T} + 3\ln(1 - e^{-\theta T}) - D(\theta/T)\right]^{(2)}$$
  
where  $D(\theta/T)$  represents the Debye integral,  $n$ 

is the number of atoms per formula unit, k is the Boltzmann constant and T is absolute temperature. The nonequilibrium Gibbs function  $G^*(V, P, T)$  as a function of V, P and T can be minimized with respect to volume V:

$$\left(\frac{\partial G^*(V; p, T)}{\partial V}\right)_{p,T} = 0 \tag{3}$$

By solving Eq.(3), we get the associated thermal properties for different temperatures and pressures.

# **III. RESULTS AND DISCUSSION**

In order to determine the static structural parameters at 0 K, a set of total energy versus total volume calculations was carried out for the Fe<sub>2</sub>Nb and Fe<sub>2</sub>Zr compounds in C15, C14 and C36. The corresponding values were fitted with the universal Birch-Murnaghan equation of state [12]. The lattice parameters, (c/a) ratios, Bulk Modulii, as well as the heats of formation of the studied compounds are presented in Tables 1, and when available, some experimental and theoretical data are given.

TABLE I: STRUCTURAL PROPERTIES of Fe<sub>2</sub>Nb AND Fe<sub>2</sub>Zr COMPOUNDS

Phases	Structure	a (Bohr)	c/a	B (Gpa)	$\Delta H_f$
Fe2Nb	C14	9.14	1.627	153	-0.11(7)
		Exp 9.11 <sup>13, 14</sup>	1.633	/	/
	C15	12.90	/	166.1	-0.11(1)
	C36	9.13	3.262	159.2	-0.11(2)
	C14	9.45	1.629	143.6	-0.27(0)
Fe <sub>2</sub> Zr		13.34			
	C15	<b>Theo</b> 13.25 <sup>15</sup>	/	139.5	-0.28(8)
		Exp13.31 <sup>13, 14</sup>			
	C36	9.44	3.262	141.3	-0.276(7)
		Exp 9.36 <sup>13,14</sup>	3.256		

From this TABLE I, the predicted lattice parameters of the C14-Fe<sub>2</sub>Nb, C15-Fe<sub>2</sub>Zr and the C36-Fe<sub>2</sub>Zr are very consistent with those measured experimentally by X-ray diffraction [13,14] as well as with those calculated from theoretical methods [15] (FP-LAPW). However, according to our knowledge, there have been no available experimental values of the bulk modulii of the named compounds. But considering our results, we remark that Fe<sub>2</sub>Zr is less compressible than Fe<sub>2</sub>Nb on the same kind of structure.

The groundstate of each compound, was determined from the evaluation of the formation energies. The formation energy is calculated as the difference between the Laves phase total energy and the sum of the total energy of the constituents in their corresponding ground state structure (bcc-Fe, hcp-Zr and bcc-Nb). For each structure (C14, C15 and C36), the last collumn in Table I, lists the formation energies of the two studied compounds. As a first remark, the obtained values are always negatives, showing that the formation of the C14, C15 or C36 phases is favored compared to the phase separation. Moreover, the C15 structure is the most stable for the Fe<sub>2</sub>Zr compound, while it is difficult to predict from our results (At 0 K), which of the three Laves structures is the most stable for the Fe<sub>2</sub>Nb one, because the heats of formation values are very closes. This small variation is related to the close relation between the structures of the three Laves phase polytypes, which results in very similar nearest neighbor geometries [4]. This is in good agreement with farmer calculations of the structural energies, by various pseudo potential or model potential calculations and e.m.f measurements [4]. Therefore, the stability of a particular crystal structure type depends sensitively on temperature, composition (deviation from stoichiometry or presence of impurities) and also different Laves phase types may coexist in a single system.

Using a given set of total energy versus molar volume values, E(V), and a numerical EOS, the thermal properties of Fe<sub>2</sub>Nb and Fe<sub>2</sub>Zr within the three laves phases structures were obtained. As a first remark, the lattice parameters and bulk modulii obtained with a numerical EOS, at T = 0 K (static case), for the three Laves Structures are in fairly good agreement with the murnaghan EOS and the available experimental and theoretical results (see Table.1). The effects of temperature on the volume and Bulk modulii of each compound are shown in fig.1 (a) and (b) respectively



Fig. 1. Temperature effects on the a) Volume and b) Bulk modulii of  $Fe_2Zr$  and  $Fe_2Nb$  Laves Phases

As shown in the Figure I, The volume of Fe<sub>2</sub>Nb and Fe<sub>2</sub>Zr within the different structures increases with increasing temperature but the rate of increase is more important in the case of C36 structure of the First compound, whereas, the effect of temperature on the C14-Fe<sub>2</sub>Nb

compound volume are very moderate. In terms of bulk modulus, the unique remark is that the C14-Fe<sub>2</sub>Nb compound is insensitive to the temperature variations, as its value at 1000 K is still about 154 Gpa.

Another vibrational property is the heat capacity. The Figure 2, shows the evolution of this parameter with temperature of the two compounds  $Fe_2Nb$  and  $Fe_2Zr$ , within the three Laves phases structures.



Fig.2 Heat capacity variation with temperature of  $Fe_2Nb$  and  $Fe_2Zr$  Laves phases.

We remark that, the evolution is similar for the different alloys. It follows the Debye model at low temperature (0-300 K) (CV(T)  $\sim$  T3) and the classical behavior (CV(T)  $\sim$  3R For mono-atomic solids) is found at sufficient high temperatures; obeying Dulong and petit's Rule. The specific heat capacity of the different materials at sufficient high temperature does not depend much on temperature and tends to approach 75 J mol-1 K-1.

#### **IV. CONCLUSION**

The pseudopotential DFT method as well as the Debye model is used to give new insights on structural and thermal properties of three Laves phases of Fe-Nb-Zr system. The main results can be summarized as follows:

- The calculated lattice parameters and bulk modulus of Fe<sub>2</sub>Zr and Fe<sub>2</sub>Nb are in good agreement with available experimental data.
- Fe2Zr is less compressible than Fe<sub>2</sub>Nb on the same kind of structure.
- The calculated heats of formations show that the Fe-Zr-Nb system favors binary Fe<sub>2</sub>Zr and Fe<sub>2</sub>Nb compounds formation over phase separation. In addition, it is difficult (At 0 K) to predict, which of the three Laves structures is the most stable for a given compound, because the heats of formation values are very closes.
- The use of the quasi-harmonic Debye model was successfully applied to determine the thermal properties of the Fe<sub>2</sub>Nb and Fe<sub>2</sub>Zr Laves phases in the 0-1000 K temperature range. Significant differences in properties are obtained Above 300 K.
- The C14-Fe<sub>2</sub>Nb compound is insensitive to the temperature variations.

Finally, the approach presented here is a promising alternative for the determination of several properties in complex structures that are difficult and/or costly to obtain from experiments.

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