EFFECTS OF BUOYANCY RATIO ON CONVECTIVE HEAT AND SOLUTE TRANSFER IN NEWTONIAN FLUID SATURATED INCLINED POROUS CAVITY

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Abstract

This paper summarizes a numerical study of the effects of buoyancy ratio on double-diffusive natural convection in square inclined cavity filled with fluid saturated porous media. Transverse gradients of heat and solute are applied on the two horizontal walls of the cavity, while the other two walls are impermeable and adiabatic. The Darcy model with the Boussinesq approximation is used to solve the governing equations. The flow is driven by a combined buoyancy effect due to both temperature and concentration variations. A finite volume approach has been used to solve the non-dimensional governing equations. The results are presented in streamline, isothermal, iso-concentration, Nusselt and Sherwood contours for different values of the non-dimensional governing parameters.

Keywords : *Boussinesq approximation, Darcy model, Double-diffusive natural convection, Porous media.*

Résumé

Cet article résume une étude numérique des effets du rapport des forces de volume sur la convection naturelle dans une cavité carré inclinée remplie d'un milieu poreux saturé de fluide. Les gradients transversaux de chaleur et de soluté sont appliqués sur les deux parois horizontales de la cavité, tandis que les deux autres parois sont imperméables et adiabatique. L'écoulement convectif résultant de la présence simultanée de gradients de température et de concentration dans un milieu poreux est régi par les lois classiques de Darcy. Les équations de conservation de la masse, de la quantité de mouvement, de l'énergie et de la concentration ont été déduites en tenant compte de l'approximation de Boussinesq. Pour la résolution du système d'équations couplées, obtenu, et les conditions aux limites associées, nous considérons une solution numérique par la méthode des volumes finis. Les résultats sont présentés par : les lignes de courant, les isothermes, les iso-concentrations, les variations des nombres de Nusselt et Sherwood, pour différentes valeurs des paramètres de contrôle non-dimensionnelles.

Mots clés: *convection naturelle, milieu poreux, approximation de Boussinesq, modèle de Darcy, volumes finis, cavité inclinée.*

ملخص

يلخص هذا العمل دراسة عددية لتأثير معامل الطفو على الحمل الحراري الطبيعي المزدوج داخل تجويف مسامي مائل مشبع بمائع. يتم تطبيق التدرج في للحرارة والمذاب على الوجهين السفلي والعلوي للتجويف، في حين أن الوجهين الآخرين يعتبران منيعين وكظومين. لحل المعادلات الموافقة نستخدم نموذج دارسي ومقاربة بوسيناسك. يتحرك المائع داخل الفجوة تحت تأثير معامل الطفو الناتج عن تغيرات درجة الحرارة التركيز. لحل المعادلات الموجهة اللابعدية نستخدم طريقة الحجوم المحدودة. يتم عرض النتائج على شكل خطوط الجريان، الخطوط المتساوية الحرارة والتركيز، تغيرات عدد نسلت وشيروود الموافقة لمختلف القيم للعوامل اللابعدية الموجهة.

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الكلمات المفتاحية : الحمل الحراري الطبيعي، الوسط المسامي، مقاربة بوسيناسك، نموذج دارسي، الحجوم المحدودة، تجويف مائل.

I. INTRODUCTION

Heat and mass transfer and fluid flow induced by double diffusive natural convection in fluid saturated porous media have been the object of considerable efforts owing to their practical importance in many engineering applications such as the migration of moisture through air contained in fibrous insulations, chemical reactors and transport of contaminants in saturated soil and electrochemical processes. The published numerical, experimental and analytical results concerning convective heat and mass transfert in fluid saturated porous media, represent an important bibliography may be found in the book by Nield and Bejan [1] and the second one by Ingham and Pop [2]. Most of the existing studies in the literature on double diffusive convection are dealing with horizontal square cavities. Trevisan and Bejan[3], studied analytically and numerically the mass transfer resulting from high convection in a porous medium heated from below they indicated the existence of different scaling laws for the dependence of the Nusselt number versus the Rayleigh and Lewis numbers. The thermo-solutal bifurcation phenomena in porous enclosures subject to vertical temperature and concentration gradients has been studied numerically and theoretically by Mamou and Vasseur [4]. In the case of a horizontal porous cavity partially heated from below and differentially salted, Bourich et al. [5] reported numerical results of thermo-solutal natural convection. They found multiple solutions in pure thermal convection vanish in the presence of horizontal solutal gradients when critical conditions, depending on the Rayleigh and Lewis number, are reached. The same author [6] demonstrated that the solutal buoyancy force induced by horizontal concentration gradients eliminates the multiplicity of solutions obtained in pure thermal convection when N exceeds some critical value, which depends on Le and Ra, when they studied the double diffusive convection in a porous enclosure submitted to cross gradients of temperature and concentration. They also analyzed the effects of the governing parameters on the flow structure and heat and mass transfer. The existence of multiple solutions in a horizontal porous enclosure heated horizontally and salted from the bottom has been studied numerically by Mohamad and Bennacer [7]. It was demonstrated that the multiplicity of solution obtained when Gr_m (modified Grashof number) =1000 and $0.8 \le N \le 1$. The bifurcation from monocellular dominating flow to bicellular dominating flow in this range of N, and in the case of thermally driven flow has been observed and the concentration gradient reversal was possible. Mahidjiba et al. [8] studied numerically by using linear stability analysis the onset of double diffusive convection in a horizontal porous cavity. They specified mixed boundary conditions for heat and solute on the horizontal walls of the enclosure while the two vertical ones are impermeable and adiabatic. It is shown that there exists a supercritical Rayleigh number for the onset of the supercritical convection and an over stable Rayleigh number, at which over stability may arise. The over stable regime is shown to exist up to a critical Rayleigh number at which the transition from the oscillatory to direct mode convection occurs. In a numerical and analytical study by Kalla et al. [9] of double-diffusive natural convection within a horizontal porous layer, where the vertical and the horizontal walls are submitted respectively to uniform heat and mass fluxes, they demonstrated the existence of multiple steady-state solutions, for a given set of the governing parameters. The double-diffusive natural convection problem in parallelogrammic enclosures filled with fluid-saturated porous media has been studied numerically by Costa [10]. Vertical walls are maintained at constant deferent levels of temperature and concentration, and the inclined walls are adiabatic and impermeable. It is shown that in terms of flow structure, temperature levels and concentration levels, strong changes occur in the parallelogrammic enclosure when changes are made on the Darcy-modified Rayleigh number, on the inclination angle and on the aspect ratio of the enclosure. Very different behaviors are obtained for the combined or opposite global heat and mass flows that cross the parallelogrammic enclosure. A literature review shows that relatively little work is available on the case of natural convection in inclined enclosures. Therefore, the present paper investigates numerically double diffusive natural convection within a porous inclined cavity with localized heating and salting from below. The complete system of governing equations is solved numerically and results are obtained for a large range of the governing parameters. The global Nusselt and Sherwood numbers dependence on the dimensionless governing parameters and boundary conditions is explored in detail.

II. MATHEMATICAL FORMULATION

The studied configuration, depicted in Fig.1, is a square-saturated porous cavity with length H. The cavity is tilted at an angle α with respect to the horizontal plane. The wall at Y = H represents the low-temperature (Tl) and low-concentration (Sl) boundary, and the wall at Y = 0 denotes the high temperature (Th) and high concentration (Sh) boundary. The other two walls are regarded as being insulated and impermeable. It is assumed that the third dimension of the cavity is large enough so that the fluid flow and heat and mass transfer can be considered two-dimensional. Hypotheses of incompressible and laminar flow are considered, and the saturated porous medium is assumed isotropic and homogeneous with constant thermophysical properties. Interaction between the thermal and concentration gradients, (Soret and Dufour effects) are neglected. The binary fluid that saturates the porous matrix is modelled as a Boussinesq incompressible fluid whose density variation can be expressed as:

$$\rho = \rho_0\left[1 - \beta_T\left(T - T_0\right) - \beta_S\left(S - S_0\right)\right] \qquad (1)$$

Where β_T and β_s are the thermal and concentration expansion coefficients. Subscript 0 stands for a reference state.

$$\beta_T = -\frac{1}{\rho_0}\left(\frac{\partial \rho}{\partial T}\right)_{P,S}, \quad \beta_S = -\frac{1}{\rho_0}\left(\frac{\partial \rho}{\partial S}\right)_{P,T} \qquad (2)$$

The following non dimensional variables are introduced:

$$x^+ = \frac{x}{H}, \quad y^+ = \frac{y}{H}, \quad u^+ = \frac{u}{a/H}, \quad v^+ = \frac{v}{a/H}$$

$$\psi^+ = \frac{\psi}{a}, \quad T^+ = \frac{T - T_L}{T_H - T_L}, \quad S^+ = \frac{S - S_L}{S_H - S_L} \qquad (3)$$

The dimensionless governing equations, based on the above definitions, are as follows:

$$\frac{\partial u^+}{\partial x^+}+\frac{\partial v^+}{\partial y^+}=0 \qquad (4)$$

$$\left(\frac{\partial^2}{\partial x^{+2}}+\frac{\partial^2}{\partial y^{+2}}\right)\psi^+ = -\mathrm{Ra}^*\left(\cos\alpha\left(\frac{\partial T^+}{\partial x^+}+\mathrm{N}\frac{\partial S^+}{\partial x^+}\right)-\sin\alpha\left(\frac{\partial T^+}{\partial y^+}+\mathrm{N}\frac{\partial S^+}{\partial y^+}\right)\right)$$

(5)

$$u^+\frac{\partial T^+}{\partial x^+}+v^+\frac{\partial T^+}{\partial y^+}=\left(\frac{\partial^2 T^+}{\partial x^{+2}}+\frac{\partial^2 T^+}{\partial y^{+2}}\right) \qquad (6)$$

$$u^+\frac{\partial S^+}{\partial x^+}+v^+\frac{\partial S^+}{\partial y^+}=\frac{1}{Le}\left(\frac{\partial^2 S^+}{\partial x^{+2}}+\frac{\partial^2 S^+}{\partial y^{+2}}\right) \qquad (7)$$

Where ψ^+, T^+ and S^+ are dimensionless stream function, temperature and concentration, respectively.

The dimensionless boundary conditions are:

$$\forall x^+, y^+=0:\ T^+=1,\ S^+=1,\ \psi^+=0$$
$$\forall x^+, y^+=1: T^+=0, S^+=0, \psi^+=0 \qquad (8)$$
$$\forall y^+, x^+=0 \text{ and } 1: \frac{\partial T^+}{\partial x^+}=\frac{\partial S^+}{\partial x^+}=\psi^+=0$$

From the dimensionless equations it is seen that the present problem is governed by three dimensionless parameters: the buoyancy ratio N, the Lewis number Le and the thermal Rayleigh number Ra_T defined as:

$$Ra_T=\frac{gH\beta_T K\Delta T}{a\upsilon},\quad N=\frac{\beta_S\Delta S}{\beta_T\Delta T},\ \ Le=\frac{a}{D}$$

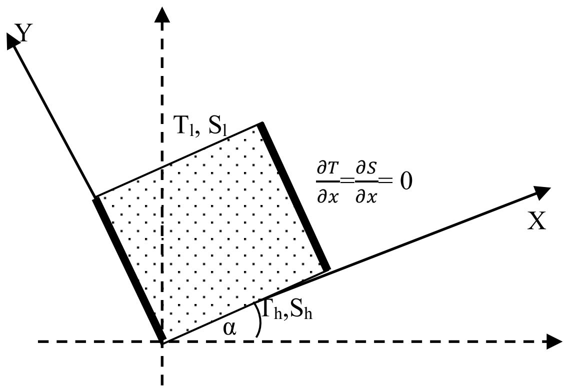


Fig. 1.Physical model and geometry

The average values of Nusselt and Sherwood numbers, evaluated on the bottom wall are given by:

$$Nu=-\int_0^1\left.\frac{\partial T^+}{\partial y^+}\right|_{Y^+=0}dx^+\ ,\qquad Sh=-\int_0^1\left.\frac{\partial S^+}{\partial y^+}\right|_{Y^+=0}dx^+ \qquad (09)$$

III. NUMERICAL SOLUTION

The numerical solution of governing equations (4) – (7) with specified boundary conditions equations (8), is obtained using the volume finite method described by Patankar [11]. The computation domain is divided into rectangular control volumes with one grid located at the centre of the control volume that forms a basic cell. The set of conservation equations are integrated over the control volumes, leading to a balance equation for the fluxes at the interface.

The iterative process, employed to find the stream function, temperature and concentration fields, was repeated until the following convergence criterion was satisfied:

$$\frac{\sum_i\sum_j(\Phi_{i,j}^{new}-\Phi_{i,j}^{old})}{\sum_i\sum_j\Phi_{i,j}^{new}}\le 10^{-6}$$

Where Φ stand for Ψ, T and S. The subscripts I and j denote grid locations in the (x, y) plane. A further decrease of the convergence criteria 10−6 does not cause any significant change in the final results. Numerical tests, using various mesh sizes, were done for the same conditions in order to determine the best compromise between accuracy of the results and computer time. A mesh size of 61 ×61 was adopted. The accuracy of the code was checked, modifying the thermal and solutale boundary conditions, to reproduce the results reported in [6]. Good agreement can be seen from Table 1 with a maximum deviation of about 3.4%.

TABLE I: VALIDATION OF THE NUMERICAL CODE, FOR α=0, Ra* = 200, N=0.3 AND VARIOUS LE, IN TERMS OF ψMAX, NU AND SH

Le	ψ_{max}		Nu		Sh	
	Present work	Ref.[6]	Present work	Ref.[6]	Present work	Ref.[6]
0.1	11.625	11.706	4.484	4.633	1.209	1.221
1	9.505	9.609	4.130	4.276	4.840	5.086
10	9.104	9.171	3.983	4.078	15.870	17.02

IV. RESULTS AND DISCUSSION

A. Considered situations

There are 4 dimensionless parameters governing the problem under analysis: Le, N, Ra*, α. All the presented results refer to moist air saturating the porous medium, with a low concentration of water vapor, thus fixing Le =0.8. Many values for N, Ra* and α in this work being taken N = (5, 2, 0.5 and 0), Ra* = (100 and 70) and (0°≤α≤ 90°).

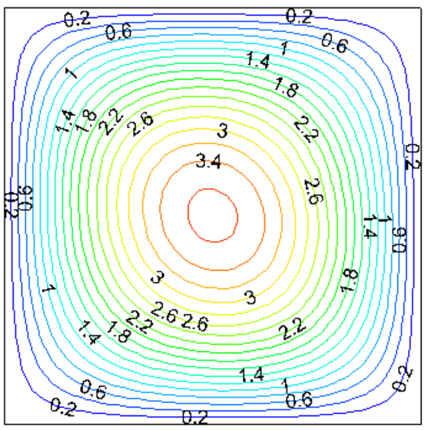
B. Flow structure, temperature and concentration fields, heat and mass transfer visualization

The appearance of single or multiple cell flows in the porous material is an obvious characteristic of convection. The physical notion of a cell is associated with an identifiable body of fluid rotating in the same sense. Therefore, it has to be bounded by a closed streamline within which the vorticity is of the same sign. Positive and negative streamlines Ψ correspond to counter clockwise and clockwise circulations, respectively. The convective motion will be referred to as natural flow whenever the fluid is ascendant above the heated element. On the other hand the convective pattern will be called antinatural when the fluid is descendant above the heated element.

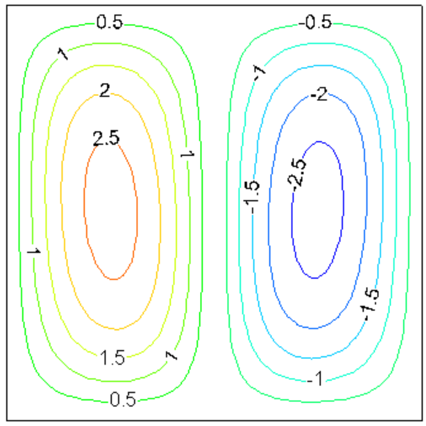
In the case of no solute transfer N = 0 single or multiple cell convection was found. When α=0° and Ra*=100 the single cell mode was obtained. An example of this flow is given in Fig. 2 where the streamlines are shown.

The stream function shows a single extremum value whose magnitude becomes larger as Ra* increases, indicating a more vigorous motion, as expected. As a function of the tilt angle, the ψ extremum value presents a maximum around 45°.

Results for α= 45°, Ra*=100 and combined global heat and mass flows are presented in Fig. 3a for N = 0.5 and in Fig. 3b for N = 2. Main changes from Fig. 2 to 3 are due to the increase on the buoyancy term. Flow is more intense, the temperature and concentration gradients are higher near the horizontal walls, and heat and mass transfer increases as N increases. As N increases, heat flows in an arrow region close to the right wall of the enclosure. In what concerns temperature and concentration fields, as Le = 0.8~ 1, there are no major differences on these fields.



(a)



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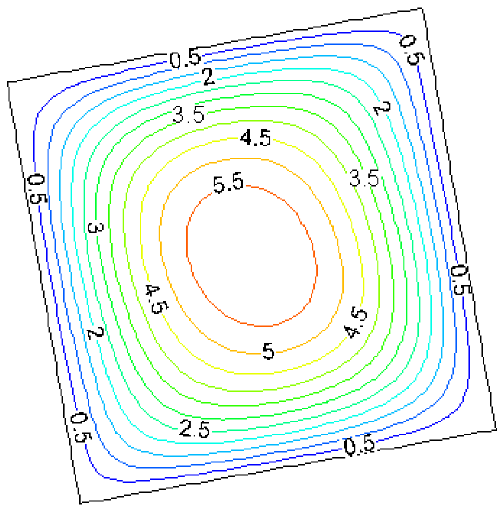
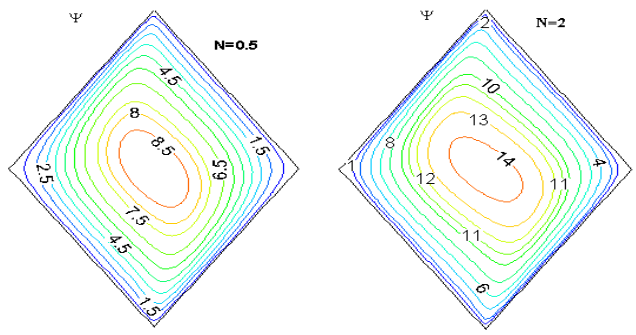


(c)

Fig. 2. streamlines for N = 0 (a) Ra=70, α= 0, Ψmax= 3.7, (b) α= 0, Ra = 100, Ψmax= 2.7, (c) α=30°, Ra = 100, Ψmax= 6.



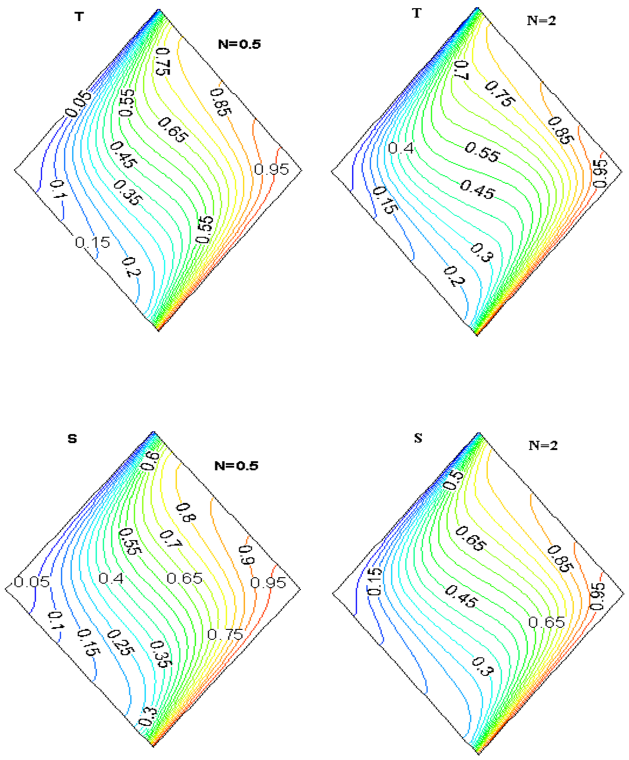


Fig. 3.Isotherms, iso-concentration and streamlines for combined global heat and mass flows, Le = 0,8,α= 45°, Ra = 100, for N =0.5, Ψmax= 8.9 and N=2, Ψmax= 14.4.

C. Heat and mass transfer parameters

In general terms, it can be observed that as Le=0.8≈ 1, there are no significant differences between the behavior and the numeric values of the global Nusselt and Sherwood numbers. Fig. 4.

Global Nusselt number is presented in Fig. 5. as function of the inclination angle α for different values of the Buoyancy ratio N when Ra*=100. For low values of α (α ≈ 0°) and high values of α(α ≈ 90°) the global Nusselt number is nearly the same. When (0 ≤ α ≤ 45°) the Nusselt number increases for any value of N. It is observed the existence of a maximum Nusselt number for α near 45° and a minimum for α near 0°. A physical explanation can be given for the thermal diode effect. The hot fluid moves upwards and reaches the right wall, which has a favorable inclination, allowing some tangentiality to the flow flowing along the wall towards the cold wall. The same applies also for the descending cold fluid on the neighboring of the opposite horizontal wall. The flow is intense and the thermal gradients near the horizontal walls are high, thus resulting into high global heat transfer rates.

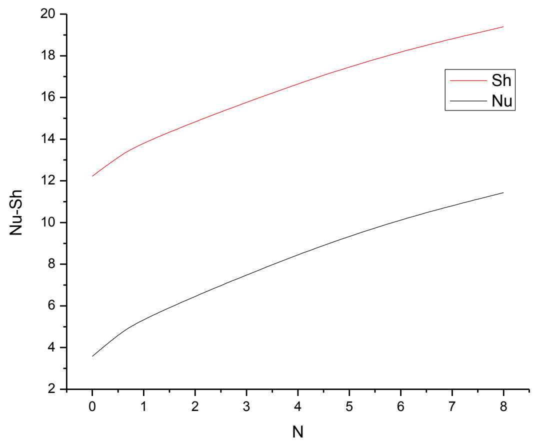


Fig. 4. Global Nusselt and Sherwood numbers as function of the Buoyancy ratio N for α = 30° and Ra= 100.

The highest heat and mass transfer parameters occurring for the range40° ≤ α ≤ 60°) and the minimum Nusselt and Sherwood numbers correspond to α = 0°. It is also observed that the increasing of the buoyancy ratio, always leads to increases on the heat and mass transfer performances of the enclosure.

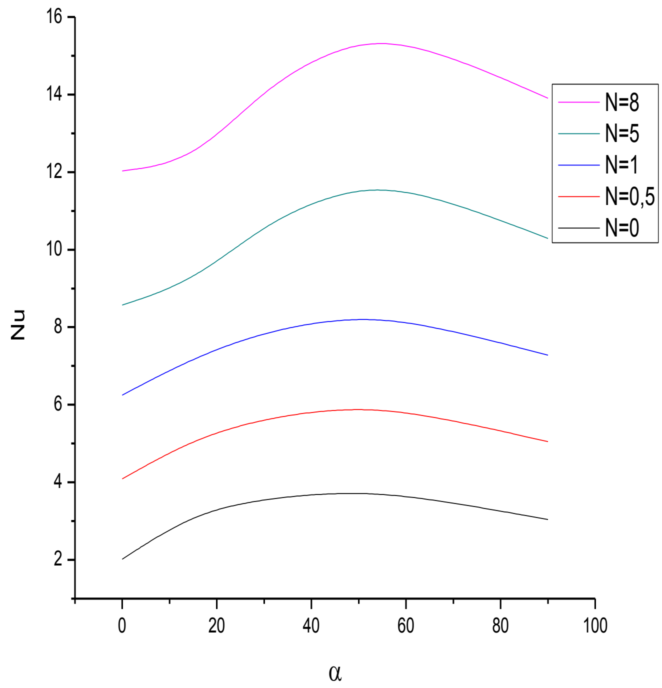


Fig. 5. Global Nusselt number versus inclination angle α for different values of the Buoyancy ratio N.

CONCLUSIONS

In terms of flow structure, temperature levels and concentration levels, strong changes occur in the square enclosure when changes are made on the Darcy-modified Rayleigh number, Buoyancy ratio N and on the inclination angle of the enclosure. Increasing the source term of the vertical momentum equation, by increasing the Darcy-modified Rayleigh number or by increasing the buoyancy ratio, always leads to increases on the heat and mass transfer performances of the enclosure. Very different behaviors are obtained for the combined global heat and mass flows that cross the enclosure. In what concerns the heat and mass transfer performances of the square enclosure, some main aspects should be mentioned. Selected combinations of the buoyancy ratio and inclination angle can lead to considerably high heat and mass flows through the enclosure, and some combinations of these parameters can even lead to the maximum allowable heat and mass transfer. It is thus present a maximum transfer performance, which is of crucial importance when the enclosure is to be used as a transfer promoter. However, other selected inclination angles from he foregoing ones, can lead to essentially unchanged poor transfer performances of the enclosure.

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