

# NUMERICAL STUDY OF DEVELOPING LAMINAR FORCED CONVECTION OF A NANOFLUID HEAT TRANSFER IN AN ANNULAR HORIZONTAL PIPE

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## Abstract

This study reports numerical simulation for 3D laminar forced convection of a nanofluid flow in horizontal annulus with constant heat flux at the outer cylinder while the inner cylinder is considered adiabatic. The numerical model is carried out by solving the governing equation of continuity, momentum and energy using take account for the finite volume method, with the assistance of SIMPLER algorithm. The results show that for the Reynolds numbers and Prandtl fixed, the dimensionless velocity profile for the laminar forced convection of a nanofluid consisting of water does not vary with the volume concentration of nanoparticles while the effect of the concentration of nanoparticles on the temperature of the mass is significant nanofluid. These results are consistent with those found in the literature. In general the use of nanofluid with a volume concentration of nanoparticles causes an increase in the coefficient of heat transfer by convection.

**Keywords:** *nanofluid, annulus, forced convection, numerical simulation, single-phase*

## Résumé

Cette étude rapporte la simulation numérique laminaire en 3D pour la convection forcée d'un nanofluide dans un anneau horizontal avec flux de chaleur constant au cylindre extérieur tandis que le cylindre intérieur est considéré comme adiabatique. Le modèle numérique est réalisé pour la résolution des équations de continuité, quantité de mouvement et l'énergie en utilisant la méthode des volumes finis et l'algorithme SIMPLER. Les résultats montrent que pour les nombres de Reynolds et de Prandtl fixés, le profil de la vitesse adimensionnelle pour la convection forcée laminaire d'un nanofluide constitué par l'eau ne varie pas avec la concentration volumique des nanoparticules tandis que l'effet de la concentration des nanoparticules sur la température du nanofluide est important. Ces résultats sont conformes à ceux trouvés dans la littérature. En général, l'utilisation des nanofluides avec une concentration volumique de nanoparticules provoque une augmentation du coefficient de transfert de chaleur par convection.

**Mots clés:** *Nanofluides, Annulaire, Convection forcée, Simulation numérique, Monophasé.*

## ملخص

هذه الدراسة تقرير لمحاكاة رقمية ثلاثية الأبعاد بالحمل القسري الرقائقي داخل الحلقة الأفقية مع تدفق حرارة ثابتة على الاسطوانة الخارجية وتعتبر الاسطوانة الداخلية كظومة. النموذج الرقمي لمعادلات الإحفاظ للكتلة، الحركة والطاقة تحل عددياً بطريقة الحجم المنتهية بمساعدة الخوارزمية SIMPLER. تبين النتائج أن عدد رينولدز و برانداال الثابتين أن ملمح السرعة اللابعدية بالحمل القسري الرقائقي للمائع النانوي يتألف من الماء لا تختلف مع إختلاف التركيز الحجمي في حين أن تأثير تركيز الجسيمات النانوية على درجة حرارة المائع النانوي هامة. هذه النتائج تتفق مع تلك التي وجدت في الأدبيات. بشكل عام استخدام الموائع النانوية مع تركيز حجم الجسيمات النانوية يؤدي إلى زيادة في معامل نقل الحرارة عن طريق الحمل الحراري.

**الكلمات المفتاحية:** مائع نانوي، الحلقة، الحمل القسري، المحاكاة الرقمية، أحادية الطور.

## I. INTRODUCTION

Enhancement of the thermal characteristic of liquid has been achieved by adding micrometer particles to a base fluid Maxwell [1]. These micron-sized particles cause some problems such as corrosion, clogging, rapid sedimentation, and high-pressure drop, all these problems have been solved by using solid nanoparticles dispersed uniformly and suspended stably in conventional liquids. This fluid was termed a “nanofluid” by Choi [2] in 1995 at the Argonne National Laboratory to characterize the new class of fluids with superior thermal properties to prevalent base fluids. Nanoparticles used in nanofluids have been made of various materials, such as oxide ceramics ( $Al_2O_3$ , CuO), nitride ceramics (AlN, SiN), carbide ceramics (SiC, TiC), metals (Cu, Ag, Au), semiconductors ( $TiO_2$ , SiC), and carbon nanotubes. Also, many types of liquids, such as water, ethylene glycol (EG), and oil, have been used as base liquids in nanofluid. The volumetric fraction of the nanoparticles,  $\phi$ , is usually below 5 %. With respect which can provide effective improvements in the thermal conductivity and convective heat transfer of base fluids [2-7]. Roy et al. [8] Investigated numerical study of laminar flow heat transfer for ( $Al_2O_3$  – EG) and ( $Al_2O_3$  – water) and reported an improvement in heat transfer rate. Also they showed that wall shear stress increases with increasing nanoparticles concentration and Reynolds number. Despite the fact that nanofluid is a two phase mixture, since the solid particles are very small size they are easily fluidized and can be approximately considered to behave as a fluid Xuan and Li [9]. Therefore, considering the ultrafine ( $< 100nm$ ) and low volume fraction of the solid particles, it might be reasonable to treat nanofluid as single phase flow in certain conditions Yang et al. [10]. As this approach is simpler to use several theoretical studies were done based on this approach [11–13] mixed convection in inclined tubes appears in many industries such as heat exchangers and solar energy collectors, Buoyancy force has an important role on the hydrodynamic and thermal behaviors of a fluid flow throughout the ducts Behzadmehr et al. [15]. The annuluses are a common and important geometry for fluid flow and heat transfer device. They have a lot of engineering application such as double pipe heat exchanger, gas turbines, reactors, turbo machinery, ventilation and air conditioning system and cooling of electronic component. Investigation of the heat transfer enhancement in the annuluses is essential. Moghari et al. [16] also studied mixed convection in an annular space and showed the effects of some important parameters such as nanoparticles volume fraction, aspect ratio, Grashof number, and heat flux, it is observed that the local Nusselt number increases with increase in nanoparticles concentration, Grashof number, and radius ratio. Izadi et al. [17] have also investigated laminar forced convection of a nanofluid consisting of  $Al_2O_3$  and water numerically in a two dimensional annulus with single phase approach.

## II. MATHEMATICAL MODEL

Figure 1 shows the schematic representation of the model used in this study. We consider the annulus consists of two long horizontal duct, having inner diameter  $D_i = 0.5$  diameter is  $D_o = 1$  and length of  $L = 100$ . The length of the

tube is very large compared to its hydraulic diameter, the hydraulic diameter is defined ( $D_h = D_o - D_i$ ). The problem study is 3D, the flow is laminar and forced convection of nanofluid ( $Al_2O_3$ /water) through a horizontal annulus duct. The outer cylinder is heated by uniform heat flux while the inner cylinder is adiabatic. The single-phase model is used, simplifying hypothesis be applied to solved this problem. The nanofluid is assumed incompressible and Newtonian with negligible viscous dissipation and pressure working. The fluid and solid phases are in thermal equilibrium with the same velocity of movement and the Boussinesq approximation is adopted.

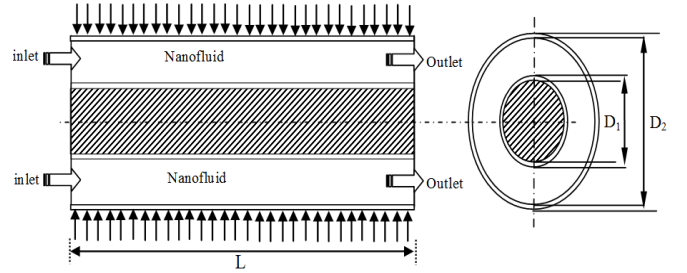


Fig. 1. The physical model and geometry corresponding

## III. DIMENSIONLESS CONSERVATION EQUATIONS

In this article the star superscript (\*) make reference to a dimensionless quantity.

$$\text{At } t^* = 0, \quad u^* = v^* = w^* = T^* = 0 \quad (1)$$

$u^*, v^*, w^*$  are the radial, angular and axial velocity component  $= (u/v_0), (v/v_0), (w/v_0)$ , The dimensionless temperature  $T^* = (T - T_0) / (q_w D_h / k_{nf})$ , and  $t^* = (v_0 t / D_h)$

At  $t^* \geq 0$ ,

### A. Mass conservation equation

$$\frac{1}{r^*} \frac{\partial (r^* u^*)}{\partial r^*} + \frac{1}{r^*} \frac{\partial w^*}{\partial \theta} + \frac{\partial v^*}{\partial z^*} = 0 \quad (2)$$

$r^*, \theta, z^*$  are the radial, angular and axial coordinates

### B. Radial momentum equation

$$\frac{\partial u^*}{\partial t^*} + \left[ \frac{1}{r^*} \frac{\partial (r^* u^* u^*)}{\partial r^*} + \frac{1}{r^*} \frac{\partial (u^* w^*)}{\partial \theta} + \frac{\partial (v^* u^*)}{\partial z^*} - \frac{w^{*2}}{r^*} \right] = \frac{\partial p^*}{\partial r^*} + \frac{Gr}{Re^2} \frac{\beta_{nf}}{\beta_f} \cos \theta T^* + \frac{1}{Re} \frac{1}{(1-\phi)^{2.5}} \times \quad (3)$$

$$\frac{\rho_f}{\rho_{nf}} \left[ \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* \frac{\partial u^*}{\partial r^*} \right) + \frac{1}{r^{*2}} \frac{\partial}{\partial \theta} \left( \frac{\partial u^*}{\partial \theta} \right) + \frac{\partial}{\partial z^*} \left( \frac{\partial u^*}{\partial z^*} \right) - \frac{u^*}{r^{*2}} - \frac{2}{r^{*2}} \frac{\partial w^*}{\partial \theta} \right]$$

Where the Reynolds number  $Re$  is defined by following  $Re = (v_0 D_h / \nu_0)$  and the Grashof number is defined by The following statement  $Gr = (g \beta_f q_w D_h^4 / k_{nf} \nu_{nf}^2)$

### C. Angular momentum equation

$$\frac{\partial u^*}{\partial t^*} + \left[ \frac{1}{r^*} \frac{\partial (r^* u^* w^*)}{\partial r^*} + \frac{1}{r^*} \frac{\partial}{\partial \theta} (w^* w^*) \right] = -\frac{1}{r^*} \frac{\partial p^*}{\partial \theta} + \frac{Gr}{Re^2} \frac{\beta_{nf}}{\beta_f} \sin \theta T^* + \frac{1}{Re} \frac{1}{(1-\phi)^{2.5}} \times$$

$$\frac{\rho_f}{\rho_{nf}} \left[ \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* \frac{\partial w^*}{\partial r^*} \right) + \frac{1}{r^{*2}} \frac{\partial}{\partial \theta} \left( \frac{\partial w^*}{\partial \theta} \right) + \frac{\partial}{\partial z^*} \left( \frac{\partial w^*}{\partial z^*} \right) - \frac{w^*}{r^{*2}} - \frac{2}{r^{*2}} \frac{\partial u^*}{\partial \theta} \right]$$

Where  $p^*$  is the pressure,  $p^* = (p / \rho_{nf} \nu_0^2)$

### D. Axial momentum equation

$$\frac{\partial u^*}{\partial t^*} + \left[ \frac{1}{r^*} \frac{\partial (r^* u^* v^*)}{\partial r^*} + \frac{1}{r^*} \frac{\partial}{\partial \theta} (w^* v^*) \right] = -\frac{\partial p^*}{\partial z^*} + \frac{1}{Re} \times \frac{1}{(1-\phi)^{2.5}} \times \frac{\rho_f}{\rho_{nf}} \left[ \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* \frac{\partial v^*}{\partial r^*} \right) + \frac{1}{r^{*2}} \frac{\partial}{\partial \theta} \left( \frac{\partial v^*}{\partial \theta} \right) + \frac{\partial}{\partial z^*} \left( \frac{\partial v^*}{\partial z^*} \right) \right]$$

### E. Energy conservation Equation

$$\frac{\partial T^*}{\partial t^*} + \left[ \frac{1}{r^*} \frac{\partial}{\partial r^*} (r^* u^* T^*) + \frac{1}{r^*} \frac{\partial}{\partial \theta} (w^* T^*) \right] = \frac{k_{nf}}{k_f} \times \frac{(\rho C p)_f}{(\rho C p)_{nf}} \frac{1}{Re \cdot Pr} \left[ \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* \frac{\partial T^*}{\partial r^*} \right) + \frac{1}{r^{*2}} \frac{\partial}{\partial \theta} \left( \frac{\partial T^*}{\partial \theta} \right) + \frac{\partial}{\partial z^*} \frac{\partial T^*}{\partial z^*} \right] \quad (6)$$

### F. Thermal and physical Properties of Nanofluid

The density of the nanofluids is calculated according to Pack and Cho's equation [18]:

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \quad (7)$$

The specific heat of nanofluid. Xuan et al.[7] is:

$$(\rho C p)_{nf} = (1-\phi)(\rho C p)_f + \phi(\rho C p)_s \quad (8)$$

The Hamilton-Crosser equation [19]:

$$k_{nf} = \frac{k_s + (n-1)k_f - (n-1)\phi(k_f - k_s)}{k_s + (n-1)k_f - (n-1)\phi(k_f - k_s)} k_f \quad (9)$$

Where  $n$  the shape factor is defined by  $n = 3/\Psi$  and  $\Psi$  is

ratio of the sphericity.  $n = 3$  for spherical nanoparticles.

The Brinkman [20] model for spherical nanoparticles

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \quad (10)$$

Khanafar et al. [11] provides a model which calculates the coefficient of thermal expansion:

$$(\rho\beta)_{nf} = \left[ \frac{1}{1 + \frac{(1-\phi)\rho_f}{\phi\rho_s}} \frac{\beta_s}{\beta_f} + \frac{1}{1 + \frac{\phi}{(1-\phi)} \frac{\rho_f}{\rho_s}} \right] \beta_f \quad (11)$$

The table 1 shows the thermo-Physical properties of the base fluid and nanoparticle solid  $Al_2O_3$  at  $20^\circ C$

TABLE I: THERMO-PHYSICAL PROPERTIES AT  $20^\circ C$

Physical Properties	Fluid phase water	Solid phase $Al_2O_3$
$Cp(Jkg^{-1}K^{-1})$	4181.8	761.55
$\rho(kg.m^{-3})$	998.2	3960.14
$k(Wm^{-1}K^{-1})$	0.593	37.14
$\beta(K^{-1})$	21	0.75
$\mu(Ns.m^{-2})$	1.002	/

### G. Boundary Conditions

Governing equations has been solved with the following boundary conditions:

At the inlet of the Annular duct :  $z^* = 0$

$$0.5 \leq r \leq 1.0 \text{ and } 0 \leq \theta \leq 2\pi$$

$$u^* = w^* = T^* = 0, v^* = 1 \quad (12)$$

At the outlet of the Annular duct :  $z^* = 200$

$$0.5 \leq r \leq 1.0 \text{ and } 0 \leq \theta \leq 2\pi$$

$$\frac{\partial u^*}{\partial z^*} = \frac{\partial v^*}{\partial z^*} = \frac{\partial w^*}{\partial z^*} = \frac{\partial}{\partial z^*} \left( \frac{\partial T^*}{\partial z^*} \right) = 0 \quad (13)$$

(The duct length  $L$  is 200 time of the hydraulic diameter  $D_h$  to insure that the fully developed condition is reached at the outlet)

At the internal pipe:  $r^* = 0.5$

$$u^* = v^* = w^* = 0 \text{ and } \frac{\partial T^*}{\partial r^*} = 0 \quad (14)$$

At the outer wall of the inner cylinder:  $r^* = 1.0$

$$u^* = v^* = w^* = 0 \text{ and } \frac{\partial T^*}{\partial r^*} = 1 \quad (15)$$

Along the angular direction, the periodic conditions are imposed.

The heat transfer is notified by the Nusselt number, which reflects the relative ration of convective to conductive heat transfer. Since the surface of the inner cylinder is adiabatic, the Nusselt number will be reported to the outer surface of the outer cylinder.

At steady state, the local Nusselt number depending on angular and axial position is expressed by the following equation:

### H. The Nusselt number

At the solid-nanofluid interface ( $r^* = 1.0$ ) the local Nusselt number is defined as:

$$Nu(\theta, z^*) = \frac{h_o(\theta, z)D}{k_{nf}} = \left[ \frac{(k_{nf}/k_f)(\partial T^*/\partial r^*)}{T^*(1, \theta, z^*) - T_b^*(z^*)} \right]_{r^*=1} \quad (16)$$

where the dimensionless bulk fluid temperature is:

$$T_b^*(z^*) = \frac{\int_{0.5}^1 \int_0^{2\pi} v^*(r^*, \theta, z^*) T^*(r^*, \theta, z^*) r^* dr^* d\theta}{\int_{0.5}^1 \int_0^{2\pi} v^*(r^*, \theta, z^*) r^* dr^* d\theta} \quad (17)$$

The axial Nusselt  $Nu(z^*)$  number is defined as:

$$Nu(z^*) = \frac{1}{2\pi} \int_0^{2\pi} Nu(\theta, z^*) d\theta \quad (18)$$

The average Nusselt number for the full solid-nanofluid interface is defined as:

$$Nu_A = \frac{1}{(2\pi)(200)} \int_0^{2\pi} \int_0^{100} Nu(\theta, z^*) dz^* d\theta \quad (19)$$

$Nu_A$  is the average Nusselt number.

#### IV. NUMERICAL RESOLUTION

For solution of the governing equations are discretized by the finite volume method. The temporal discretization of the derivation terms follows the backward Euler scheme whereas the convective and the non-linear terms follow the Adams-Bashfort scheme whose the truncation error is of  $\Delta t^{*2}$ . The spatial discretization of the diffusive terms and the pressure gradient follows the fully implicit central difference scheme. A SIMPLER algorithm was applied for pressure-velocity coupling similar to that described by Patankar [21]. With step time of  $\Delta t^* = 10^{-3}$ , the mesh used contains  $26 \times 44 \times 162$  point in the radial, angular and axial directions successively. A validation concerning the forced convection is verified by the comparison of our results with those of Nazrul and al. [22]. The controlling parameters used in this study problem are:  $Re = 500, 1000$ ,  $Pr = 6.79$ ,  $Gr = 0$ , radius ratio  $r_o/r_i = 2$  and one volume fraction  $\phi = 1\%$ . In figure 2 the numerical code used, is a transformation of the code developed in the first step by Boufendi and Afrid[23] and in the second step by Touahri and Boufendi[24,25]

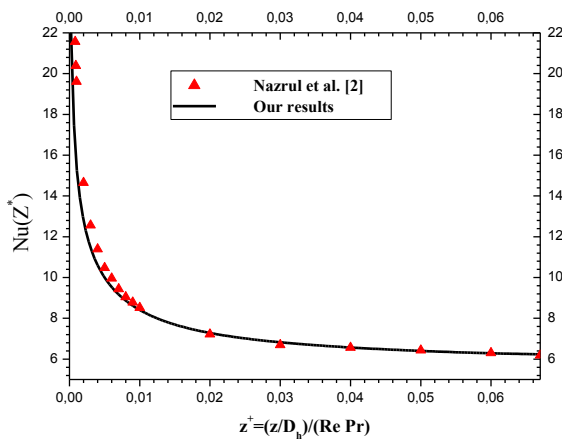


Fig. 2. Axial evolution of the mean Nusselt number; Comparison with the result of [21]

#### V. RESULT AND DISCUSSION

The present study for different Reynolds number  $Re=500$  and  $1000$ , a fixed Grashof number ( $Gr = 0$ ) and a volume concentration of  $1\%$ . The result shows:

##### A. The hydrodynamic and thermal fields

The hydrodynamic and the thermal fields are represented in Fig.3 and Fig. 4 for the forced convection regime with cases of the Reynolds number  $Re = 1000, Gr = 0$ , the velocity distribution show a central area where it is high and areas where velocities are low located on either side of this central part. or the maximum velocity equal to center  $v^* = 1.4307$

At radial position  $r^* = 0.3780$  following a parabolic profile in any axial section of the speed profile is axisymmetric is the stream lines of the axial velocity are concentric circles with a variation in the radial direction. The thermal fields are also show in fig. 4 for the same above case the distribution of the speed from the entrance to the exit, this distribution shows that the isotherms are circular and concentric in a radial stratification in any section which decreases from the worm outside the inner cylinder. the absolute maximum is reached at the output that is equal to  $0.2588$ . This distribution is characteristic of heat transfer by forced convection.

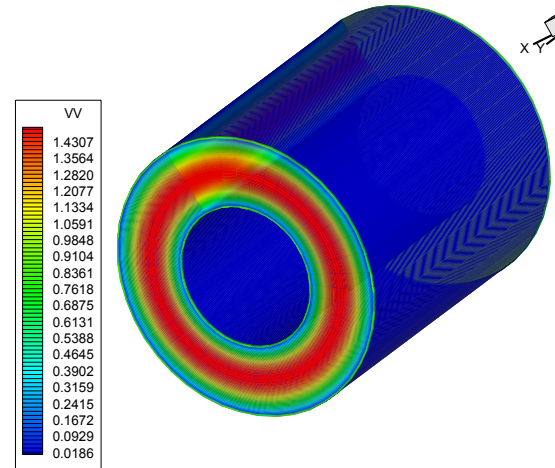


Fig. 3. Axial velocity field along the annular along duct for  $Al_2O_3$  nanofluid and for  $Re = 1000, Gr = 0$

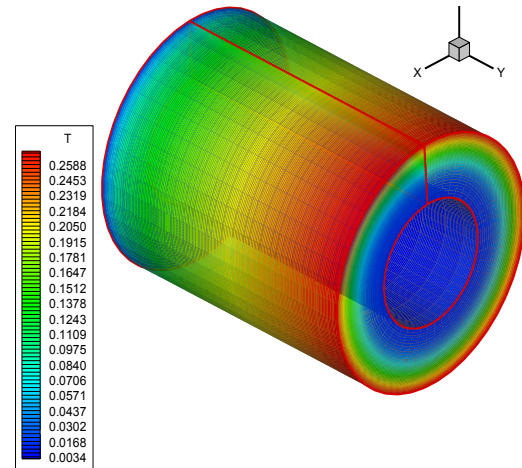


Fig. 4. Thermal field along the annular duct for  $Al_2O_3$  nanofluid and for  $Re = 1000, Gr = 0$

and for  $\phi = 1\%$

Figure. 5 illustrates the wall and fluid mean bulk temperature versus local angular position at two Reynolds number ( $Re = 500, 1000$ ) and volume fraction 1%, one can observed that the wall temperature and bulk fluid temperature decrease Along the annulus, the average temperature profile of the nanofluid between the inlet and the outlet. Qualitatively this graph undergoes a monotonic increase from the inlet to the outlet. This growth follows a constant heating, the maximum mean bulk temperature  $T_b^* = 0.16$  and  $T_b^* = 0.08$  for  $Re=500$  and  $Re= 1000$  respectively. As well as for the wall temperature the maximum at two different position the top  $\theta=0$  and the bottom  $\theta=\pi$ ; The maximum at the exit duct for different Reynolds number  $Re = 500$  are the same  $T^* = 0.355$ , but for the same previous positions at  $Re= 1000$  the maximum at the exit is equal to  $T_w^* = 0.275$

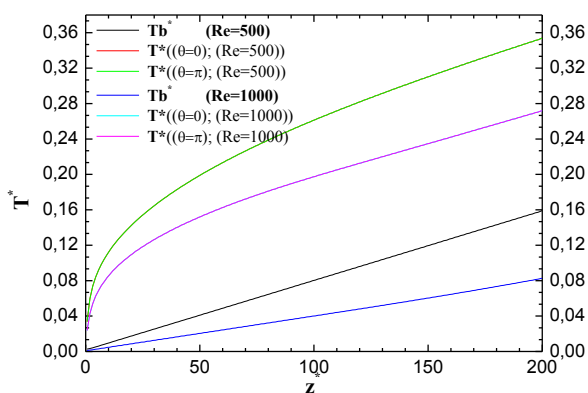


Fig. 5 Axial variation of the bulk nanofluid temperature  $T_b^*$  and the wall temperature  $T_w^*$  at  $\theta = 0$  and  $\theta = \pi$  for the two Reynolds numbers studied and  $\phi = 1\%$

## B. The Nusselt numbers

Circumferentially Nusselt number characterizes the heat transfer between the interface hot wall and the nanofluid, which is obtained by (16). The variation of the local Nusselt number is presented in fig. 6 for two different Reynolds numbers 500 and 1000 and  $Gr=0$  from the entrance to exit duct, it is clear that these variation with abrupt decrease in the short entrance zone and a very slow diminution and asymptotic, constant at the exit zone is physically acceptable with same behaviour a fluid flow in forced convection. the value of the Nusselt number at the outlet of the duct is 5.28

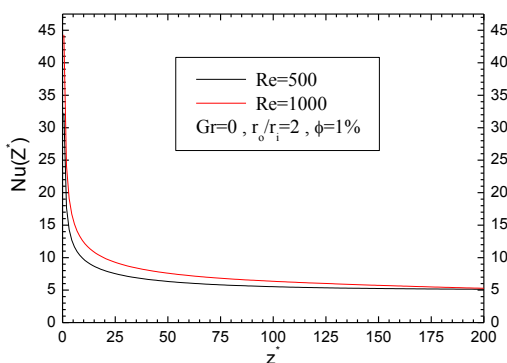


Fig. 6. Axial profile of the Nusselt number for  $Re=500$  and  $1000$

## CONCLUSION

The three dimensional Laminar forced convection heat transfer of ( $Al_2O_3 / water$ ) nanofluid inside a horizontal annular pipe under constant heat flux in the outer wall of cylinder will the inner cylinder is considered adiabatic. The obtained results for the range Reynolds number and single concentration 1% show that:

- The existence of the nanoparticle in the base fluid causes a decrease in the average temperature and the wall temperature. but for the small Reynolds number of the maximum average temperature and wall temperature and greater than the temperature of the largest Reynolds number
- The average temperature along the pipe at the bottom and at the top has the same value at the exit for the same Reynolds number.
- For the forced convection study, the Nusselt number increases for any value of the Reynolds number or until he attains 5.28 at the exit.

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