Evaporation of a thin binary liquid film covering one plate of a vertical Channel

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Résumé – *Le présent travail présente une étude numérique des transferts couplés de chaleur et de masse dans un canal vertical formé par deux plaques isothermes. La première plaque est couverte par un film liquide binaire extrêmement mince d'eau et d'éthylène glycol. La seconde plaque est sèche. Le changement de phase (évaporation et ou condensation) associé à l'un ou aux deux composants peut avoir lieu dépendamment de plusieurs facteurs dont les conditions opératoires. La formulation mathématique de ce problème est basée sur les équations de conservations de masse, de quantité de mouvement, de l'énergie et de conservation d'espèces. La variabilité de propriétés thermo-physiques du mélange liquide et du mélange gazeux ainsi que l'effet des forces de flottabilité ont été pris en compte. En utilisant la méthode de différence finie, un modèle numérique a été développé et testé systématiquement. Une analyse de l'effet des différents paramètres tels que les températures des plaques et les conditions d'entrées du mélange gazeux sur le processus de changement de phase et sur le transfert de chaleur et de masse a été effectuée.*

Abstract – *The present work is a numerical study of the heat and mass transfer in a vertical channel with isothermal plates. The first plate is covered with an extremely thin binary film of water and ethylene glycol. The second one is dry. Due to the heating effects and the forced (mixed) convection flow of air containing the respective vapors of film constituents, phase change can occur. The mathematical formulation of the problem is based on the conservation equations of mass, momentum, energy and species subjected to the appropriate boundary conditions. The variability of the thermo physical properties of the liquid and the gas mixtures as well as the effect of the buoyancy forces in the momentum equations were taken into account. A numerical model using the finite difference method was developed and tested systematically. A detailed parametric analysis on the effects of several operating variables such as the temperatures of the plates and the inlet conditions of the gas mixture on the phase change process and on the heat and mass transfers was conducted.*

Key words: Evaporation – Condensation - Binary liquid film - Mixed convection - Combined heat - Mass transfer.

1. INTRODUCTION

The phase change of a binary or a multi component liquid film flowing on an isothermal or heated solid surface is a very interesting research subject due to its several industrial applications. We cite the heavy chemistry, the thermal desalting and the mixture separation processes in general. Despite the main importance of the binary film

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current utilization, only a few studies were conducted to investigate the phase change phenomena and the heat and mass exchanges. For example, Baumann and Thiele [1] studied a mixture of methanol and benzene flowing in the internal surface of a tube. The forecast of the evaporation of a benzene/methanol mixture in a turbulent jet of hot air shows the influence of the phase equilibrium and its interaction with the transfers.

Hoke *et al.* [2] supposed that the shear stress, at the liquid-gas interface, is negligible during the evaporation of a binary liquid film of water-ethylene glycol. Minkowyz and Sparrow [3] presented a theoretical investigation of laminar film condensation by natural convection on an isothermal vertical plate. Results corresponding to a pure liquid film are obtained for a wide range of governing parameters such as ambient pressure, concentration and temperature. They show in particular that the influence of non condensable gas is accented at lower pressure levels. Concerning the theoretical treatment of flows with multi component mixtures.

Kotake [4] carried out a numerical study on film condensation of a binary mixture inside a cylindrical duct with variable section. The author used the integral method for gas flow and the Nusselt model for the film one. He analysed the effects of several important parameters such as the geometry of the cylinder on the condensation process.

The same author [5] analyzed the effects of the presence of non condensable gas on film condensation of multi-component mixtures. The equations governing the heat and mass transfer on the liquid/gas interface were solved by incorporating a disturbance due to the addition of a small quantity of non condensable gas to the mixture. In addition, it is reported that diffusion occurs in the flow saturated with gas causing the transport of the most volatile component far from the interface.

Schroppel [6] studied the film condensation of a binary mixture on a vertical plate. He analysed the influence of a linear variation of the plate temperature compared to the case of uniform wall temperature.

Vijary and Slattery [7] analysed the isothermal evaporation of a binary liquid mixture into the air. Their study is extended to measure the binary liquid diffusion coefficient by observing the position of liquid-gas interface.

Agunaoun *et al.* [8] considered a liquid film with binary mixture flowing on one of the plates of a vertical channel and evaporating by the mixed convection of the surrounding humid air. The wet plate is subjected to a constant heat flux and the other plate is adiabatic.

Cherif and Daif [9] studied the evaporation of a binary film flowing on one plate of a vertical channel. They solved the coupled parabolic equations of the momentum, heat and mass transfers. Results corresponding to ethylene glycol/ water and ethanol/water mixtures are presented and analysed.

Liu *et al.* [10] presented an approximate analytical solution of the boiling of a binary film flowing on a horizontal roll by employing an integral method. By including the contribution of the radiative heat transfer, they proposed a general correlation for the coefficient of the thermal transfer during the boiling process.

The objective of this work is to perform a numerical study on the evaporation (the condensation) of an extremely thin binary liquid film (of a vapor mixture) in a vertical channel. A particular attention is addressed to analyze the effect of the film plate temperature on the rate of the phase change.

2. ANALYSIS

The studied physical model (Fig. 1) shows the flow and transfers in a vertical channel of height H and width d. The humid plate is maintained at two different temperatures: T_{p1} on its higher part and T_{p2} on its lower one. The second plate is isothermal and dry. The gas mixture enters the channel with a temperature T_0, a water vapour concentration c_{01}, an ethylene glycol vapour concentration c_{02}, a pressure p_0 and a velocity u_0.

For the mathematical formulation of the problem, the following simplifying assumptions were taking into consideration:

i. Liquid and gas flows are laminar, steady and two dimensional.
ii. The liquid film is supposed to be extremely thin.
iii. Boundary layer approximations are supposed valuable for the gas stream.
iv. Humid air is an ideal mixture of water and ethylene glycol vapours and dry air. It is considered as an ideal gas.
v. The gas-liquid interface is in the thermodynamic equilibrium.
vi. The effect of surface tension is neglected. The Soret and Duffour effects are also ignored.
vii. Radiation heat transfer, viscous dissipation and pressure work terms are neglected in the energy equation.

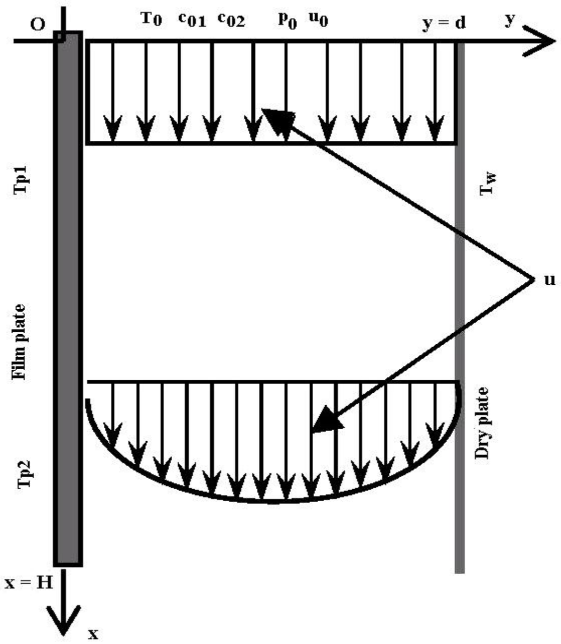

Fig. 1: Physical model

2.1 Governing equations

Following the above approximations, the governing equations in the gas mixture are:

Continuity equation

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial x} = 0 \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\left(\frac{\rho - \rho_0}{\rho}\right)g - \frac{1}{\rho}\frac{dP}{dx} + \frac{1}{\rho}\left(\mu\frac{\partial u}{\partial y}\right) \tag{2}$$

Energy equation

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) + \rho D_1 (C_{pv1} - C_{pa})\frac{\partial T}{\partial y}\frac{\partial c_1}{\partial y} + \rho D_2 (C_{pv2} - C_{pa})\frac{\partial T}{\partial y}\frac{\partial c_2}{\partial y} \tag{3}$$

Diffusion equations

$$u\frac{\partial c_1}{\partial x} + v\frac{\partial c_1}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\rho D_1\frac{\partial c_1}{\partial y}\right) \tag{4}$$

$$u\frac{\partial c_2}{\partial x} + v\frac{\partial c_2}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\rho D_2\frac{\partial c_2}{\partial y}\right) \tag{5}$$

Global mass balance at each section

$$\int_0^d \rho u(x,y)\, dy = d\rho_0 u_0 + \int_0^x \rho v(x,0)\, dx \tag{6}$$

2.2 Boundary conditions

Channel entry

$$T = T_0;\ p = p_0;\ c_1 = c_{01};\ c_2 = c_{02};\ u = u_0 \tag{7}$$

Dry plate

$$T(x,d) = T_w;\quad \begin{cases} u = 0 \\ v = 0 \end{cases};\quad \begin{cases} \left.\dfrac{\partial c_1}{\partial y}\right|_{y=d} = 0 \\ \left.\dfrac{\partial c_2}{\partial y}\right|_{y=d} = 0 \end{cases} \tag{8}$$

$c_1(x,0)$ and $c_2(x,0)$ are given by [9]:

$$c_1(x,0) = \frac{p^*_{vs1}}{p^*_{vs1} + \left[p^*_{vs2}\dfrac{M_1}{M_2}\right] + \left[p - p^*_{vs1} - p^*_{vs2}\right].\dfrac{M_a}{M_1}} \tag{9a}$$

$$c_2(x,0) = \frac{p^*_{vs2}}{p^*_{vs2} + \left[p^*_{vs1}\dfrac{M_1}{M_2}\right] + \left[p - p^*_{vs1} - p^*_{vs2}\right].\dfrac{M_a}{M_1}} \tag{9b}$$

Where $p^*_{vs,1}$ and $p^*_{vs,2}$ are the saturation pressures respectively for water and ethylene glycol. They are expressed as:

$$p^*_{vs,i} = P^*_{vs,i}(X_{Li}, T) = X_{Li} P_{vs,i}(T) \qquad i = 1, 2 \tag{10}$$

$$X_{Li} = c_{Li} \frac{M}{M_i} \quad \text{with} \quad \frac{1}{M} = \frac{c_{L1}}{M_1} + \frac{c_{L2}}{M_2}$$

$$P_{vs,1} = \left[10^5 \times 10^{17.443-(2975/T + 3.686 \log T)}\right]^{0.5} \tag{11}$$

$$P_{vs,2} = 6894.8 \exp(16.44 - 10978.8/(9T/5 - 49)) \tag{12}$$

The units used in equations (11 and 12) are Pa for the pressure and K for the temperature [8].

In order to describe the mass and energy magnitude transported between the channel walls and the moist air, the following dimensionless coefficients are used:

- The local Sherwood number defined as:

$$Sh_x = -\frac{2d\left[(\partial C/\partial y)_{y=0}\right]_x}{C(x,0) - C_m}$$

C_m is the fluid bulk concentration at a cross section:

$$C_m = \int_0^d \rho u \,.\, C \,.\, dy \Big/ \int_0^d \rho u \,.\, dy$$

- The total evaporation (condensation) rate given by [9]:

$$\dot{m}_{evap,condt}(x) / 0.004 = \frac{1}{H} \int_{à}^{x} \rho\, v(x,0)\, dx$$

$$m_{evap,condt}(X) : \text{total evaporation (condensation) rate} \quad (kg/ms)$$

The thermo-physical properties of the gas and the liquid mixtures are considered as variable with temperature and composition. The correlations used in this study are given in [7].

3. SOLUTION METHOD

The present problem defined by the governing equations and the boundary conditions is solved numerically using a finite difference marching procedure in the downstream direction. A fully implicit scheme where the axial convection terms are approximated by the upstream difference and the transverse convection and diffusion terms by the central difference is employed. The discrete equations are resolved line by line from the inlet to the outlet of the channel.

Several grid sizes have been tested to ensure that the results are grid independent (**Table 1**). The grid distribution adopted in this study consists of 100 x 31 nodes respectively in the axial and transverse direction of the gas region.

Table 1: Comparisons of the difference of temperature ($T_p - T_0$) for different grid systems. {case of heated plates, T_p = wall temperature ; $T_0 = 20\,°C$; $p_0 = 1\,atm$; $u_0 = 1\,m/s$; $d = 0.015\,m$; $H = 1\,m$; $c_{0eau} = 0$; $c_{0ethy} = 0$; $c_{liq,1} = c_{liq,2} = 0{,}5$ (50 % water and 50 % ethylene glycol); $q_{w2} = 0\,W/m^2$; $q_{w1} = 3000\,W/m^2$ }

$N_x \times N_y$	x = 0.2	x = 0.5	x = 1
51 × 31	51.94	60.08	65.91
100 × 31	51.83	59.98	65.81
150 × 31	51.63	59.86	65.74
100 × 21	52.31	60.45	66.27
100 × 51	51.55	59.76	85.68

In order to verify the validity of the numerical model, we conducted several comparison tests. In particular, the present results were compared to those of [9] for the case of the evaporation of a binary film mixture by mixed convection in a vertical heated channel. A good agreement was found (Fig. 2).

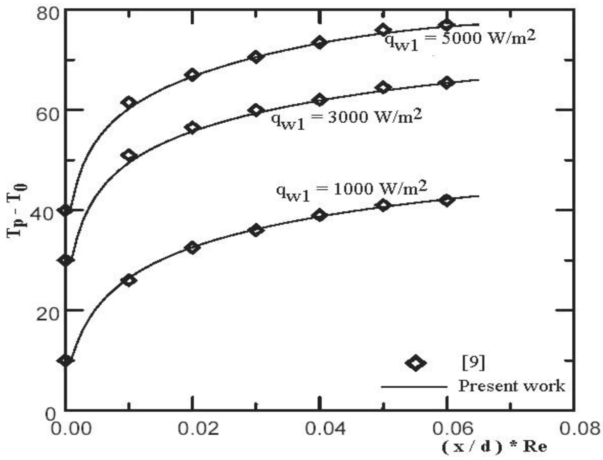

Fig. 2: Comparison with the results of Cherif *et al.* [9] for the case of heated channel

4. RESULTS AND DISCUSSION

In this theoretical study, we analyze the binary film (mixture of water and ethylene glycol) evaporation-condensation phenomenon by mixed convection of a gas mixture in a vertical channel. We investigate the influence of the humid wall temperature on the phase change process. Four cases are considered:

Case 1 ($T_{p1} = 20\,°C$, $T_{p2} = 40\,°C$)

Case 2 ($T_{p1} = 40\,°C$, $T_{p2} = 20\,°C$)

Case 3 ($T_{p1} = 20\,°C$, $T_{p2} = 60\,°C$)

Case 4 ($T_{p1} = 60\,°C$, $T_{p2} = 20\,°C$)

Results concern an air-water/ethylene glycol system with $d / H = 0.02$, $T_0 = 25\,°C$, $c_{01} = 0$, $c_{02} = 0$, $c_{L1} = c_{L2} = 0.5$, $p_0 = 1\,atm$, $T_w = 25\,°C$ and $u_0 = 1\,m/s$.

Figures 3 and 4 present the transversal distribution of vapours concentrations respectively corresponding to the water (c_1) and the ethylene glycol (c_2) at two axial positions ($x / L = 0.5$ and 1.0). The wall temperatures are those of case 1 and case 2.

These figures show that when the first part of the channel is at a temperature lower than that of the second part, evaporation always takes place. That is indicated by the positive gradients of concentration profiles. When the order of the temperature is reversed (Fig. 4), the situation changes at the channel half. The condensation phenomenon takes place since the gas is enriched with the respectively vapours in the first part of the channel. It is also noticed that this phenomenon is more effective for the least volatile component (ethylene glycol).

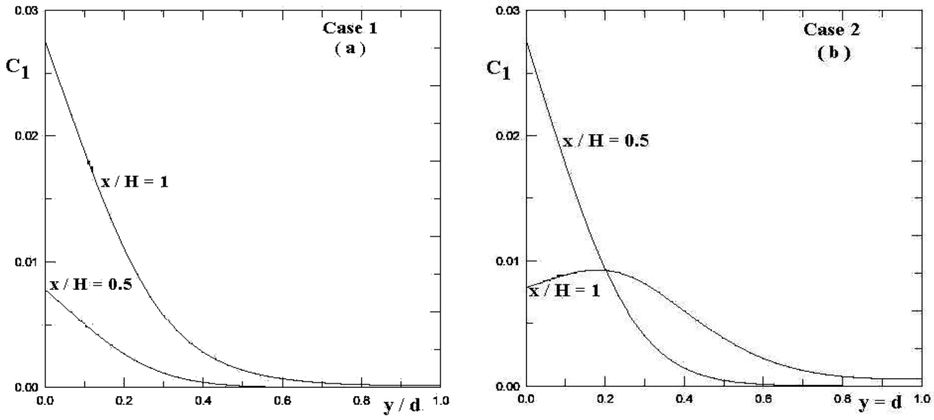

Fig. 3: Transversal variation of the water vapour concentration c_1 at two axial positions (*a-case 1 and b-case 2*)

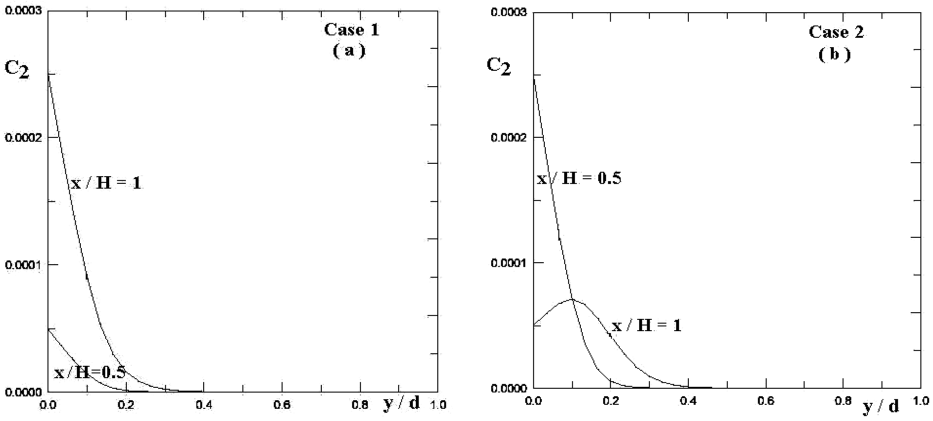

Fig. 4: Transversal variation of the ethylene glycol vapour concentration c_2 at two axial positions (*a-case 1 and b-case2*)

Figures 5-a and 5-b show the transversal distribution of the gas temperature at two axial positions for cases 1 and 2. Similar profiles are observed for the two cases. Besides, the axial velocity distribution is not affected by the wall temperature value (not shown here). This result can be justified by the fact that the thermal and mass buoyancy forces are negligible. However, the effect of the wall temperature on the mass transfer coefficient and the evaporation rate is more important as shown in figure 6 and 7.

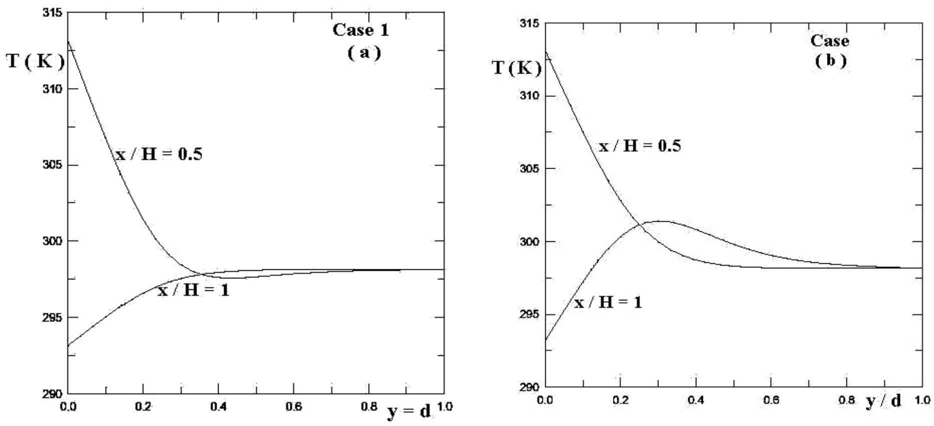

Fig. 5: Transversal variation of gas temperature at two sections (*a-case 1 and b-case2*)

Figure 6 shows that the local Sherwood number decreases and remains positive for case 1. For case 2, it becomes negative for second half of the channel ($x/H > 0.5$) indicating that condensation is taking place.

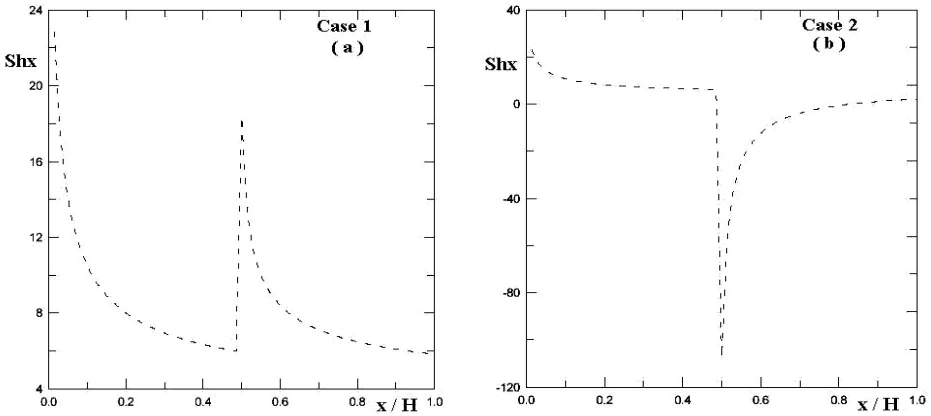

Fig. 6: Axial variation of the local Sherwood number
for cases 1 and 2 (*a-case 1 and b-case2*)

Figure 7-a illustrates the axial variation of the total evaporating (condensing) rate for cases 1 and 2. For case 1, it is observed as mentioned earlier that only evaporation takes place; however, condensation occurs on the second part of the plate for case 2.

It is of interest also to note that the total evaporating rate (at the channel exit) is higher for case 1 than that for case 2. Figure 7-b which presents a similar behaviour shows a striking improvement of the condensation in case 4 and of the evaporation in case 3. It is seen that the rate of phase change is enhanced for cases 3 and 4.

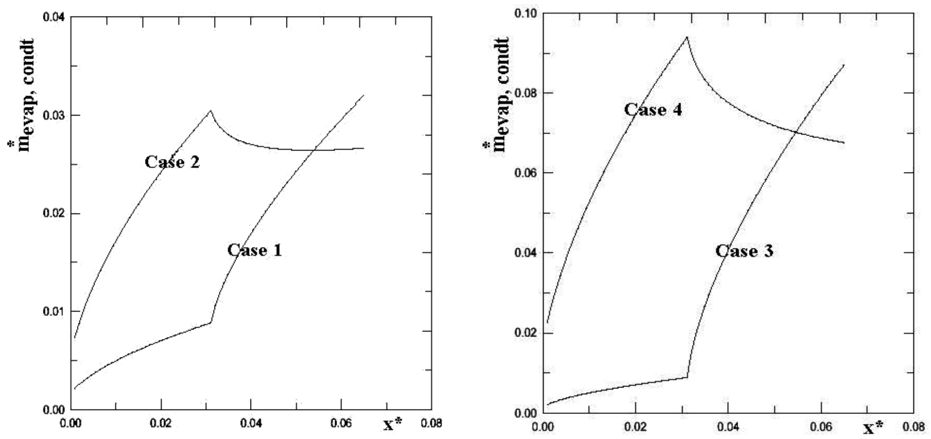

Fig. 7: Axial distribution of the rate of condensation (evaporation) – (*cases 1, 2, 3 and 4*)

Figures 8, 9, and 10 analyze for cases 1 and 2 the influence of the inlet gas parameters (temperature, concentrations c_{01} and c_{02}).

Figure 8 shows that the total condensation (evaporation) rate is enhanced for higher values of the inlet temperature of the gas, T_0 .

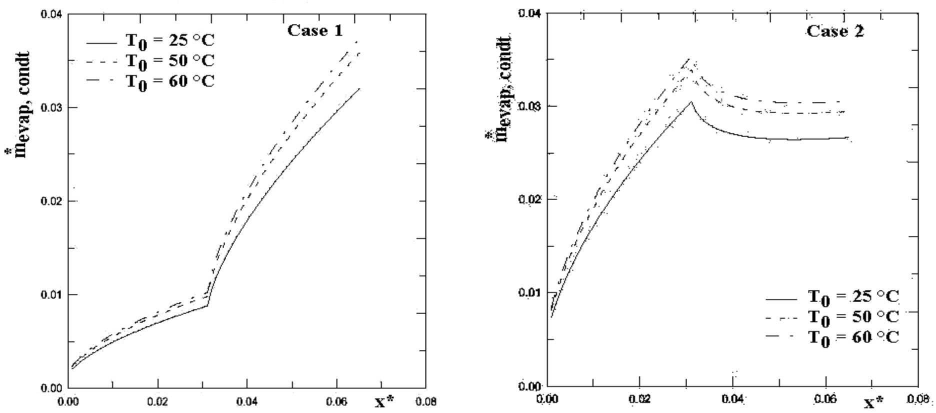

Fig. 8: Effect of the inlet gas temperature on the rate of condensation (evaporation) - (*case 1 and case 2*)

Figure 9 indicates that the condensation (evaporation) rate becomes more important for the lower values of c_{01} .

Figure 10 illustrates that when one increases the concentration of ethylene glycol vapour c_{02} , the total condensation (evaporation) rate decreases.

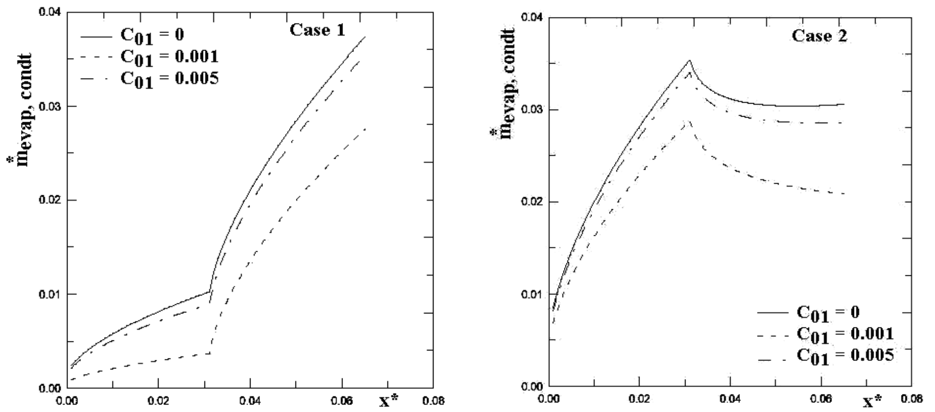

Fig. 9: Effect of the inlet gas water concentration on the rate of condensation (evaporation) - (*case 1 and case 2*)

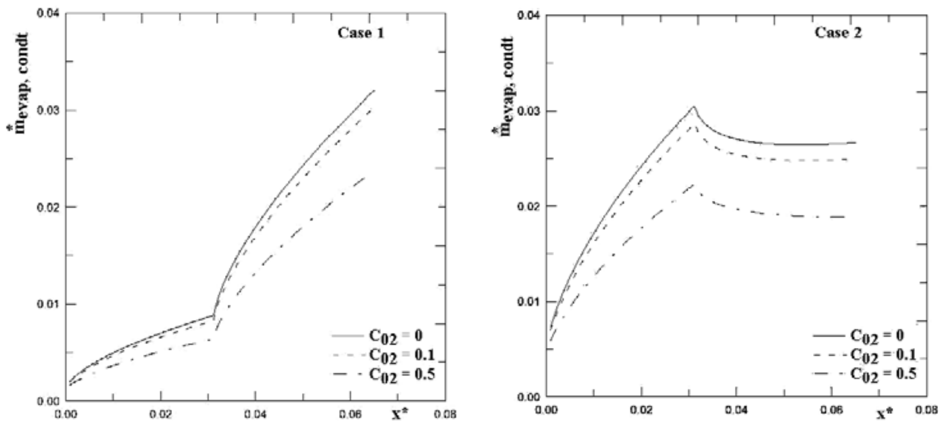

Fig. 10: Effect of the inlet gas ethylene glycol concentration on the rate of condensation (evaporation) - (*case 1 and case 2*)

5. CONCLUSION

The problem of condensation (evaporation) by mixed convection heat and mass transfer in a vertical channel has been numerically analyzed for an air water/ethylene glycol system.

One channel plate is covered by the binary film (water and ethylene glycol) and the second is dry. The two plates are isothermal.

The effects of the temperatures of the plates on the phase change process and on the heat and mass transfers are analyzed.

The influence of the inlet conditions of the gas mixture on the total condensation (evaporation) rate is also investigated.

It is observed that the nature of the phase change (condensation or evaporation) is directly dependent on the values of the wall temperatures.

NOMENCLATURE

c_i	Mass fraction vapour for species i	H	Channel length, [m]
c_{Li}	Mass fraction liquid for species i	L_v	Latent heat per mass unit, [kJ/kg]
X_{Li}	Molar concentrations for species i	$\dot{m}_{evap,condt}(X)$	: Total evaporation (condensation) rate, [kg/ms]
c_p	Specific heat for constant pressure, [kJ.kg^{-1}.K^{-1}]	p	Pressure, [Pa]
c_{pa}	Specific heat for air, [kJ.kg^{-1}.K^{-1}]	T	Temperature, [K]
c_{pv}	Specific heat for water vapour, [kJ.kg^{-1}.K^{-1}]	u	Axial velocity, [m/s]
M_i	Molar mass of species i vapour	v	Transversal velocity, [m/s]
D	Mass diffusivity [m^2/s]	x	Axial coordinate [m] direction
d	Channel width [m]	y	Transversal coordinate [m]
g	Gravitational acceleration [m/s^2]		

Greek symbols

μ	Dynamic viscosity, [kg/ms]
ρ	Density, [kg/m^3]
λ	Thermal conductivity, [W/mK]

Subscripts

sat	Saturation
0	Inlet condition
1	Water
2	Ethylene glycol

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