

Numerical simulation of a nanofluid flow in mixed convection inside a heated square cavity

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Abstract

In this work, we have numerically studied the two-dimensional and laminar flow of water, and nanofluid (Cu-water) in mixed convection inside a square cavity, where right and left walls are heated with a constant temperature and contain conductive fins. The lower and upper walls are adiabatic. The numerical resolution of the mathematical model was made by the finite volume method. The results obtained show that the Richardson number variations ($Ri = 0.1, 0.5, 1, 5$ and 10), the volume fraction (from 0.0 to 0.1), the aspect ratio as well as the angle of the cavity inclination have a considerable influence on the increase of the Nusselt number at the fins, and consequently on the improvement of the heat transfer inside the cavity.

Keywords: mixed convection, cavity, fins, cooling of electronic components, nanofluids.

1. Introduction

Following the recent technological advances, industrial equipments are becoming more powerful and smaller. It is at the nanometric level of the matter of the convective medium that the recent works have been concentrated. The idea is then to increase heat transfer by introducing into the pure fluids a low concentration of nanoscale particles, hence the appearance of the nanofluid term introduced by Choi [1]. In this context, many authors have carried out works. Among them, Pishkar and Ghasemi [2] numerically studied mixed convection in a horizontal channel. Two conductive fins are mounted on the bottom wall which is heated at a constant temperature in the presence of a nanofluid. They found that the heat transfer rate is significantly influenced by the distance between the fins and their thermal conductivities. On the other hand, when the Reynolds number and the volume fraction increase, the heat transfer increases remarkably. Molaei and Dekhordi [3] studied mixed convection inside a rectangular channel subjected to a constant heat flux, partially filled with a metal foam. The study is made with a nanofluid water- Al_2O_3 with different concentrations. The results of the simulation were validated against those obtained experimentally and an acceptable agreement was found. Adriana [4] who numerically studied the forced convection using a conventional fluid and a nanofluid in a laminar regime, in a tube of two zones, an isothermal zone and a zone with a constant heat flux. His results show that the nanofluid heat transfer coefficient increased from 3.4% to 27.8% over that of pure water under a fixed Reynolds number ($Re = 1500$), which clearly shows that nanoparticles suspended in water improve convective heat

transfer. Kherbeet et al [5] studied the effect of using different types of nanoparticles on the heat transfer caused by heating the bottom wall of a horizontal channel. nanoparticles have diameters between 25 and 70 nm and the volume fraction varies from 1 to 4%. They found that the nanofluids that have the lowest density, have the highest speed at the same section. The results also indicated that the Nusselt number increases by increasing the volume fraction and the size of the nanoparticles.

In the present study, the effect of the Richardson number, of the nanoparticle volume fraction, of inclination angle and of the aspect ratio on heat transfer rate and flow pattern inside of a heated square cavity containing conductive fins.

2. Problem definition and Mathematical formulation

The configuration under study, together with the system of coordinates is depicted in Fig. 1. It consists of a ventilated square cavity. The right and left vertical walls are uniformly heated with a constant temperature, and on each wall is placed a conductive fin. The upper and the bottom horizontal walls are considered insulated. The cavity is submitted to an imposed flow of a nanofluid through an opening located on the lower part of left vertical wall. The forced flow leaves the cavity through an outflow opening placed on the upper part of the right vertical wall.

Using the following reduced variables, one can write the system of dimensional equations of continuity, conservation of mass, momentum, and energy.

$$X = \frac{x}{d}; Y = \frac{y}{d}; \quad U = \frac{u}{u_0}; \quad v = \frac{v}{u_0};$$

$$P = \frac{p}{\rho_{nf} u_0^2}; \quad \theta = \frac{T - T_0}{T_c - T_0}$$

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\mu_{nf} v^*}{\rho_{nf} \nu_f Re} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \tag{2}$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\mu_{nf} v^*}{\rho_{nf} \nu_f Re} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{Ri}{\rho_{nf}} \left((1 - \varphi) \rho_f + \varphi \rho_s \frac{\beta_s}{\beta_f} \right) \theta \tag{3}$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = k^* \frac{\alpha_{nf}}{\alpha_f} \frac{1}{Re Pr} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \tag{4}$$

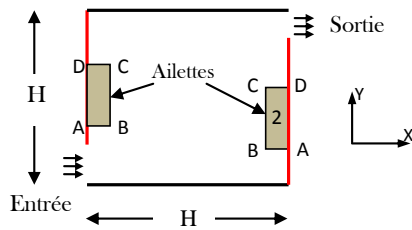


Figure 1. Physical model in a square cavity

The dynamic viscosity as well as the effective thermal conductivity of the nanofluid are determined from the Brinkman [7] and Maxwell [8] models, respectively:

$$\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}} \tag{5}$$

$$\frac{k_{nf}}{k_f} = \frac{k_s + (n-1)k_f + (n-1)\varphi(k_f - k_s)}{k_s + (n-1)k_f + \varphi(k_f - k_s)} \tag{6}$$

the density ρ_{nf} , the heat capacity (ρC_p) $_{nf}$, the coefficient of thermal expansion ($\rho \beta$) $_{nf}$, and the thermal diffusivity of the nanofluid α_{nf} are defined as follows [6]:

$$\rho_{nf} = (1 - \varphi) \rho_f + \varphi \rho_s \tag{7}$$

$$(\rho c_p)_{nf} = (1 - \varphi) (\rho c_p)_f + \varphi (\rho c_p)_s \tag{8}$$

$$\rho \beta_{nf} = (1 - \varphi) (\rho \beta)_f + \varphi (\rho \beta)_s \tag{9}$$

The local and mean Nusselt numbers are respectively defined along a fin wall as follows:

$$Nu(N) = - \left(\frac{k_{nf}}{k_f} \right) \frac{\partial \theta}{\partial N} \Big|_{ABC D} \tag{10}$$

$$\overline{Nu} = \frac{1}{AB + BC + CD} \int_{ABC D} Nu(N) dN \tag{11}$$

Where N represents dimensionless coordinates (X or Y). The boundary conditions are summarized in Table 1.

Table 1: Boundary conditions

Condition	U	V	θ
inlet	U_0	0	0
outlet	$\partial U / \partial x = 0$	$\partial V / \partial x = 0$	$\partial \theta / \partial x = 0$
Left wall	0	0	1
Right wall	0	0	1

Upper wall	0	0	$\partial \theta / \partial y = 0$
Bottom wall	0	0	$\partial \theta / \partial y = 0$

3. Numerical Method and Validation

The previous partial differential equations are discretized by the finite volume method developed by Patankar, and the SIMPLE algorithm [9] has been used for solving the different equations. For $Re = 10$ and $Gr = 10^5$, the mesh of 40401 nodes was chosen since it is considered to have the best compromise between computation time and precision as shown in Table 2. Validation of the mathematical model was done by comparing our results with those obtained by Pishkar and Ghasemi [2], and a good agreement was found, as shown in figures 2 and 3.

Table 2. Average Nusselt number for the grid examination

Nodes number	Nu_{moy} (aillette 1,2)	Erreur (%)
961	15.27764	
2601	14.3994	5.748532
5041	14.06328	2.334264
10201	13.89849	1.171775
22801	13.86978	0.206569
40401	13.84686	0.165251
63001	13.82784	0.13736

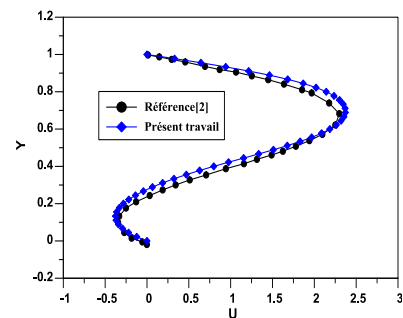


Figure 2. Horizontal velocity profile U as a function of Y, X = 3.25 for Re = 100

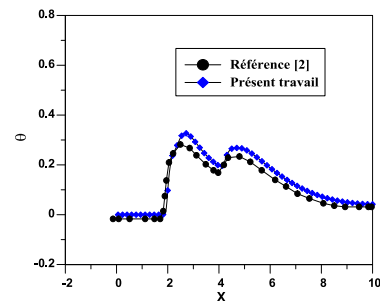


Figure 3. Temperature profiles θ in function of X, Y = 0.5 for Re = 100

4. Results and discussions

4.1. Effect of the Richardson number and the nanoparticles volume fraction (ϕ)

Figures 4 and 5 allow to examine the effect of the Richardson number on the flow and thermal fields. Our simulations were made for a Grashof number ($Gr = 5000$), for different values of the Richardson number ($Ri = 0.1, 0.5, 1, 5$ and 10), for different values of the nanoparticles volume fraction ($\phi = 0, 0.03, 0.05$ and 0.1). For $\phi = 0.05$, and $Ri = 0.1, 0.5$ and 1 , we notice a large recirculating zone in the neighborhood of the right fin and small recirculating zones above and below the two fins. Then for $Ri = 5$ and 10 we notice the disappearance of all the recirculation zones. For these last two values of the Richardson number where the speeds are low, since for $Ri = 5$ the streamlines presents ripples in the middle of the cavity, which increase for $Ri = 10$ contrary to $Ri = 1$ where they are more or less steep. The thermal field of the flow is presented by the contours of the isotherms which are located essentially on the right and left sides of the cavity in the vicinity of the fins. By increasing Richardson number, we remark a considerable increase in the thermal boundary layer especially in the vicinity of the left fin. This is undoubtedly due to the increase in buoyancy forces, compared to the inertial forces, where a low heat transfer is expected in the

vicinity of the fin 1. By consulting figure 10 which shows the variation of average Nusselt number in function of Ri number, for each fin and for both together, we see that the mean Nusselt number increases by decreasing the Richardson number, and that an increase in rate of 10 time of the speed flow (Ri from 10 to 0.1) improves the heat transfer about 57%.

The effect of the volume fraction was examined for $\phi = 0, 0.03, 0.05$ and 0.1 and for $Ri = 0.1, 1, 5$ and 10 , with $Gr = 5000$. Figure 6 shows the variation of the average Nusselt number for each fin as a function of the solid volume fraction ($0 \leq \phi \leq 0.1$) of the nanofluid Cu-water, for different Richardson numbers ($0.1 \leq Ri \leq 10$) and $Gr = 5000$. Figure 5 clearly indicates that the decrease in the number of Richardson leads to an increase in heat transfer. It also shows that the fin 2 is better cooled than the fin 1 for each fraction (ϕ) and for each value of Ri . For $Ri = 0.1$, an increase in the volume fraction of 10% improves heat transfer by about 44% for both fins together. However, when Ri decreases, and for the same fraction ($\phi = 10\%$), the heat transfer improves by about 19% on average compared to that of pure water. This, shows the importance of the presence of nanoparticles in a pure fluid and their influence on heat transfer.

For each value of the Richardson number, correlations of the average Nusselt numbers of the fins are obtained in table3.

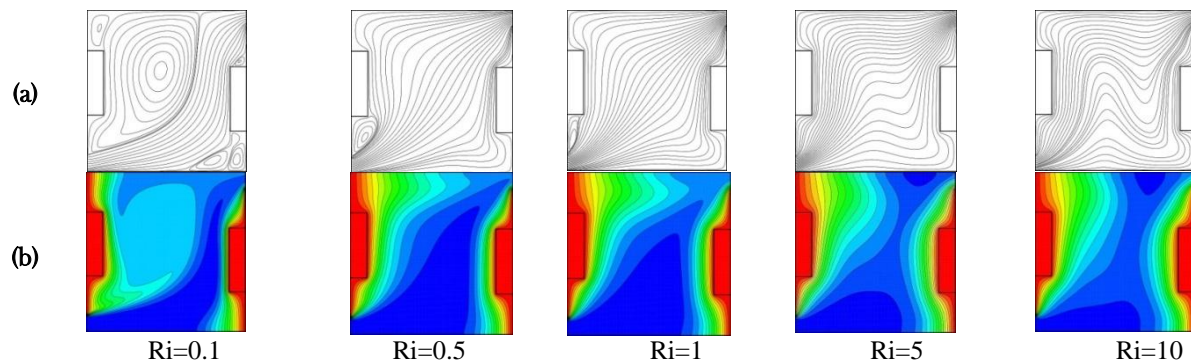


Figure 4. Variations of streamlines (a) and isotherms (b) for water-copper nanofluid for ($\phi = 0.05$), $Gr = 5000$ and different values of the Richardson number

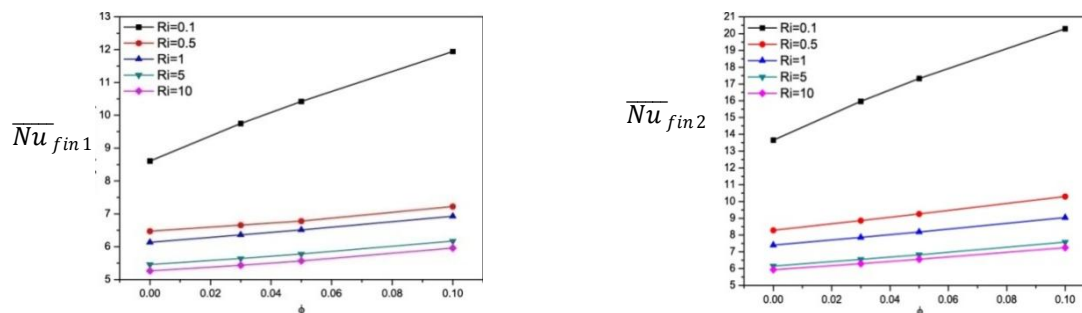


Figure 5. Variation of the mean Nusselt number for each fin as a function of the solid volume fraction of the Cu-water nanofluid, at different values of the Richardson number and $Gr = 5000$.

4.2. Effect of the inclination angle of the cavity

Figures 6 and 7 respectively illustrate the contours of the isotherms and the average number of Nusselt for different values of the inclination angle. The effect of the angle of inclination is only observed for $Ri = 10$, allowing

the buoyancy forces to be manifested when the value of the angle φ is changed. For $Ri = 0.1$, the results showed that, apart from a slight improvement for $\varphi = -90^\circ$ and 90° , the inclination has no effect on the improvement of the heat transfer. On the other hand for $Ri = 10$, the optimal heat transfer corresponds to the $\varphi = 0^\circ$.

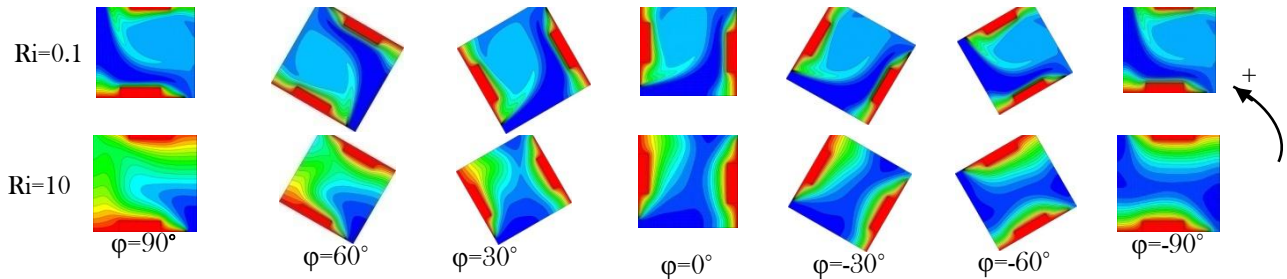


Figure 6. Variations of isotherms at different angles of inclination, for the nanofluid Cu-water ($\phi = 0.05$) and $Gr = 5000$ $Ri = 1$ and 10 .

4.3. Effect of aspect ratio

In this part the effect of the aspect ratio on the dynamic and thermal field, and then on the cooling of the fins, was examined. The study was done for the nanofluid (Cu-water) of volume fraction ($\phi = 0.05$), $Gr = 5000$, for different aspect ratios ($R = 1, 7/10, 5/10$ and $4/10$), and for different values of the Richardson number ($Ri = 0.1, 1$ and 10), for which figure 8 shows the

contours of the isotherms. Figure 9 shows that the influence of cavity restriction on the cooling improvement is greater for high velocity flows where the forced convection is dominant. However for $Ri = 10$, where there is dominance of the natural convection, we notice a slight improvement. The improvement rate is as follows: for $Ri = 0.1$, it is of 56%, for $Ri = 1$ an optimal improvement of 78%, and for $Ri = 10$ an improvement is of 22.4%.

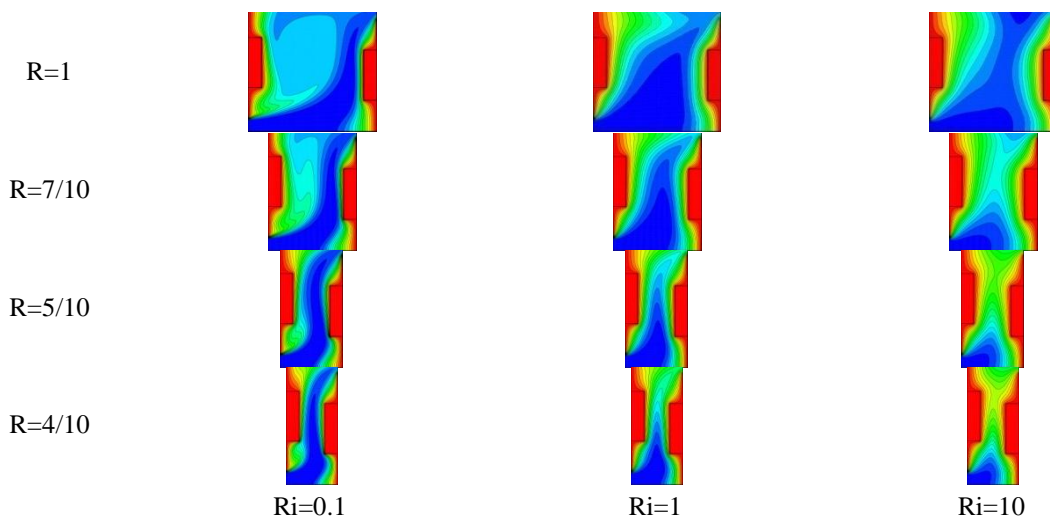


Figure 8. Contours of streamlines (a) and isotherms (b) for the water-copper nanofluid for ($\phi = 0.05$), $Gr = 5000$ and different values of the Richardson number

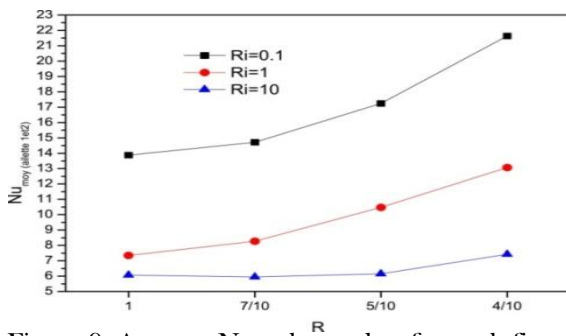


Figure 9. Average Nusselt number for each fin and for both together versus R, for the Cu-water nanofluid ($\phi = 0.05$), $Gr = 5000$

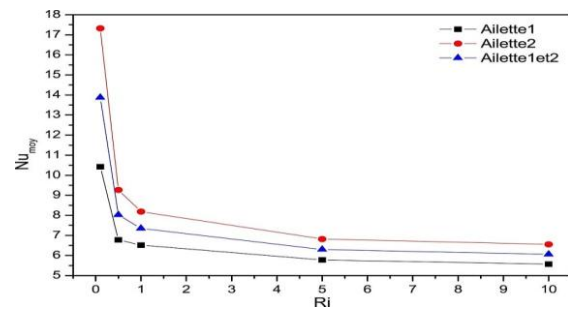


Figure 10. Average Nusselt number for each fin and for both together versus Ri for the Cu-water nanofluid ($\phi = 0.05$), $Gr = 5000$

Table 3: Correlation of \overline{Nu}_{fin} as a function of ϕ ($\phi = 0$ to 0.1)

Ri	\overline{Nu}_{fin1}	\overline{Nu}_{fin2}
0.1	$-64.27\phi^2+40.22\phi+8.5$	$-144.6\phi^2+79.9\phi+13.66$
0.5	$19.43\phi^2+5.72\phi+6.45$	$7.17\phi^2+19.49\phi+8.26$
1	$-6.95\phi^2+8.64\phi+6.1$	$19.87\phi^2+14.16\phi+7.44$
5	$24.71\phi^2+4.42\phi+5.45$	$32.82\phi^2+10.71\phi+6.19$
10	$3.02\phi^2+7.16\phi+5.2$	$27.92\phi^2+10.43\phi+5.94$

5. Conclusion

In this work, we presented a numerical study of mixed convection in a two-dimensional inside a heated square cavity containing conductive fins using copper-water nanofluid. The results obtained indicate that the decrease of the Richardson number and the increase of the fraction leads to the increase of the heat transfer essentially at the level of the fin 2. The best heat transfer is when $\phi = 0^\circ$, whereas the inclination of the cavity has no influence. The heat transfer rate increases with a small aspect ratio but as Ri increases.

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