RIGOROUS FULL-WAVE SPETRAL SOLUTION FOR MODELING MICROSTRIP DISCONTINUITIES

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Abstract: The algorithm we have developed in this work gives the scattering parameters for microwave planar transmission lines structures such as gap and right bend discontinuities through a selection of localized Roof-Top basis functions. The original work concerns mainly, the introduction of triangular sources which allow a better description of the problem within the base of Roof-Top trial functions and a coupling two-port network between the applied source and the considered structure to model the mismatch between the computed and the actual values of Z_{in}. The results we have obtained using the developed algorithm, compare well with those published by other authors. And a large computing time has been saved using of a 2D-FFT which in addition has greatly simplified the analysis.

I- INTRODUCTION

Besides the finite difference and finite element methods, the integral methods are very powerful and flexible numerical tools which are well adapted to the analysis and characterization of MIC and MMIC structures [1-5]. Within this direction we have developped an algorithm based on an integral method to obtain the scattering parameters of gap and right bend discontinuities which are of great importance for MMIC's. The scattering parameters are found through the determination of localized Roof-top trial functions, used to descritize the metal strips. The use of triangular excitation sources allow not only a better description within a Roof-top basis, but also to eliminate the bouncing effect due to the abrupt change of the impulse source and to obtain a faster convergence for the value of the input impedance for different dimensions of the excitation source.

Furthermore, the use of a coupling two port network, composed of a series impedance Z_s, a shunt impedance Z_p and a transformer n, models very well the mismatch between the computed characteristics of the discontinuities and the actual ones. It is shown through an intensive study that the series impedance Z_s remains very small for a wide range of frequencies. Hence, only a shunt impedance Z_p and a transformer n can model accurately the impedance mismatch. A short-circuited microstrip line of a known impedance has been used to validate the experimental data for Z_{in}. On the other hand, the two-dimentional Fast Fourier Transform has been applied to our system matrix, which transforms the moments matrix into a series of simple expressions.

II- DESCRIPTION OF THE METHOD

The tangential field components $\tilde{E}_x$ et $\tilde{E}_y$ in terms of the surface current density components Jx and Jy can be obtained from the solutions of Helmoltz equations as

$$E_x = \sum_m \sum_n \tilde{G}_{11} \int J_x(x',y') \cos K_{xm} x' \sin K_{yn} y' dx' dy' \cos K_{xm} x \sin K_{yn} y + \sum_m \sum_n \tilde{G}_{12} \int J_y(x',y') \sin K_{xm} x' \cos K_{yn} y' dx' dy' \cos K_{xm} x \sin K_{yn} y \qquad (1-a)$$

$$E_y = \sum_m \sum_n \tilde{G}_{21} \int J_x(x',y') \cos K_{xm}x' \sin K_{yn}y' dx'dy' \sin K_{xm}x \cos K_{yn}y + \sum_m \sum_n \tilde{G}_{22} \int J_y(x',y') \sin K_{xm}x' \cos K_{yn}y' dx'dy' \sin K_{xm}x \cos K_{yn}y \qquad (1-b)$$

Where $\tilde{G}_{ij}$ are the Fourier transforms of the dyadic Green's functions [10-12]. The descritization procedure is performed using the roof-tops which are bi-dimentional rectangular functions obtained from a product of triangular and rectangular functions (Refer to figures-1, a, b). The descritization scheme for the conductor strip in the x and y directions is shown in figure1-c, in which the center point (x_k , y_k) of each roof-top is represented by a circle in the x-direction and by a cross sign in the y-direction.

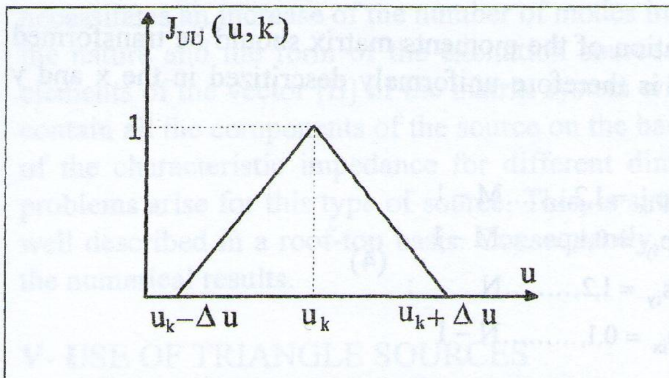

Fig.1.a: Roof-tops: Triangular function.

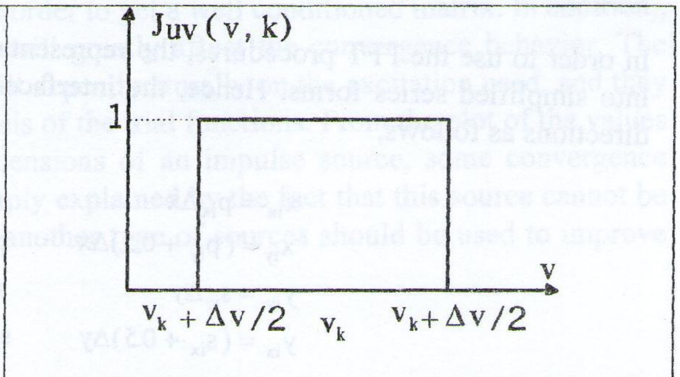

Fig.1.b: Roof-tops: Rectangular function.

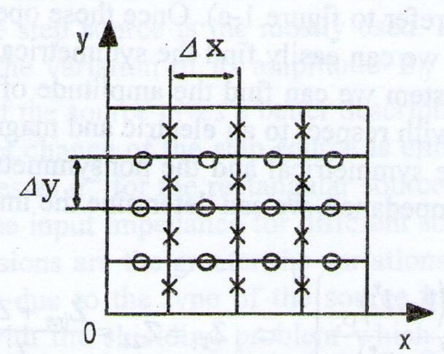

Fig.1.c: Descritization scheme for the conducting strip

The numerical analysis is carried out using the Galerkin's procedure in conjunction with the moments method as it is shown by equations (1). The surface current density on the metallic strip in both x and y directions can be expressed in terms of a series of trial functions NX and NY respectively, as

$$J_{sx}(x,y) = \sum_{k=1}^{NX} a_{xk} J_{xk}(x,y)$$
$$J_{sy}(x,y) = \sum_{i=1}^{NY} a_{yi} J_{yi}(x,y) \qquad (2)$$

which can be written in the spectral domain as follows,

$$\tilde{E}_x = \tilde{G}_{11}(\alpha_n,\beta)\tilde{J}_x + \tilde{G}_{12}(\alpha_n,\beta)\tilde{J}y$$
$$\tilde{E}_y = \tilde{G}_{21}(\alpha_n,\beta)\tilde{J}_x + \tilde{G}_{22}(\alpha_n,\beta)\tilde{J}_y \qquad (3)$$

Using the expression of J_{sx} and J_{sy} into equations (3) and applying the internal product, we can obtain a matrix system of the form [A][x]=[B] , where [x] is a vector containing the coefficients of the current density J. The matrix [A] is constructed from both the testing products and the dyadic Green's functions associated to the boundary conditions and [B] is a column vector where its elements are the scalar products of the sources fields and the super subdivision test function.

III- USE OF THE TWO DIMENSIONAL FAST FOURIER TRANSFORM

To solve the obtained system of equations, the approach we have used is based on two techniques which reduce considerably the computing time. The first technique uses the 2D-FFT for the computation of the index tables, and the second one is based on the application of the indexed equations to calculate all the elements of the moments matrix by a direct combination of the elements of the index tables.

In order to use the FFT procedures, the representation of the moments matrix should be transformed into simplified series forms. Hence, the interface is therefore uniformely descritized in the x and y directions as follows,

$$\begin{aligned} x_{ix} &= p_{ix}\Delta x & p_{ix} &= 1,2,\ldots\ldots M-1 \;; \\ x_{iy} &= (p_{iy}+0.5)\Delta x & p_{iy} &= 0,1,\ldots\ldots M-1 \\ y_{iy} &= s_{iy}\Delta y & s_{iy} &= 1,2,\ldots\ldots N \;; \\ y_{ix} &= (s_{ix}+0.5)\Delta y & s_{ix} &= 0,1,\ldots\ldots N-1 \end{aligned} \qquad (4)$$

Where (x_{ix}, y_{ix}) et (x_{iy}, y_{iy}) are the Roof-top coordinates for the points that are represented by the circle and the cross-sign respectively (refer to figure 1-c). Once these operators have been determined from the Fredholm integral equations, we can easily find the symmetrical moments matrix for which $A_{yx} = A_{xy}$. From the solution of this system we can find the amplitude of the current densities. Then, if we impose some kind of symmetry with respect to an electric and magnetic walls, we will determine two impedances Z_{ine} and Z_{inm} for the symmetrical and the nonsymmetrical cases respectively. Using the general expression of the input impedance, we can determine the impedance matrix [Z] as,

$$Z_{in} = \left\{ \frac{\langle E_0, J^* \rangle_{D_s}}{\langle E_0, E_0^* \rangle_{D_s}} \right\}^{-1} \Rightarrow Z_{11} = Z_{22} = \frac{Z_{ine} + Z_{inm}}{2} \qquad (5)$$

$$Z_{12} = Z_{21} = \frac{Z_{ine} - Z_{inm}}{2}$$

We should note here that the use of the 2D-FFT for our matrix system has allowed a considerable reduction in computing time by a factor of 5 as compared to classical technique.

IV- STUDY OF CONVERGENCE

The convergence and the reduction of computing time to determine the circuit parameters depend essentially on the nature of the trial functions used. The number of these functions that are necessary to obtain a good convergence of the results represents the dimension of our matrix system, and the number of terms for the summation (m,n) ensures a good decomposition of the trial functions in the basis of the cavity modes. The convergence curve for the the characteristic impedance versus the number of modes is shown in figure-2. It is interesting to note that a very good convergence is reached (saturation part of the curve) for 80 roof-top trial functions and 10000 modes

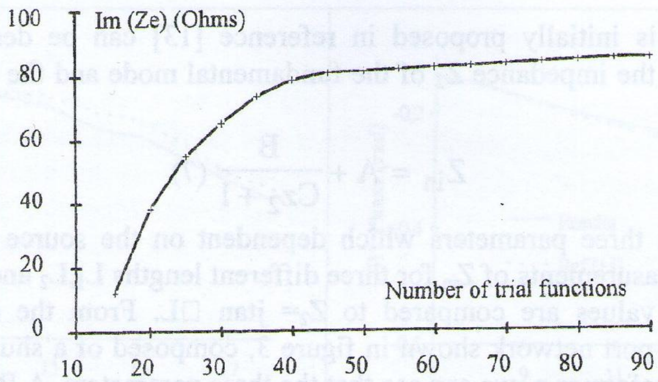

Fig.2. Convergence of the input impedance $Im(Z_{in})$ versus the number of trial basis functions and the modes TE and TM

We have observed during our computations that increasing the number of the trial functions, necessitates an increase of the number of modes in order to get a well conditioned matrix. In addition,, the nature and the form of the excitation source will greatly affect the convergence behavior. The elements of the vector [B] of the matrix system will depend naturally on the excitation used, and they contain all the components of the source on the basis of the trial functions. From the plot of the values of the characteristic impedance for different dimensions of an impulse source, some convergence problems arise for this type of source. This, is simply explained by the fact that this source cannot be well described in a roof-top basis. Consequently, another type of sources should be used to improve the numerical results.

V- USE OF TRIANGLE SOURCES

A number of excitation sources have been considered in the litterature for the analysis of MMIC structures [13] among which the step source is the mostly used. In our case we have used a source with a triangular shape, where the variation of its amplitude E_0 decreases linearly and it becomes zero at x = d. In fact, this form of the source gives a better description in the roof-top basis, where the bouncing effect due to the abrupt change of the step source is eliminated. It has been found that the solution of the problem converges as n^{-2} for the rectangular source, while it diverges as n^{-1} for a step source. From the variations of the input impedance for different source dimensions, we can conclude that the larger the source dimensions are the greater the variations in the impedance will be. Hence, we should know the net effects due to the type of the source in order to model properly a given structure. We have also dealt with the shielding problem which contributes to the generation of a propagating higher order mode. The only solution to eliminate this effect is to reduce the structure dimensions in order to work at higher frequencies. In connection to this idea, we have used a transmission line of the order of the wavelength and a ratio w/b of the order of a tenth of the wavelengh to reduce considerably both the system matrix dimensions and the number of the terms tobe summed up.

VI- DESIGN OF THE COUPLING TWO-PORT NETWORK

To validate the expression of the input impedance seen from the source , we have studied a circuited microstrip line by considering this structure as a transmission line operating at the fundamental mode., It has been possible to compare the numerical values given by the relationship

$$Z_{in} = jZ_C \tan\beta L \quad (6)$$

where Zc is the characteristic impedance of the line and □ the constant of propagating of the fundamental mode.

A simple numerical computation can show that equations (5) and (6) are not comparable. It is the raison why it is necessary to introduce a coupling two-port network between the source and the structure to correct the numerical results.

This approach ,which is initially proposed in reference [13] can be described by a homograph relationship connecting the impedance Z_2 of the fundamental mode and the impedance seen from the source, as

$$Z_{in} = A + \frac{B}{Cz_2 + 1} \quad (7)$$

where A, B and C are three parameters which dependent on the source properties. They can be determined from the measurements of Z_{in} for three different lengths L_1,L_2 and L_3 of the short circuited microstrip line. These values are compared to Z_2= jtan □L. From the equivalent circuit of the proposed matching two-port network shown in figure 3, composed of a shunt impedance Z_p ,a series impedance Zs and a transformer n, we can see that the three parameters A,B and C will be a function of Z_s, Z_p and n. Once this two-port has been specified, the computation of the propagation constant is deduced from the solution of the system [A][X]=[0],where[A] is the moments matrix.

The solution of the system goes through the test of the resonance condition (det[A]=0) to avoid the trivial solution. Since the two main parameters of interest are the frequency and the length of the resonator c, the idea is to keep one of these two parameters constant while changing the other one until the determinant becomes zero. Once a solution for the system has been determined for a given frequency, the length c will be taken as a multiple of the guided mode wavelength;

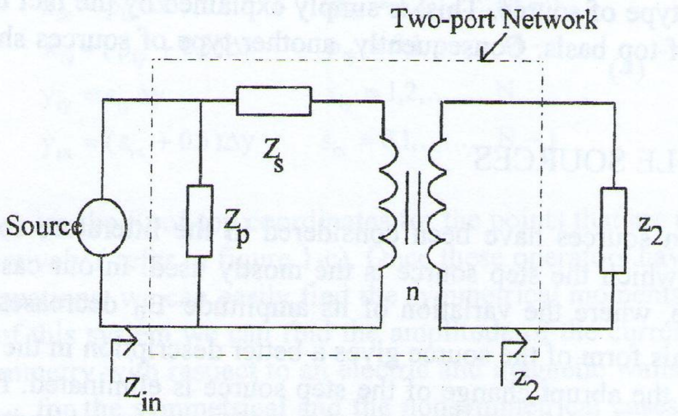

Fig. 3. Equivalent circuit for the coupling two-port network

VII- NUMERICAL RESULTS

A mathematical algorithm has been implemented on a PC for the computation of the parameters of the two port network for a short-circuited line of length L and a=6.985mm, b=6.35mm, w/d=1, □r=9.7 (refer to figures 4a, b). The results we have obtained for a frequency of 20 Ghz using the developed algorithm are : Z_p = 152.459 j (□), Z_s = - 0.0039 j (□) and n^2 = 58.407. We have noted through an intensive study that the series impedance Z_s remains very small for a very wide range of working frequencies. Hence, only the shunt impedance Zp should be considered to accurately model the source/circuit discontinuity. The scattering parameters can be easily deduced from the coupling two port. The comparison of our results with those published in [8] and [11] for gap and right bend discontinuities are shown in figures 5a and 5b respectively. As it is shown in these two sets of figures, our results compare well with the published ones in case of 80 trial functions and 10000 modes.

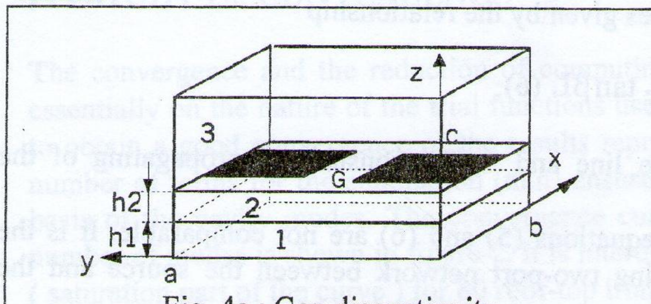

Fig. 4a : Gap discontinuity

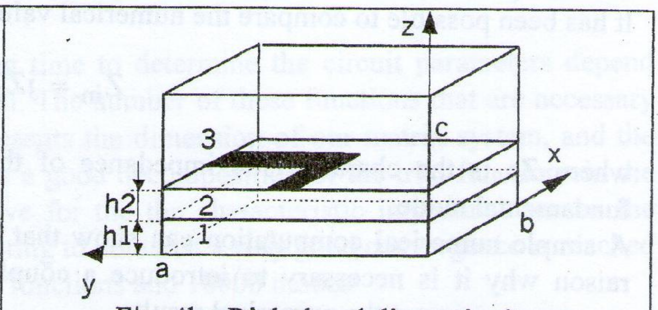

Fig. 4b : Right bend discontinuity

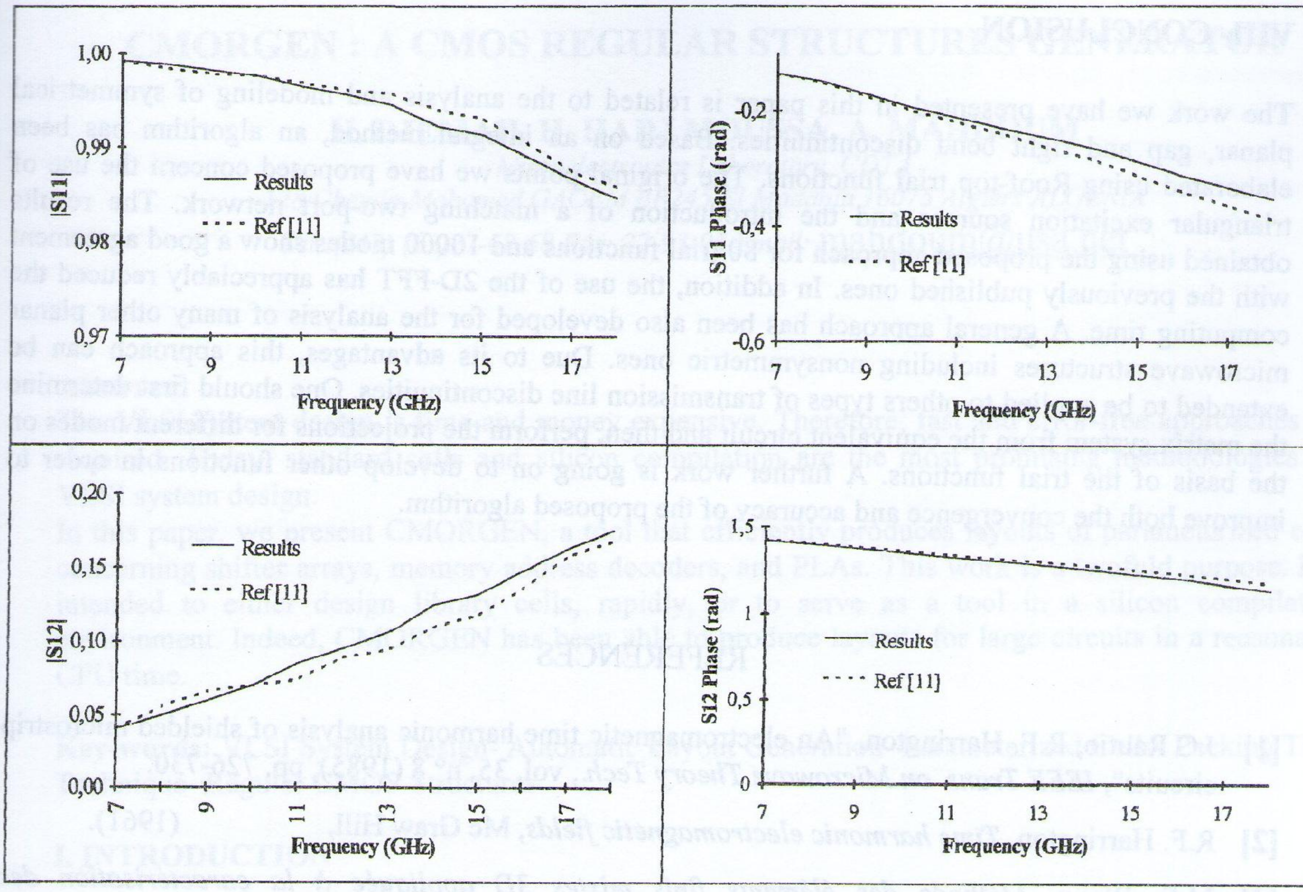

Fig. 5. S parameters for Gap discontinuities (Dimensions (mm) :
a = c = 6.35 ;h = 0.635 ;w/h = 1; g =0.381)

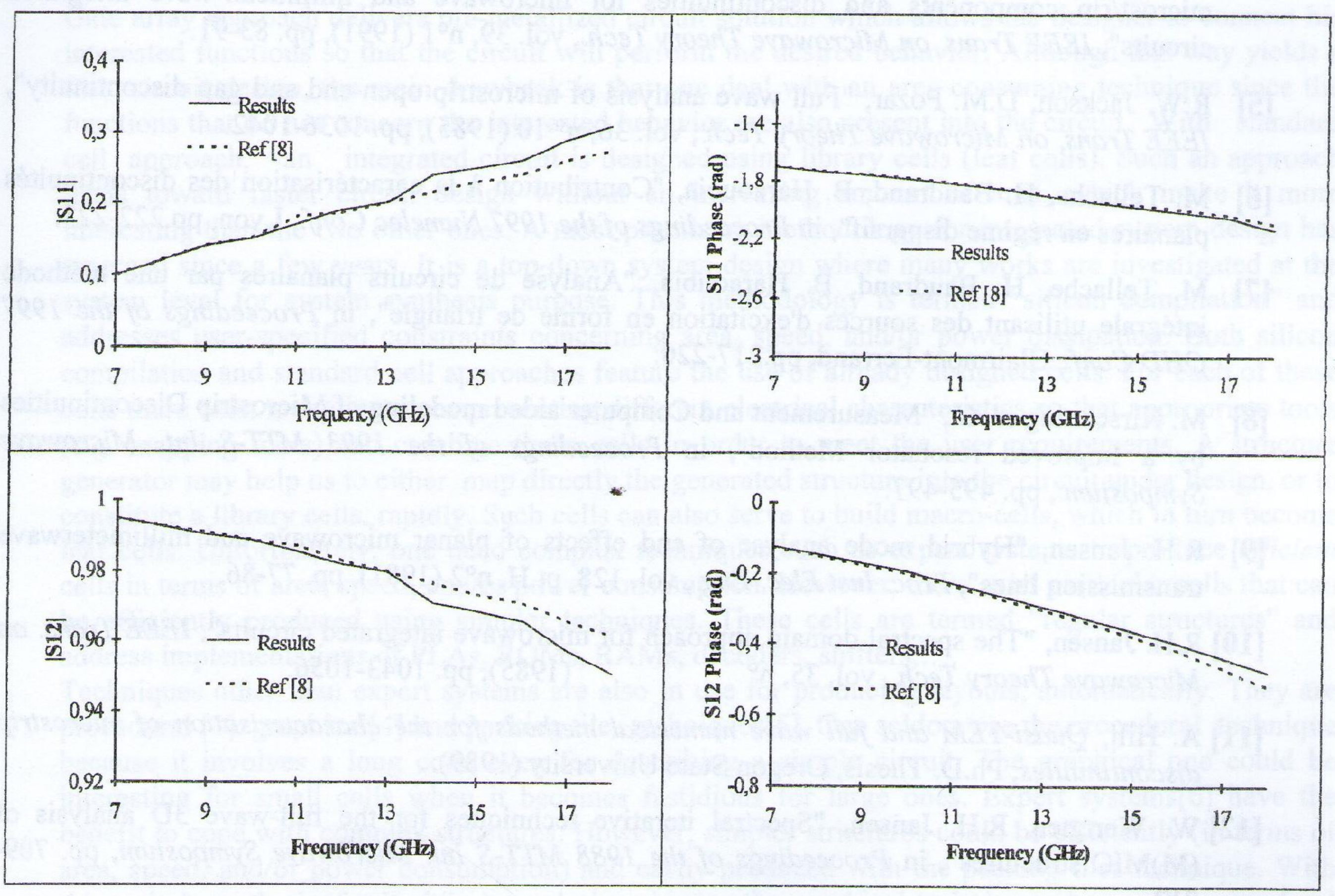

Fig.6. S parameters for right bend discontinuities (Dimensions in mm) :
a = b = 12.7 ; c = 3.18 ;h = 0.635 ; w/h = 1)

VIII- CONCLUSION

The work we have presented in this paper is related to the analysis and modeling of symmetrical planar, gap and right bend discontinuities. Based on an integral method, an algorithm has been elaborated using Roof-top trial functions. The original points we have proposed concern the use of triangular excitation sources and the introduction of a matching two-port network. The results obtained using the proposed approach for 80 trial functions and 10000 modes show a good agreement with the previously published ones. In addition, the use of the 2D-FFT has appreciably reduced the computing time. A general approach has been also developed for the analysis of many other planar microwave structures including nonsymmetric ones. Due to its advantages, this approach can be extended to be applied to others types of transmission line discontinuities. One should first determine the matrix system from the equivalent circuit and then, perform the projections for different modes on the basis of the trial functions. A further work is going on to develop other functions in order to improve both the convergence and accuracy of the proposed algorithm.

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