Market risk estimation using non-parametric Value at Risk and Conditional Value at Risk: An empirical study of the Algerian stock market

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Abstract :

The aim of this study is to estimate the Algerian market risk, where the non-parametric VaR and CVaR are used to examine the portfolio of four Algerian companies during the period 28-04-2019 to 26-04-2020 using daily returns with equal weights, we also calculated the VaR and CVaR for the same portfolio but with optimal weights, and we found that these methods are useful in estimating the risk of our portfolio and they also can be affected by the portfolio optimization.

Mots clés:
risque de marché
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Conditional Value at Risk
Codes de Classification JEL: G10, G11, G19.

Le résumé :

L'objectif de cette étude est d'estimer le risque du marché algérien, où nous avons calculé la VaR et la CVaR non paramétriques pour un portefeuille de quatre sociétés algériennes au cours de la période 28-04-2019 au 26-04-2020 en utilisant des rendements quotidiens à poids égal,nous avons également calculé la VaR et la CVaR pour le même portefeuille mais avec des pondérations optimales, et nous avons constaté que ces méthodes sont utiles pour estimer le risque de notre portefeuille et qu'elles peuvent également être affectées par l'optimisation du portefeuille.

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1- The introduction

The financial risk management has become one of the most important field in finance's topics. it gives the institution the opportunity whether accepting these financial risks or exposing it-self to a bigger risk to achieve a competitive advantage.

When creating a financial portfolio, the manager has to manage a lot of risks, one of these is market risk, it refers to the changing in the value of financial instrument that is contained in our portfolio due to unpredictable fluctuation in the price, however when the core activity of a business is to hold portfolio of assets, it would be dangerous to ignore their potential change in value. (Koenig, 2004)

The Value-at-Risk model is known as the most used method to evaluate the market risk, it can gives one number that reflect your risks, this number is the maximum loss that can a portfolio face at a certain level of confidence.

After VaR failed to be a coherent risk measure in one hand, and to give a number about the risk exceeding its level of confidence in the other hand, Conditional-Value-at-Risk (CVaR) that introduced by Rockafellar and Uryasev (2000) can solve these problems, it equals to the average losses exceeding the value at risk at a certain level of confidence.

The present study focuses on measuring the Value at Risk and the Conditional Value at Risk of a portfolio containing various financial assets listed on the Algerian stock market, we took a sample of four assets using their daily prices from the period 28/04/2019 to 26/04/2020.

Beside our aim to estimate the market risk of the Algerian stock market using VaR and CVaR, we also build an optimization model to see the effect of optimizing portfolio weights on the estimation of risks.

1-2 Research question:

The application of the Value at Risk and the Conditional Value at is widely use among academics and professionals. However, its application on the Algerian stock market is very humble. Which makes us questioning as follow:

-Can Value at Risk and Conditional Value at Risk estimate Algerian market risk?

We can also address one sub question as follow:

- Can VaR and CVaR be affected by the optimization of the portfolio?

1-3 Research hypotheses:

A- Although the Algerian stock market is not active, but based on the historical data extracted we can estimate the VaR and CVaR of the Algerian portfolio.
B- Since the optimization of the portfolio reduces its risks, this means that it will affect the VaR and the CVaR.

The purpose of this study is trying to give an overlook of the risk in the Algerian stock market, this is due to the lack of studies dealing with this topic. On the other hand, showing the importance of the Value at Risk and the Conditional Value at Risk in measuring market risk.

2-Theoretical background

Markowitz (1959) explored in his work of portfolio selection the appropriate definition and measurement of risk. From then, the researchers in the fields of financial economics have long recognized the importance of measuring risk of a portfolio of financial assets or securities. (Hendricks, 1996)

One of the tools measurements of the portfolio risk is the Value at Risk, it measures the market risks of the financial portfolio by giving one quantitative measure of the changes in interest rates, exchange rates, or changes in equity prices over a specific time horizon.

a. Value at Risk

Darryll Hendricks (1996) has defines VaR as a tool of measuring market risks by determining how much the value of the portfolio could decline over a given period of time with a given probability as a result of changes in market prices or rates, he also explained that there are two most important components of Value-at-Risk, these components are the length of time over which market risk is to be measured and the confidence level at which market risk is measured. The choice of these components by risk managers greatly affects the nature of the Value-at-Risk model.

VaR measures the worst expected loss over a given time interval under normal market conditions at a given confidence interval. (Dowd & Kevin, 1999)

VaR provides users with a summary measures of market risk. VaR is method of assessing risk that uses standard statistical technique routinely used in other technical fields. (Jorion P. , 2007)

VaR measure is the highest possible loss of L that is incurred by maintaining the current portfolio over a given period of time at a certain confidence level: (Jorion P. , 2006)

$$Prob(L > VaR) \leq 1 - c$$

Where: c is the confidence level, typically 90%, 95%, 99%,

$L = -\Delta X(\tau)$, where $\Delta X(\tau)$ is the relative change in returns over the time horizon τ.

For example, a bank might say that the daily Value at Risk of its portfolio is $10 million at 95% confidence level, this means that there is only 5% probability for loss greater than $10 million.

Value at Risk has divided into three categories: Parametric, Semi-parametric, Non-parametric. (R. F. Engle , S. Manganelli, 2004)

The parametric approaches are the parametrization of the behavior of prices, the semi-parametric VaR category is a method based on Extreme value theory. (see James W Taylor, 2008)

The Non-parametric approach requires no distributional assumption and it estimates the VaR as the quantile of the empirical distribution of historical returns. For this approach, CVaR can be estimated as the mean of the returns that exceeds the VaR estimation. (James W Taylor). One of the most used method in non-parametric approach is the historical simulation (HS), it is a very simple method in simulating Value at Risk, Michael B Miller (2019) mentioned in his book (Quantitative financial risk management) as follow: “In this approach we calculate VaR directly from past returns. For example, suppose we want to calculate the one-day 95% VaR for an equity using 100 days of data. The 95 th percentile would correspond to the least worst of the worst 5% of returns. In this case, because we are using 100days of data, the VaR simply corresponds to the fifth worst day.” He completes saying: “…The historical approach is non-parametric. We have not made any assumptions about the distribution of historical returns. There are advantages and disadvantages to both approaches. The historical approach easily reproduces all the quirks that we see in historical data: changing standard deviation, skewness, kurtosis, jumps, etc.

Developing a parametric model that reproduces all of the observed features of financial markets can be very difficult. At the same time, models based on distributions often make it easier to draw general conclusions. In the case of the historical approach, it is difficult to say if the data used for the model are unusual because the model does not define usual.”

b. Conditional Value-at-Risk

Although VaR has become the standard measure of market risk, it has been criticized for reporting only a quantile, and cannot report outcomes beyond the quantile. On the other hands, VaR sub additive risk measure, this means that the total risk on a portfolio should not be greater than the sum of the risks of the part of the portfolio. (James W Taylor, 2008), VaR does not tell us anything about the tail distribution, two portfolios could have the exact same 95 % VaR but very different distributions beyond the 95% confidence level. (Miller M. B., 2019)

The conditional value at risk is a risk measure that overcomes these weaknesses, it defined as the conditional expectation of the return given that exceeds the VaR.

Using the concept of conditional probability, we can define the expected value of a loss, given an exceedance, as: (Michael B Miller, 2019)

$$E[L|L \geq VaR_y] = S$$

Where: **S** is the CVaR or Expected Shortfall

If the expected profit of a fund can be described by a probability density function given byf (x), and VaR is the VaR at the γ confidence level, we can find the CVaR as:

$$S = -\frac{1}{1-y}\int_{-\infty}^{VaR} xf(x)dx$$

3- Calculating Value at Risk and the Conditional Value at Risk on the Algerian Stock Exchange:

In this section of our study, we calculate VaR and CVaR of the portfolio which contains 4 Algerian companies using the non-parametric method, this study is a comparison between calculating VaR and CVaR for equal-weighted portfolio and for optimal one.

This research used stock sample with daily data retrieval over one year from 28 April 2019 to 26 April 2020:

- Stock Alliance Assurance (ALL);
- Stock Rouiba (RUI);
- Stock Biopharm (BIO);
- Stock Saidal (SAI).

The actual start of the process of establishing the Algiers Stock Exchange dates back to the year 1993, after the issuance of Legislative Decree No.08-93 dated 25 April 1993, Amending and complementing the commercial law. (M.Kennouche, 2020)

a. Equal-weighted portfolio:

1 - Before calculating the VaR and CVaR, we calculate the daily returns of the portfolio with equal weights (0.25, 0.25, 0.25, 0.25) because using these historical returns non-parametrically, VaR can be calculated using a percentile with the desired level of confidence.

The table (01) summarize our daily returns by transforming them into monthly returns using R

Table (01) : Monthly returns of equal-weighted portfolio

Date	Monthly returns
4/30/2019	0
5/30/2019	0.020141611
6/30/2019	0.016716429
7/30/2019	-0.024606798
8/29/2019	-0.013860958
9/29/2019	-0.009948608
10/31/2019	-0.00217291
11/28/2019	0.002326969
12/31/2019	0.000587168
1/30/2020	0.00043607
2/27/2020	-0.039060399
3/29/2020	0.000285192
4/26/2020	0

Source: data from Algerian stocks exchange.

Figure (01): monthly returns

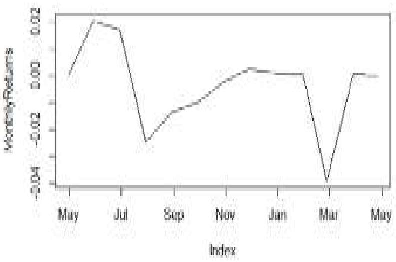
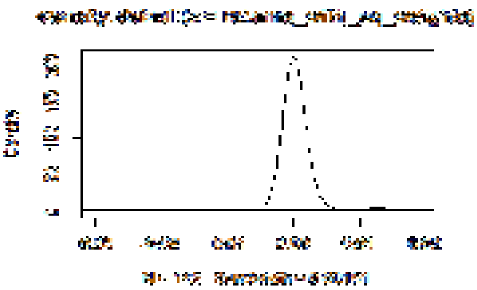


Figure (02): return distribution



Source: Author's estimation using R program
(Skewness = -2.816001).

By looking at the Figure (01), we notice that the value of the returns is negative for most of the time, it means that this portfolio is facing losses almost over all the period of the study, and by using this information we can conclude that the distribution of the return is left-skewed, the graph besides Figure (02) demonstrates the shape of the returns distribution:

2- calculating the return and the volatility of the portfolio using R program:

If we have a portfolio with n assets, the i_{th} of which delivers a return R_i at time t, this return has a mean $\mu_{t,i}$ and standard deviation $\sigma^2_{t,i}$ as follows:

library(PerformanceAnalytics)
a- Create a vector of returns
means <- apply(rp_Algeria, 2, "mean")

b- Create a vector of standard deviation
sds <- apply(rp_Algeria, 2, "sd")

c- Create a matrix with variances on the diagonal
diag_cov <- diag(sds^2)

d- Create a covariance matrix of returns
cov_matrix <- cov(rp_Algeria)

e- Create a correlation matrix of returns
cor_matrix <- cor(rp_Algeria)

f- Create a weight matrix w
w <- as.matrix(weights)

g- Create a matrix of returns
mu <- as.matrix(means)

h- Calculate portfolio mean of daily returns
t(w) %*% mu

i- Calculate portfolio volatility

sqrt(t(w) %*% cov_matrix %*% w)

Where: rp_Algeria is Dataframe of the five Algerian assets.

The tables below show the covariance matrix of our portfolio and the mean return for each asset:

Table (02): Covariance Matrix of the portfolio

	ALL	BIO	ROUI	SAI
ALL	8.31E-05	1.43E-07	-1.43E-06	6.75E-06
BIO	1.43E-07	8.65E-05	2.26E-07	3.45E-07
ROUI	-1.43E-06	2.26E-07	4.82E-05	-1.17E-06
SAI	6.75E-06	3.45E-07	-1.17E-06	0.000137

Source: Author's estimation using R program

Table (03): Mean return of each asset

	ALL	BIO	ROUI	SAI
Returns	-3.82E-05	-3.47E-04	-6.12E-04	-9.86E-04

Source: Author's estimation using R program

And now that we have the covariance matrix and the return of each asset matrix with equal weights, we can compute the return and the volatility of our portfolio, the table below shows the results extracted from R program:

Table (04): the return and volatility of the equal-weighted portfolio

Portfolio return	-0.000330585
Portfolio volatility	0.00477648

Source: Author's estimation using R program

And the table below showing the percentage risk contribution of each stock in the portfolio:

Table (05): risk contribution

	WEIGHTS	% OF CONT
ALL	0.25	0.2425
BIO	0.25	0.239
ROUI	0.25	0.1256
SAI	0.25	0.3928

Source: Author's estimation using R program

We notice from the table (05) that the risk contribution of Saidal stock is bigger than its weight, on the contrary, the Rouiba stock's risk contribution is smaller than its weight, it will be a wise investing decision to reduce the investment weight in Saidal stock and compensate it by raising the investment weight in Rouiba stock so that we can reduce the risk of the portfolio as a whole.

3- Calculating VaR and CVaR of the equal-weighted portfolio:

- VaR:

In the case of our portfolio, we see that almost all the returns are taking negative values which reflecting losses, but as an investor we need one single value that can determine these losses, so to solve this problem we use the Value at Risk which can gives us one single number reflecting the maximum loss can any portfolio face with a desired interval of confidence in a given temporal horizon.
The Value at Risk is one of the most important methods for evaluation the exposure to the market risks that arises from the changes in prices of equities, commodities, exchange rates and interest rates, it allows the non-expert investors to discover how risky a portfolio is and the appropriate strategy could take. (Almira Biglova et al , 2004)
Non-parametric method is used to calculate VaR regardless of the shape of distribution it has. the historical simulation (HS) is one of the most used method in non-parametric approach, it is a very simple method in simulating Value at Risk.
using R software, we calculated the daily value at risk of the Algerian portfolio at 95% confidence level as follow:

```
library(PerformanceAnalytics)
var95 <- VaR(Algerian_Portfolio , p=0.95, method = "historical")
```

we found that the 95 % VaR of the portfolio is <u>-0.01003255</u>, that means we are 95% confidence that our portfolio losses will not exceed -0.01003255 in a day.
We can notice that the VaR can summarize the losses of the portfolio in one number under a certain confidence of interval, and even if there is no doubt that VaR provides useful information, it is not a coherent risk measure, and it cannot offer an exhaustive representation of an investor's preferences. To overcome these limitations and problems, Artzner et al. [1999] propose the conditional value at risk (CVaR) as an alternative risk measure. The CVaR,

also called expected shortfall or expected tail loss, measures the expected value of portfolio returns, given that the VaR has been exceeded. (Almira Biglova et al , 2004)

- **CvaR:**

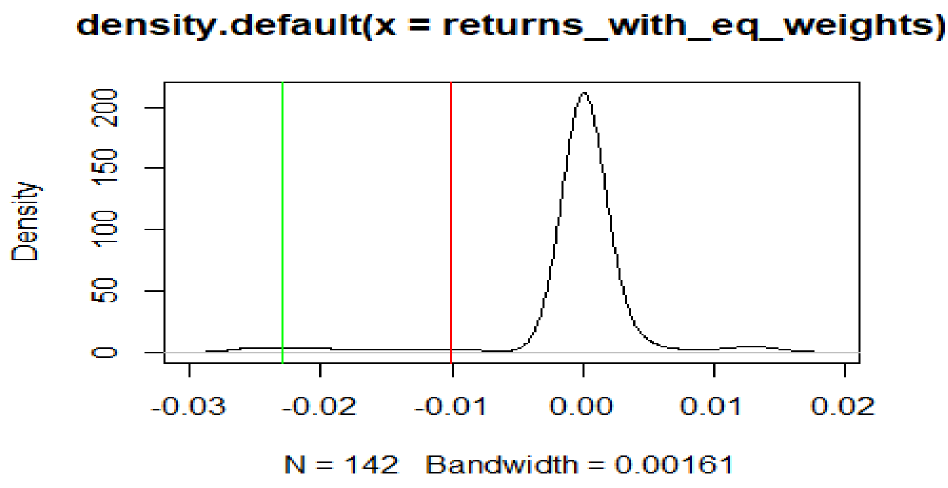
The easiest estimates of coherent risk measures is the Conditional Value at Risk, it is the probability weighted average of tail losses suggests that we can estimate CVaR as an average of "Tail's VaR".

Using R program, we calculated the daily CVaR at 95% confidence level as follow:

```
library(PerformanceAnalytics)
CvaR95 <- CVaR(Algerian_portfolio, p=0.95, method = "historical")
```

We found that 95% CVaR equals -0.02292115, that means we are 95% confidence that the average losses of our portfolio exceeding VaR is -0.0229215 in a day.

Figure (03): Value at Risk and CVaR, (Red = VaR , Yellow = CVaR)



Source: by the researchers using R program

b. Optimized portfolio:

After using equal weights in building our portfolio, we will build the same portfolio with optimized weights based on mean-variance Markowitz theory, The MV portfolio is proposed under the two constraints:

1) full-investment constraint, where, all the investible fund is used for the asset-investment. The summation of allocation weights should be equal to 1.

$$W_{n=} \sum_{i}^{n} = 1 \ , Wi = 1$$

2) The long-only constraint, where, all the allocation weights are positive, i.e. no short-selling is admissible.

$$Wi \geq 0$$

Achieving an optimal portfolio corresponds to quadratic modelling which is considered a type of nonlinear mathematical modelling so that it is formulated in the form of the goal function, so that it represents in the goal to be achieved by decision makers in the form of a second-degree (quadratic) mathematical picture, its goal of minimizing or maximizing in terms of significance variants under a set of restrictions. (Paul & Jonathan, 2011), So that the general form of quadratic programming is as follows (Markowitz, 2007):

$$Max\left(\delta_p^2\right) = \sum_{i=1}^{n} \frac{r_{it}}{d_t} * w_i$$

$$Min\left(\delta_p^2\right) = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j cov_{ij}$$

$$\sum_{i=1}^{n} w_i = 1$$

To solve this model, we used R program to minimize the risk as follow:

library(PortfolioAnalytics)
a- Create the portfolio specification
port_spec <- portfolio.spec(colnames(rp_Algeria))

b- Add a full investment constraint such that the weights sum to 1
port_spec <- add.constraint(portfolio = port_spec, type = "full_investment")

c- Add a long only constraint such that the weight of an asset is between 0 and 1
port_spec <- add.constraint(portfolio = port_spec, type = "long_only")

d- Add an objective to minimize portfolio standard deviation port_spec <- add.objective(portfolio = port_spec, type = "risk", name = "StdDev")

e- Solve the optimization problem
opt <- optimize.portfolio(rp_Algeria, portfolio = port_spec, optimize_method= "ROI")

We get the following results:

Table (06): optimal weights

	ALL	BIO	ROUI	SAIDAL
Weights	0.230077	0.2224	0.413667	0.133856

Source: Author's estimation using R program

The return and the volatility of the portfolio:

Table (07): the return and volatility of the optimized portfolio

Portfolio return	-0.0003305851
Portfolio volatility	0.004414097

Source: Author's estimation using R program

We notice that the risk (volatility) of the portfolio has been reduced after using optimized-weight, and by looking at the table below we can see a balanced risk contribution of each stock on the portfolio, whereas each risk contribution of each stock is almost equal to its optimized weight which have reduced the risk of the portfolio as a whole:

Table (08): risk contribution after using optimal weights

	WEIGHTS	% OF CONT
ALL	0.230077	0.2297365
BIO	0.2224	0.2216379
ROUI	0.413667	0.4143256
SAI	0.133856	0.1343

Source: Author's estimation using R program

Calculating VaR and CVaR of the portfolio:

The table below shows the 95% VaR and CVaR after using mean-variance optimization:

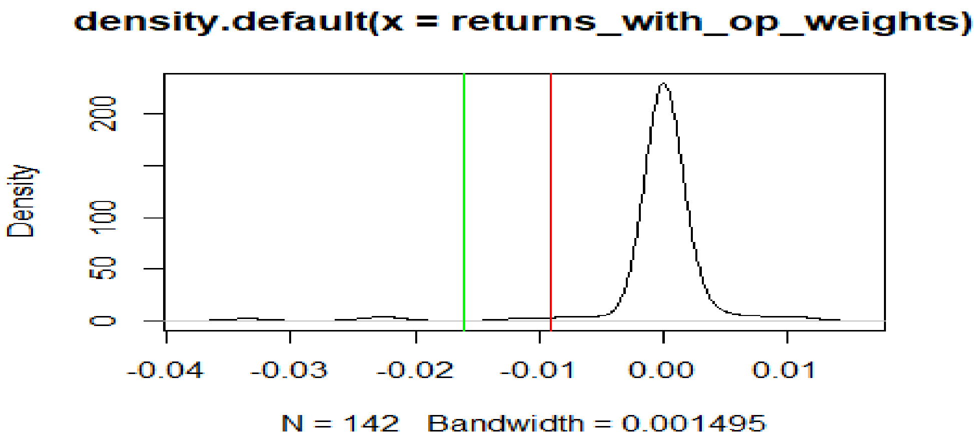
Table (09): VaR and CVaR of the optimal portfolio

VaR 95%	-0.00914
CVaR 95%	-0.01599

Source: Author's estimation using R program

It noted from the table (09) that the maximum loss of our optimized portfolio at the 95% level of confidence is -0.00914 and the average losses beyond the VaR is -0.01599 at the same level of confidence, and the two values are smaller than the VaR and the CVaR of the equal-weighted portfolio, this means that the losses have reduced after using optimized weights.

Figure (04): Value at Risk and CVaR of the optimal portfolio, (Red = VaR , Yellow = CvaR)



Source: by the researchers using R program

3- Conclusion:

This study has estimated the risk of the Algerian portfolio during the period from 28-04-2019 to 26-04-2020 using Value at Risk and conditional Value at Risk. We can notice from calculating the mean returns of each stock or the return of the portfolio (using equal-weights or optimal weights) that the results are taking negative values which mean losses, and this can be logical because of the events during the period of study, when Algeria has faced a political trouble last year and the coronavirus in this year which reflect negatively on the performance of the economy in general and on the companies that were chosen in this study in particular.

Value at Risk and Conditional Value at Risk are very useful methods for measuring and managing portfolio risks, these methods can help the risk manager by giving him one number that express the losses of a portfolio at a certain level of confidence, and based on the results of our study we notice that these methods were useful in estimating the risk of our portfolio, where if we choose to invest equally in the four companies, the losses cannot exceed -0.01003255 in a day with 95% confidence level, as for the uncertainty associated with the remaining 5%, the average losses that we can be exposed to is -0.02292115 in a day, and if we choose to invest 23% in Alliance Assurance, 41.36% in Rouiba, 22.24% in Biopharm and 13.38% in Saidal, the losses cannot exceed -0.00914 in a day with 95% confidence level, as for the uncertainty associated with the remaining 5%, the average loss that we can be exposed to is -0.01599 in a day, which we can conclude that the optimization of the portfolio affects the Value at Risk and Conditional Value at Risk, where we notice that both amount of VaR and CVaR are reduced compared to the amount of the two methods when using equal weights.

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