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## Geometry based Optimization for inductively coupled spiral coils

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### Abstract

Efficiency of inductive linking is an essential part in wireless power transfer WPT. In this paper, we propose an optimization to increase the efficiency of the system and reduce the size of coupled spiral coils implementable microelectronic devices based on geometrical characteristics. Moreover, an elimination of the splitting frequencies is achieved. The proposed approach provides much better results comparing error rates between the optimal mutual inductance and the mutual inductance obtained using calculated geometrical parameters of coils.

**Key-words :** mutual inductance; wireless power transfer; transfer coefficient; Transfer efficiency.

### Résumé

L'efficacité de couplage électromagnétique est plus importante dans les systèmes de transfert d'énergie sans fil. Dans ce travail, on propose une optimisation pour augmenter la performance du système et réduire la taille des bobines des dispositifs microélectroniques implantables basés sur les aspects géométriques. De plus, on arrive à éliminer l'effet de la fréquence fractionnée. La méthode proposée donne de bons résultats obtenus en comparant les taux d'erreurs entre l'inductance mutuelle optimale et l'inductance mutuelle calculée à partir des paramètres géométriques des bobines.

**Key Words :** Mutuel inductance ; Wireless power conversion; Transmitting coefficient; Transfer efficiency.

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## 1. Introduction

In the last decades, Wireless power transfer is used in many fields including implementable microelectronic devices that are very popular and which continue drawing the attention of researchers. Wireless power transfer is considered to be a robust and safety method to transfer energy.

In such applications, the size of the transmitter and receiver is of major importance as minimizing the dimensions is needed to increase the efficiency. The mutual inductance is the key of this minimization process, whereas, all geometric parameters are related to it. Several formulas have been used for mutual inductance calculation and optimization [1] as a way to increase efficiency [2], and mutual coupling between planar inductors using Neumann's formula is one of these methods [3].

Optimization is achieved using specific algorithms such as, the genetic algorithm (GA) that has been widely used for such purposes [4-6], However, using genetic algorithms involves some critical points as well as it needs the search of optimal parameters in prior, which may lead to results which are far away from those of the optimal point.

In this paper, we propose a method of tracking the optimal point of the mutual inductance for a required distance based on the geometry of the primary coil in order to improve the transfer efficiency and eliminate the frequency splitting frequency as well as to reduce the size using an equivalent circuit that is analyzed by two port network.

## 2. Theoretical analysis

The electrical modelling of the wireless power transfer can be described as shown in Fig.1 where the transmitting coil and the receiving coil are considered as series loops consisting of capacitance ( $C_t, C_r$ ), self-inductance ( $L_t, L_r$ ) and copper wire resistance ( $R_t, R_r$ ). The characteristic impedance of port-1 is  $R_0$ , the load impedance is  $R_L$ , whereas  $M$  is the mutual inductance.

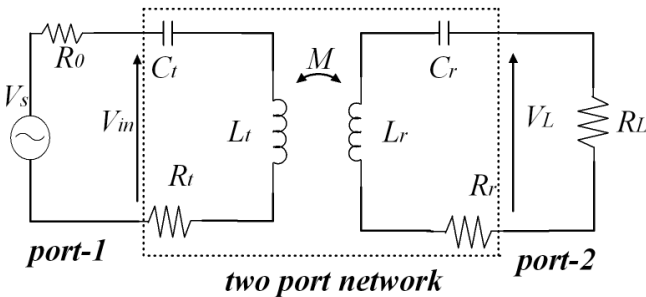


Fig. 1. The equivalent circuit model of two loop WPT system

The relationship between the current flowing through each coil and the voltage across each port is expressed by the impedance matrix  $Z$  given in Eq.1.

$$\begin{pmatrix} V_{in} \\ V_L \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I_t \\ I_r \end{pmatrix} \quad (1)$$

Where  $I_t$  and  $I_r$  are the currents passing through the primary coil and secondary coil respectively, the parameters  $Z$  can be defined by the following equations.

$$\begin{aligned} Z_{11} &= R_t + j(\omega L_t - \frac{1}{\omega C_t}) \\ Z_{22} &= R_r + j(\omega L_r - \frac{1}{\omega C_r}) \\ Z_{12} &= Z_{21} = j\omega M \end{aligned} \quad (2)$$

Where  $V_{in}$  is the voltage across the input port of the transmitting coil, and  $V_L$  is the voltage across the output port of the receiving coil, i.e. the voltage across  $R_L$ . The relationship between  $V_{in}$ , and the source voltage  $V_s$  (see Fig. 1) can be given by Eq.3, [7], where  $R_0$  and  $R_L$  are the characteristics resistances ( $R_0 = R_L = 50\Omega$ )

$$S_{21} = \frac{2V_L}{V_s} \sqrt{\frac{R_0}{R_L}} \quad (3)$$

The power transfer of the system can be described by transfer coefficient  $S_{21}$ , using the classic conversion formula as  $S_{21}$ , [8], given by:

$$S_{21} = \frac{2Z_{21}\sqrt{R_0R_L}}{(Z_{11} + R_0)(Z_{22} + R_L) - Z_{12}Z_{21}} \quad (4)$$

When the system operates at its resonance frequency  $f_0$ ,  $S_{21}$  becomes:

$$S_{21} = \frac{2}{\frac{\sqrt{(R_0 + R_t)(R_L + R_r)}}{2\pi f_0 M} + \frac{2\pi f_0 M}{\sqrt{(R_0 + R_t)(R_L + R_r)}}} \quad (5)$$

According to Eq. 5,  $S_{21}$  reaches to its maximum when the mutual inductance  $M$  equals to its optimal value, which means:

$$M_{opt} = \frac{\sqrt{(R_0 + R_t)(R_L + R_r)}}{2\pi f_0} \quad (6)$$

### 2.1 Mutual inductance model

Mutual inductance between current flow of the two coils can be calculated using Neumann's equation [3].

$$M = \frac{\mu_0}{4\pi} \iint_{c_1} \iint_{c_2} \frac{d\vec{l}_1 d\vec{l}_2}{R} \quad (7)$$

Where  $R = \sqrt{a^2 + b^2 + z^2 + 2ab \cos(\varphi_1 - \varphi_2)}$

When introducing the parameter  $\lambda$  such as:

$$\lambda = \frac{2ab}{a^2 + b^2 + z^2}$$

The mutual inductance can be expressed by the following equation:

$$M = \frac{\mu_0 ab}{2\sqrt{a^2 + b^2 + z^2}} \int_0^{2\pi} \int_0^{2\pi} \frac{\cos(\varphi_1 - \varphi_2)}{\sqrt{1 - \lambda \cos(\varphi_1 - \varphi_2)}} d\varphi_1 d\varphi_2 \quad (8)$$

Based on the solution of Neumann's equation (8),  $M$  can be simplified as:

$$M = \frac{\mu_0 \pi a^2 b^2}{2(a^2 + b^2 + z^2)^{\frac{3}{2}}} \left( 1 + \frac{15}{32} \lambda^2 + \frac{315}{1024} \lambda^4 \right) \quad (9)$$

The total mutual inductance  $M$  can be determined by adding all the combinations of each square filament  $M_{i,j}$ . For different axial separations, this can be expressed as:

$$M = \theta \sum_{i=1}^{n_p} \sum_{j=1}^{n_s} M_{ij} \quad (10)$$

Where  $a, b$ , are the radii of the primary and secondary coils respectively,  $z, \mu_0, n_p, n_s$ , are the distance between the coils, the permeability of the free space and the number of turn of primary and secondary coils respectively.

The parameter  $\theta$  depends on the shape of the planar coils. If both the primary and secondary planar coils are circular shaped,  $\theta = 1$ . If both the primary and secondary planar coils are rectangular, the parameter  $\theta$  is defined by

$$\theta = \left( \frac{4}{\pi} \right)^{1 + \frac{r_{\min}}{r_{\max}}}$$

$r_{\min}$  and  $r_{\max}$  represent the corresponding smaller and larger radii of the rectangular coils, respectively.

### 2.2 self-inductance model

The self-inductances of circular and rectangular coils are expressed by the following equation [9].

$$L = \frac{1.27 \mu_0 n^2 d_{avg}}{2} \left[ \ln \left( \frac{2.07}{\rho} \right) + 0.18\rho + 0.13\rho^2 \right] \quad (11)$$

Where  $\rho$  is the file factor

$$\rho = \frac{d_i - d_{out}}{d_i + d_{out}} \quad d_{avg} = \frac{d_i + d_{out}}{2}$$

and  $d_{out}$  and  $d_{in}$  are the outer and inner diameter respectively as shown in Fig.2

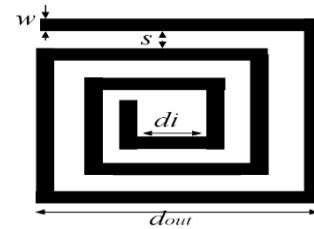


Fig. 2. Geometry of a square shaped coil

### 3. Simulation results

The objective of this paper is to maximize the efficiency of the WPT and eliminate the splitting frequency. The model simulated by MATLAB. Hence, we calculated the error between the mutual inductance which obtained by the geometry of the coils and the optimal value which obtained by (6)

Table (1) shows all parameters we have used in our simulation.

**Table 1.** Optimal inductive link coil designs using GA [6]

Parameters	Primary coil	Secondary coil
$d_{out}$ (mm)	80	20
$d_i$ (mm)	10	11
$w$ (mm)	1.5	0.25
$s$ (mm)	3	0.35

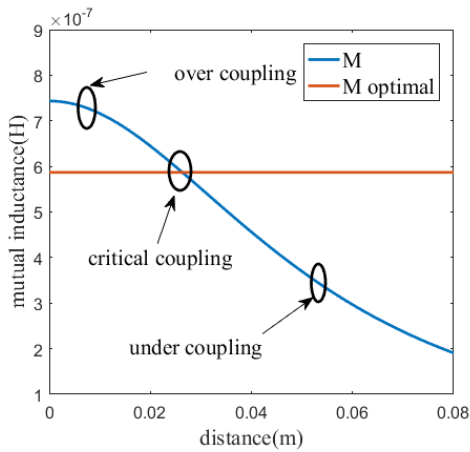
The transfer efficiency decreases at over coupling

$M > M_{optimal}$  or under coupling  $M < M_{optimal}$ . The transmission coefficient is maximum when  $M = M_{optimal}$  as shown in Fig.3

In addition, the received power can be dramatically decreased because of the magnetic over coupling between transmitting and receiving coils. This leads to a shifting of the peak power from its original resonant frequency to the other two adjacent frequencies as shown in Fig.4

Moreover, we keep the main objective that is to maximize the transmission coefficient  $S_{21}$  to be equal to 1, which means physically that the load absorbs all the transmitted wave at the port-2

This can be achieved in the critical coupling region by adjusting the error to be equal to 0 between the mutual and optimal mutual inductances. In order to reach this objective some parameters are optimized such as the outer and inner diameters as well as the number of coils in the primary coil.



**Fig. 3.** Variation of the mutual inductance versus distance at over coupling,, under coupling and in the critical region of the system .

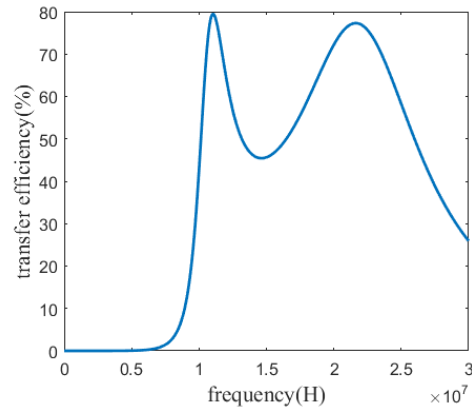
**3.1 Optimization of parameters**

Fig.5 shows the proposed algorithm which track the critical point by changing the values of outer diameter of receiver and the transmitter coils and observe the difference

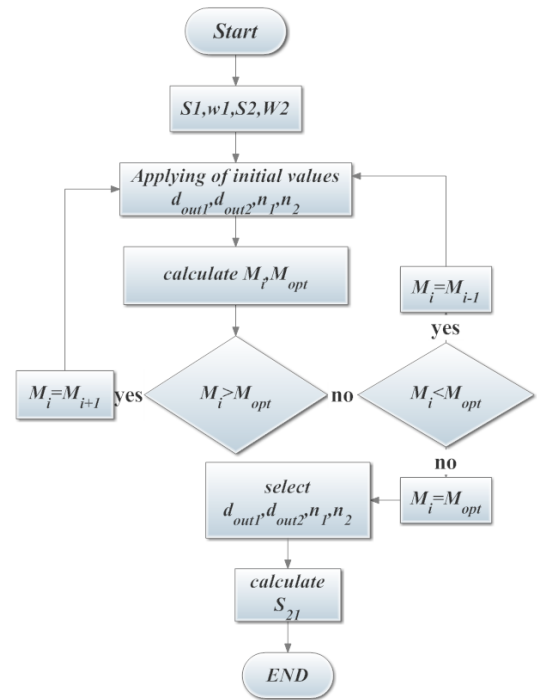
between  $M$  and  $M_{optimal}$  . When the actual value of  $M$  has large divergence to its optimal value either below or above

The curve of optimal mutual inductance , then the system operate at over and under coupling regions Fig.3 and the transfer coefficient  $S_{21}$  decreased in this regions.When

$M = M_{optimal}$  , then  $S_{21}$  has a maximum value according to the Eq.5 .



**Fig. 4.** Transfer efficiency versus frequency,splitting frequency



**Fig. 5.** Proposed algorithm

Moreover, a comparison between the results of [6] and our results is shown in Fig.7

We can see clearly that  $S_{21}$  of the proposed method stays at peak value near 0 dB at the resonant frequency 13.56 MHz and gives best results that the ones shown in [6]. Consequently, we can see a decrease of the transmission coefficient  $S_{21}$  of -3.35dB as Fig.7 reveals. Actually this is due to large error when the mutual inductance reaches the required distance that is 0.03m (refer to Fig.6). In the proposed method, the error is  $2.73 \times 10^{-7}$  while according to [6] it is  $8 \times 10^{-7}$  which is nearly 3 times greater.

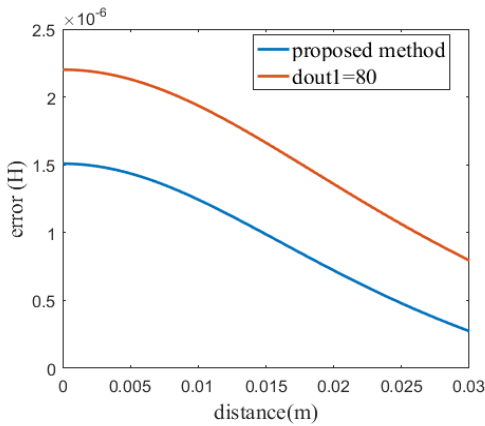


Fig. 6. Variation of the error verses deffirent distance

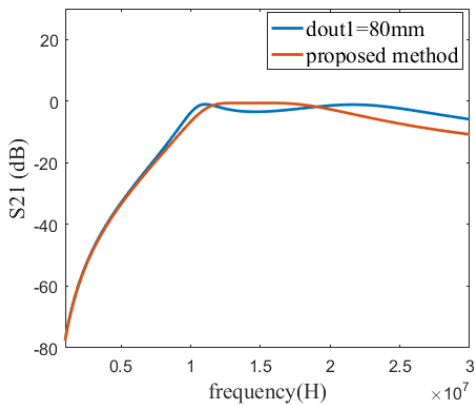


Fig. 7. The transfer coefficient S21 with respect the distance 0.03m

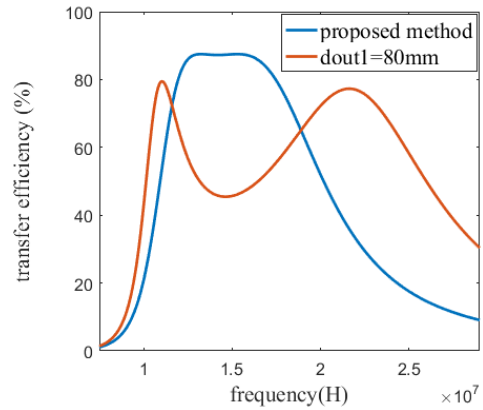


Fig. 8. Variation of transfer efficiency between receiver and transmitter

The transfer efficiency at resonance frequency can be derived from the value of  $|S_{21}|^2$  [4]. In our case, we can see clearly an increase of the efficiency to the value of 85% at original resonance 13.65 MHz, (refer to Fig.7) and the frequency splitting is eliminated by taking into account the optimal points.

Table 2. proposed algorithm comparing with [6]

Parameters	According to [6]		Proposed method	
	Primary coil	Secondary coil	Primary coil	Secondary coil
$d_{out}(mm)$	80	20	62	20
$d_i(mm)$	10	11	51.5	11
$n$	8	8	4	8
$f_0(MHz)$	13.56			
$Eeficiency(\%)$	80% not at ariginal $f_0$ 47% at $f_0=13.56$		86% at original $f_0=13.56$	

**Conclusion:**

In this paper, a new method of modeling wireless power transfer for optimized rectangular coils is presented. For this purpose, we proceed to the calculation of the mutual inductance using Neumann’s formula based on the geometry of the transmitted and the received coils. Simulation results are obtained using MATLAB. They reveal high accuracy compared to GA. The frequency splitting caused by over-coupling can also be suppressed using the proposed optimization.

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