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Prediction of Mixed-Mode Crack Propagation Paths in FGMs

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ABSTRACT — This paper investigates mixed-mode fracture behavior and crack propagation in FGMs by performing simulation of crack propagation by means of the finite element method. The displacement extrapolation technique (DET) and the strain energy density theory (SED) are used in this work. At each crack increment length, the kinking angle is evaluated as a function of stress intensity factors (SIFs). In order to show the robustness of our numerical developments, four examples of applications are presented. The effect of the defaults on the crack propagation in FGMs was highlighted.

Keywords*: Mixed mode, FGMs plate, Crack propagation path, Strain energy*

I. Introduction

Functional graded materials (FGM) have been widely used in technological application. Its mechanical behaviors, especially the fracture behaviors, have been extensively studied in recent years. A comprehensive review has been presented by Suresh and Mortensen [1]. For the fracture of the FGM, many researchers have considered various crack problems in non homogeneous materials. Carpenter et al. [2] performed fracture testing and analysis of a layered functionally graded Ti/TiB beam subjected to three-point bending. Rousseau and Tippur [3] performed experimental and numerical investigations on crack growth in an epoxy/glass FGM beam subjected to four-point bending. Benamara et al. [4, 5] investigated mode-I and mixed mode crack growth in FGMs, using the maximum tangential stress and the minimum strain energy density criteria.

Jin et al. [6, 7] investigated elastic-plastic mode I crack growth in TiB/Ti FGMs by using three-dimensional interface cohesive elements. Kim and Paulino investigated two dimensional mixed-mode crack propagation in FGMs using the finite element method and interaction integrals and also considered non-proportional loading [8]. Zhang and Paulino used cohesive zone models to simulate two-dimensional mixed mode dynamic crack propagation in FGMs [9]. Moon et al. [10] investigated crack growth resistance (*R*-curve) behavior of multilayer graded alumina-zirconia FGMs considering a crack parallel to the material gradation. Neubrand [11] performed experimental and theoretical investigations on the *R*-curve for Al/Al_2O_2 FGMs under mechanical loading. Fujimoto and Noda [12] investigated propagation of a single crack in a partially stabilized zirconia (PSZ) and T1-6A1-4V FGMs under transient thermal loads. Noda et al. [13] extended the investigation to two interacting edge cracks in FGMs. Uzun et al. [14] investigated fatigue crack growth of 2124/SiC/l0*p* single-core bulk FGMs considering mechanical loading. Forth et al. [15] investigated three-dimensional mixed-mode fatigue crack growth behavior of Ti-6A1-4V β-STOA FGM considering mechanical loadings.

Under mixed-mode loading conditions, an understanding of the crack growth path is important for fracture analysis. A number of mixed-mode fracture criteria for predicting the crack growth path have been proposed in the literature, including, for instance, the maximum

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circumferential stress (MCS) criterion [16], the maximum energy release rate (MERR) criterion [17,18], the strain-energy-density factor (SEDF) criterion [19], and the mixed-mode crack tip opening displacement(CTOD) criterion [20,21]. The strain energy density approach has been found as a powerful tool to assess the static and fatigue behavior of notched and unnotched components in structural engineering [22]. Different SED-based approaches were formulated by many researchers. Labeas et al [23], Zuo et al.[24], Nobile et al.[25], Balasubramanian and Guha [26], Ayatollahi and Sedighiani [27] and Spyropoulos [28].

The objective of this study is to present a numerical modeling of mixed-mode crack propagation in FGMs. Using the APDL code [29], the displacement extrapolation technique (DET) and the Strain energy density theory are used, to determine the stress intensity factors and the crack direction, respectively. The finite element method is used to carry out this objective. The effect of the inclusions and cavities on the crack propagation in FGMs was highlighted.

II. Evaluation of fracture parameters

II.1 Crack increment direction

Sih [19] used the strain as a critical parameter in order to propose the minimum strain energy density (S) criterion. It states that the direction of crack initiation coincides with the direction of minimum strain energy density along a constant radius around the crack tip. The (S) criterion showed a good agreement with the experimental results obtained earlier by Erdogan and Sih [6]. In addition, this criterion is the only one that shows the dependence of the initiation angle on material property represented by Poisson’s ratio ν. In mathematical form, S-criterion can be stated as:

$$\frac{\partial S}{\partial \theta} = 0 \quad and \quad \frac{\partial^2 S}{\partial \theta^2} > 0 \tag{1}$$

$$Where: \quad S = \frac{dW}{dV} r \tag{2}$$

Where r is its distance from the crack-tip and S is the strain energy density factor.

For the case of mixed modes I and II loadings, the strain energy density factor S was given by Sih as follows:

$$S = \frac{1}{\pi r}(a_{11}K_I^2 + 2a_{12}K_1 K_{II} + a_{22}K_{II}^2) \tag{3}$$

With:

$$\begin{aligned} a_{11} &= \frac{1}{16\,\mu_{tip}\,\pi}\left[(\kappa_{tip} - \cos\theta)(1+\cos\theta)\right] \\ a_{22} &= \frac{1}{16\,\mu_{tip}\,\pi}\left[(\kappa_{tip}+1)(1-\cos\theta) + (1+\cos\theta)(3\cos\theta - 1)\right] \\ a_{12} &= \frac{1}{16\,\mu_{tip}\,\pi}\left[\sin\theta\,(2\cos\theta - \kappa_{tip} + 1)\right] \end{aligned} \tag{4}$$

where: E is modulus of elasticity; v is Poisson’s ratio and $\frac{dW}{dV}$ is elastic energy per unit volume V.

II.1 Evolution of stress intensity factors

There are several techniques to obtain stress intensity factors (SIFs) in homogeneous and non-homogeneous materials, such as the displacement extrapolation technique (DET) [30-31], the displacement correlation technique (DCT) [32], the modified crack-closure integral [33] and the J^*_k Integral [34-35]. In this paper, the displacement extrapolation method is used to calculate the stress intensity factors K_I and K_{II} as follows:

$$K_I = \frac{E_{tip}}{3(1+\nu_{tip})(1+k_{tip})}\sqrt{\frac{2\pi}{L}}\,[4(v_b - v_d) - \frac{(v_c - v_e)}{2}] \tag{5a}$$

$$K_{II} = \frac{E_{tip}}{3(1+\nu_{tip})(1+k_{tip})}\sqrt{\frac{2\pi}{L}}\,(4(u_b - u_d) - \frac{(u_c - u_e)}{2}) \tag{5b}$$

where:

L is the length of the element side connected to the crack-tip.

E_{tip} and ν_{tip} are the Young’s modulus and the Poisson’s ratio at the crack tip location, respectively. $\kappa_{tip} = 3-4\nu_{tip}$ for plane strain, $\kappa_{tip} = (3-\nu_{tip})/(1+\nu_{tip})$ for plane stress.

u_i and v_i (i=b, c, d and e) are the nodal displacements at nodes b, c, d and e in the x and y directions, respectively (see Fig1).

In order to obtain a better approximation of the field near crack-tip, special quarter point finite elements proposed by Barsoum are used [36] where the mid-side node of the element in the crack-tip is moved to 1/4 of the length of the element, as shown in Fig. 1.

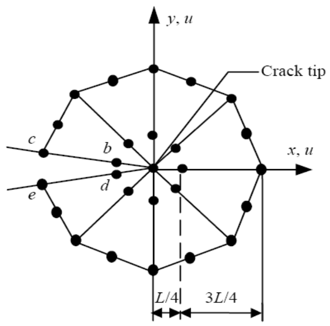

Fig. 1 Singular element around the crack tip

II. Algorithm of crack propagation

This section presents a finite element analysis for modeling fracture problems in FGMs using the re-meshing technique. Fig. 2 shows the Flow-chart of the prepared APDL code based on the combination of the finite element analysis and the strain energy density concept. According to the algorithm, after initial geometrical and physical modeling of the problem, the mesh pattern is generated around crack tip. In order to find new crack tip position at each step of propagation, the strain energy Sih's theory is employed, to obtain the kinking angle θ_0 as a function of the minimum strain energy density $(dW/dV)_{min}$.

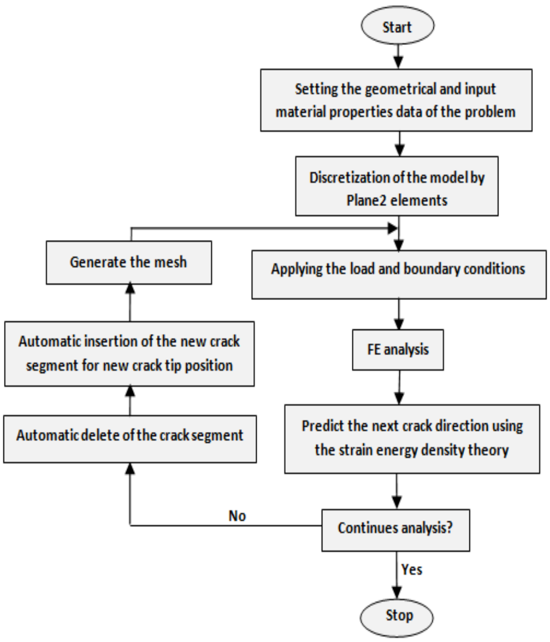

Fig. 2 A flowchart of the main operations which make the crack propagation

At each increment Δa of crack propagation, the special mesh is generated around crack tip, using the quadratic six-node triangular element. It is noted that, the same numeration of the nodes around the crack tip is taken during the crack propagation to evaluate automatically the new crack tip position. The algorithm is repeated until ultimate failure of material or by using another criterion for termination of the simulation process.

IV. Numerical results and discussion

IV.1 Evolution of SIFs

The geometry of the single edge cracked FGM plate with an initial crack of length “a” is considered for 2-Dimensional finite element analysis (Fig. 3a). This example was originally proposed by Erdogan and Wu [37], and it is one of the few theoretical fracture solutions available for a finite width FGM.

The cracked plate is submitted under a uniform tensile stress σ at the both ends. The elastic modulus was assumed to follow an exponential function given by:

$$E(x) = E_1 \exp(\lambda x); \qquad 0 \le x \le w; \qquad (6)$$

Where $E_1 = E(0)$, $E_2 = E(w)$=, and $\lambda = ln(E_2/E_1)$.

The following data were used for the finite element analysis:

a/w = 0.2, 0.3, 0.4, 0.5 and 0.6; $L/w = 8$; $\sigma=1$ unite, E_1 = 1 unite; E_2/E_1 = (0.1, 0.2, 1, 5 and 10); $\nu=0.3$; plane strain condition.

The variation continues in the elastic modulus are incorporated into the model by specifying the material parameters at the centroid of each finite element.

The Finite element standard code ANSYS [29] has been employed for modeling the problem. For the mesh generation of the cracked plate, the element type ‘PLANE183’ is used. It is a higher order two dimensional, 8-node element having two degrees of freedom at each node (translations in the nodal x and y directions), quadratic displacement behavior and the capability of forming a triangular-shaped element, which is required at the crack tip areas.

A typical FE model is shown in Fig. 3b. The special quarter point singular elements proposed by Barsoum are used for modeling the singular field near the crack tip (Fig. 4b). For a/w=0.2, the mesh discretization consists of 710 elements and 2241 nodes.

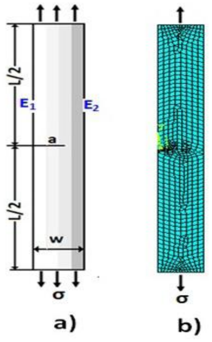

Fig. 3 Single edge cracked plate: (a) Geometry and boundary conditions; (b) FEM mesh discretization for a/w=0.2

The computed values of the SIF K_I obtained by the displacement extrapolation technique (DET) under plane strain condition are compared with the numerical results obtained by Chen et al. [38] using the element free Galerkin (EFG) method and Kim and Paulino [39] using J^*-Integral method and the displacement correlation technique (DCT), respectively (Tab. 1). The results obtained indicate reasonably good agreement between ours and other author's solution for this problem. These results allow us to conclude that the numerical model implemented in FE code (DET and Eq.6), correctly described the stress-strain field near the crack-tip and the behavior of the elastic FGM.

Table 1. Normalized stress intensity factors for edge cracked plate

Method	a/w					
	E_2/E_1	0.2	0.3	0.4	0.5	0.6
Chen et al. [38]	0.1	1.366	1.926	2.658	3.666	5.243
	0.2	1.455	1.897	2.529	3.443	4.926
	1	1.408	1.698	2.178	2.933	4.237
	5	1.158	1.392	1.794	2.446	3.611
	10	1.032	1.249	1.614	2.223	3.337
Kim and Paulino [39]	0.1	1.284	1.846	2.544	3.496	4.962
	0.2	1.39	1.831	2.431	3.292	4.669
	1	1.358	1.658	2.11	2.822	4.03
	5	1.132	1.37	1.749	2.366	3.448
	10	1.003	1.228	1.588	2.175	3.212
Kim and Paulino [39]	0.1	1.298	1.847	2.543	3.489	4.934
	0.2	1.396	1.832	2.429	3.286	4.644
	1	1.368	1.658	2.108	2.815	4.01
	5	1.132	1.366	1.744	2.375	3.426
	10	1.001	1.225	1.583	2.166	3.19
Present study	0.1	1.312	1.871	2.576	3.539	5.011
	0.2	1.406	1.850	2.455	3.328	4.714
	1	1.372	1.667	2.125	2.847	4.070
	5	1.135	1.375	1.758	2.385	3.486
	10	1.006	1.234	1.598	2.194	3.251

IV.2 Crack propagation simulation

In order to show the robustness of our numerical developments, two examples of applications are presented: rectangular plate with an oblique pre-crack and single edge cracked plate with one hole. For these examples, the variation of the elastic modulus for FGM is modeled by Eq.6.

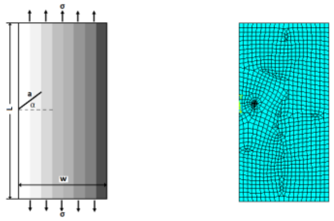

Fig. 4 Geometry model and final mesh of a rectangular plate with an oblique crack

IV.2.1 Rectangular FGM plate with an oblique pre-crack

In the present example, we consider a thin rectangular FGM plate with an oblique pre-crack (with α=30°). The plate considered is submitted under a uniform tensile load *σ*. A rectangular isotropic FGM plate with an oblique crack and final mesh for the first step of the crack propagation are shown in Fig. 4. The numerical simulation is performed in plane stress conditions.

Fig. 5 shows three steps for crack propagation trajectory obtained for FGM plate. Fig. 6 illustrates the positions of the crack-tip during the crack extension obtained for homogeneous and FGM plates. For two materials, the crack reoriented horizontally in the mode I loading.

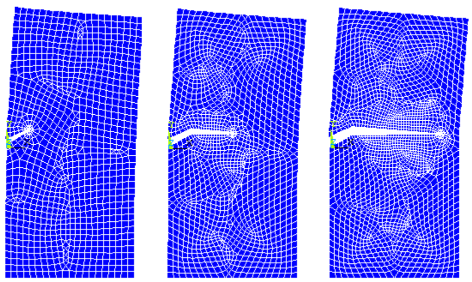

Fig. 5 Crack propagation trajectory of the inclined crack for FGM plate

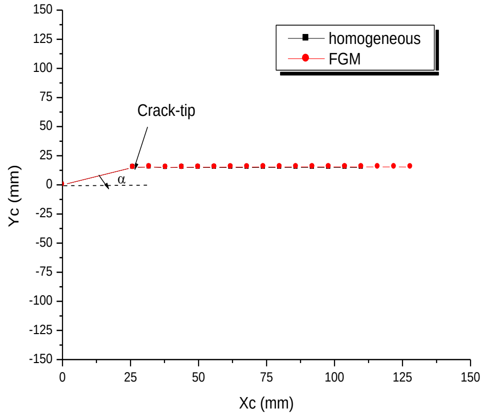

Fig. 6 Positions of the crack-tip during the crack extension obtained for homogeneous and FGM

4.2.2 Single edge cracked FGM plate with one hole

In order to determine the effect of a geometrical defect on the crack propagation in functionally graded materials, we represented in Fig. 7a the geometry of the single edge cracked plate with one hole. The single edge cracked plate is simply fixed at the bottom edge and loaded by uniform normal traction along the top edge. The structure is meshed by 8-node quadratic elements and by triangular elements concentric at crack-tip (Fig 7b). The determination of stress intensity factors, angle of direction and crack growth path are made of plane stress problem.

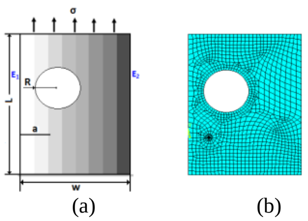

Fig. 7 a) Geometry model and b) the final mesh of the initial crack for the single edge cracked plate with one hole [5]

The numerical calculations obtained will compare with other results, for a homogeneous material case. Fig.8 shows the final configuration corresponding to the last evaluated crack length for the results obtained in references [40] and [41], and that obtained in the present study. It is clear that the crack paths obtained are seminars between them.

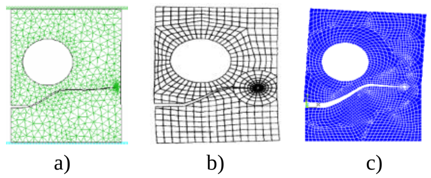

Fig. 8 Final configuration corresponding to the last evaluated crack length (with $E_2/E_1=1$):
a) Bouchard et al. [40], b) Rashid [41] and c) Present study

Fig. 9 illustrates four steps for crack extension in FGM plate. This crack would move in a straight path if there was no hole at the plate for mode-I loading (Fig 9a). However, due to the presence of the hole, the crack did not follow a straight line path, but curved towards the hole as shown in Fig. 9b. This was due to the stress concentration effect; cracks are likely to initiate at a hole boundary. Once the crack tip has moved beyond the hole, the crack reoriented horizontally in the mode I loading as shown in Figs. 9c and 9d.

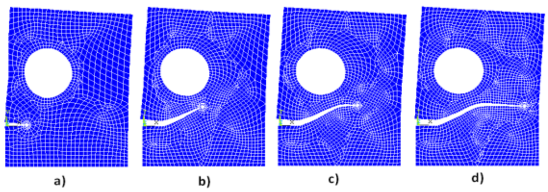

Fig. 9 Four steps of crack propagation trajectory for a single edge cracked FGM plate with one hole

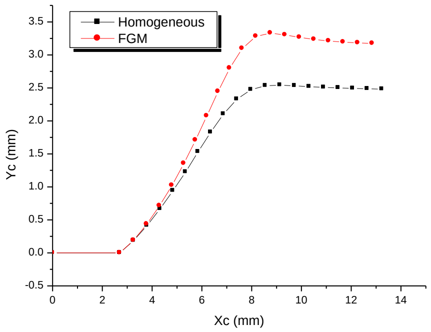

Fig. 10 Positions of the crack-tip during the crack extension obtained for homogeneous and FGM

Fig. 10 shows the crack trajectories obtained for homogeneous and FGMs plates. One can notice the same crack propagation behavior for both plates but the two crack paths are different from each other. This may explain the fact that the stress distribution around the hole is different for the two plates, which may influence directly on the propagation trajectory.

V. Conclusion

This paper investigates mixed-mode fracture behavior of FGMs by performing simulation of crack propagation by means of the finite element method. The prediction of SIFs for a single edge cracked plate was considered and compared under mode-I loading. The comparison shows that the program using the APDL Ansys Parametric Design Language is capable of demonstrating the SIF evaluation and the crack path direction satisfactorily.

The finite element modeling procedure proposed in this paper has been used successfully to simulate the propagation of cracks in FGM plate with holes and inclusions. The presence of holes and inclusions in the plates disturbed the stress and strain fields providing interesting crack trajectories. The crack simulations for mode I and mixed mode cases showed the acceptable crack path predictions. The results of the assessments strongly indicated that the finite element simulation for two-dimensional linear elastic fracture mechanics problems has been successfully employed for homogenous and FGM. Based on the results, it was recommended to add further development the APDL code to simulate crack propagation in orthotropic functionally graded materials.

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