Journal of Materials, Processes and Environment
December edition. Vol.1. N°2. (2013)
ISSN : 2335-1020

Application of First Order Shear Deformation Theory for Buckling Analysis of Functionally Graded Plates

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Abstract

In this paper, the buckling behavior of functionally graded material (FGM) plates under thermal load is investigated based on the first order shear deformation theory. The material is graded in the thickness direction according to a power-law distribution in terms of the volume fractions of the constituents. The thermal loads are assumed to be uniform, linear and non-linear distribution through-the-thickness. The derived equilibrium and buckling equations are then solved analytically for a plate with simply supported boundary conditions. Numerical examples cover the effects of the gradient index, plate aspect ratio, side-to-thickness ratio and loading type on the critical buckling for FGM plates.
Keywords: Buckling, functionally graded plate; first order shear deformation theory.

I. Introduction

Functionally graded materials (FGMs) have gained wide application in a variety of industries due to their distinctive material properties that vary continuously and smoothly through certain dimensions. Compared with common composites, FGMs avoid the inter-laminar stress gaps that are caused by mismatches in the properties of two different materials, and can be adjusted appropriately according to practical requirements.

The research on FGMs mainly focuses on thermal stress analysis, the determination of static and dynamic responses, and vibration analysis. Obata and Noda [1] studied the thermal stresses in a hollow circular cylinder and a hollow sphere of an FGM, and Fukui et al. [2] examined the stresses and strains in a functionally graded thick-walled tube under uniform thermal loading. Reddy and Chin [3] presented a finite element formulation for the analysis of the dynamic thermoelastic response of functionally graded cylinders and plates that employs the first-order shear deformation plate theory to account for the transverse shear strains and the rotations.

A generic static and dynamic finite element formulation was proposed by Liew et al. [4] for the modeling and control of piezoelectric shell laminates under coupled displacement and certain temperatures and electric potential fields. Pelletier and Vel [5] provided an analytical solution for the steady-state response of a functionally graded thick cylindrical shell subjected to thermal and mechanical loads, and Yang and Shen [6] investigated the free vibration and dynamic instability of functionally graded cylindrical panels subjected to combined static and periodic axial forces in a thermal environment.

In addition to the aforementioned linear analyses (small strains), the nonlinear response of FGMs has also attracted research interest. Praveen and Reddy [7] conducted a geometrically nonlinear transient analysis of FGM plates under thermal and mechanical loading, and Park and Kim [8] carried out a thermal postbuckling and vibration analysis of FGM plates based on the first-order shear deformation plate theory. Bouazza et al [9-14] analyzed thermal buckling of sigmoid functionally graded plates using first order shear deformation theory and classic plate theory. In that study, the material properties of functionally graded plates were assumed to vary continuously through the thickness of the plates, according to the two power law distribution in terms of the volume fractions of constituents.

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In this study, the thermal buckling analyses of P-FGM are investigated by using first order shear deformation theory. The Von Karman's nonlinear strain-displacement relation is used to account for buckling due to thermal load. Material properties are varied continuously in the thickness direction according to a simple power law distribution. The thermal buckling behaviors under uniform, linear and sinusoidal temperature rise across the thickness are analyzed. In addition, the effects of temperature field, volume fraction distributions, and system geometric parameters are investigated.

II. Functionally graded materials

Consider a case when FGM plate made up of a mixture of ceramic and metal as show in Fig1. The material properties vary continuously across the thickness according to the following, which are the same as the equations proposed by Praveen and Reddy [7]

$$
\begin{aligned}
&E(z) = E_m + E_{cm} V_f(z) \qquad E_{cm} = E_c - E_m \\
&\alpha(z) = \alpha_m + \alpha_{cm} V_f(z) \qquad \alpha_{cm} = \alpha_c - \alpha_m \\
&\nu(z) = \nu_0
\end{aligned}
\tag{1}
$$

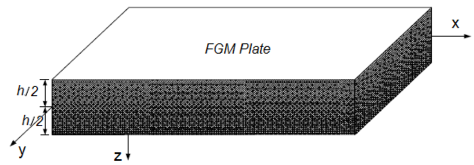

Fig. 1. Typical FGM rectangular plate.

where subscripts m and c refer to properties of metal and ceramic, respectively, and $V_f(z)$ is volume fraction of the constituents which can mostly be defined by power–law functions [15,16]. For power-law FGM, volume fraction function is expressed as

$$V_f(z) = \left(z/h + 1/2\right)^k \tag{2}$$

III.Stability equations

Assume that u, v, w denote the displacements of the neutral plane of the plate in x, y, z directions respectively; ϕ_x, ϕ_y denote the rotations of the normals to the plate midplane. According to the first order shear deformation theory, the strains of the plate can be expressed [17-19]

$$
\begin{aligned}
&\varepsilon_x = u_{,x} + z\phi_{x,x} \qquad \varepsilon_y = \upsilon_{,y} + z\phi_{y,y} \\
&\gamma_{xy} = u_{,y} + \upsilon_{,x} + z(\phi_{x,y} + \phi_{y,x}) \\
&\gamma_{xz} = \phi_x + w_{,x} \qquad \gamma_{zy} = \phi_y + w_{,y}
\end{aligned}
\tag{3}
$$

The forces and moments per unit length of the plate expressed in terms of the stress components through the thickness are

$$N_{ij} = \int_{-h/2}^{h/2} \sigma_{ij}\,dz \ ; \ M_{ij} = \int_{-h/2}^{h/2} \sigma_{ij} z\,dz \ ; \ Q_{ij} = \int_{-h/2}^{h/2} \tau_{ij}\,dz \tag{4}$$

The nonlinear equations of equilibrium according to Von Karman's theory are given by:

$$
\begin{aligned}
&N_{x,xx} + 2N_{xy,xy} + N_{y,yy} = 0 \\
&M_{x,xx} + 2M_{xy,xy} + M_{y,yy} - Q_{x,x} - Q_{y,y} = 0 \\
&Q_{x,x} + Q_{y,y} + q + N_x w_{,xx} + N_y w_{,yy} + 2N_{xy} w_{,xy} = 0
\end{aligned}
\tag{5}
$$

Using Eqs.(1) (3) and (4), and assuming that the temperature variation is either linear with respect to x- and y-directions, or constant, the equilibrium Eq. (5) may be reduced to a set of two equations as

$$
\begin{aligned}
&\nabla^4 w + \frac{2(1+\nu)}{E_1}\nabla^2 (N_x w_{,xx} + N_y w_{,yy} + 2N_{xy} w_{,xy} + q) \\
&\quad - \frac{E_1(1-\nu^2)}{E_1E_3 - E_2^2}(N_x w_{,xx} + N_y w_{,yy} + 2N_{xy} w_{,xy} + q) = 0
\end{aligned}
\tag{6}
$$

where

$$(E_1, E_2, E_3) = \int_{-h/2}^{h/2} (1, z, z^2)\, E(z)\,dz \tag{7}$$

$$(\Phi, \Theta) = \int_{-h/2}^{h/2} (1, z)\, E(z)\alpha(z) T(x,y,z)\,dz \tag{8}$$

To establish the stability equations, the critical equilibrium method is used. Assuming that the state of stable equilibrium of a general plate under thermal load may be designated by w_0. The displacement of the neighboring state is $w_0 + w_1$, where w_1 is an arbitrarily small increment of displacement. Substituting $w_0 + w_1$ into Eq. (6) and subtracting the original equation, results in the following stability equation

$$
\begin{aligned}
&\nabla^4 w_1 + \frac{2(1+\nu)}{E_1}\nabla^2 (N_x^0 w_{1,xx} + N_y^0 w_{1,yy} + 2N_{xy}^0 w_{1,xy}) \\
&\quad - \frac{E_1(1-\nu^2)}{E_1E_3 - E_2^2}(N_x^0 w_{1,xx} + N_y^0 w_{1,yy} + 2N_{xy}^0 w_{1,xy}) = 0
\end{aligned}
\tag{9}
$$

where, N_x^0, N_y^0 *and* N_{xy}^0 refer to the pre-buckling force resultants

To determine the buckling temperature difference ΔT_{cr}, the pre-buckling thermal forces should be found firstly. Solving the membrane form of equilibrium equations, gives the pre-buckling force resultants

$$N_x^0 = -\frac{\Phi}{1-\nu}, \quad N_y^0 = -\frac{\Phi}{1-\nu}, \quad N_{xy}^0 = 0 \qquad (10)$$

Substituting Eq(10) into Eq. (9), one obtains

$$\nabla^4 w_1 - \frac{2(1+\nu)}{E_1}\frac{\Phi}{1-\nu}\nabla^4 w_1 + \frac{E_1(1-\nu^2)}{E_1E_3 - E_2^2}\frac{\Phi}{1-\nu}\nabla^2 w_1 = 0 \qquad (11)$$

The simply supported boundary condition is defined as

$$w_1 = 0,\ M_{x1} = 0,\ \phi_{y1} = 0 \ \ on\ \ x = 0, a$$
$$w_1 = 0,\ M_{y1} = 0,\ \phi_{x1} = 0 \ \ on\ \ y = 0, b \qquad (12)$$

IV. Buckling analysis

In this section, the thermal buckling behaviors of fully simply supported rectangular metal-ceramic plates under thermal environment are analyzed. The thermal load is assumed to be uniform, linear and sinusoidal temperature rise through the thickness direction. The reference temperature is assumed to be 5°C. The effects of volume fraction index and geometric parameter a/h are investigated in each case.

IV.1 Uniform temperature rises

The plate initial temperature is assumed to be T_i . The temperature is uniformly raised to a final value T_f in which the plate buckles. The temperature change is

$\Delta T = T_f - T_i$.

IV.2. Linear temperature rise

The temperature field under linear temperature rise through the thickness is assumed as

$$T(z) = \frac{\Delta T}{h}(z + h/2) + T_m \qquad (14)$$

where z is the coordinate variable in the thickness direction which measured from the middle plane of the plate.

Tm is the metal temperature and ΔT is the temperature difference between ceramic surface and metal surface, i.e., $\Delta T = T_c - T_m$.

IV.3. Sinusoidal temperature rise

The temperature field under sinusoidal temperature rise across the thickness is assumed as

$$T(z) = \Delta T\left[1 - \cos\left(\frac{\pi z}{2h} + \frac{\pi}{4}\right)\right] + T_m \qquad (15)$$

V. Numerical results and discussion

a. Comparisons

In order to prove the validity of the present formulation, results were obtained for isotropic plates and compared with the existing ones in the literature. The critical temperatures of simply supported, isotropic square plates subjected to constant and linearly varying temperature distributions obtained using first order shear deformation theory are verified against the energy method based results of Gowda and Pandalai [21] and solution of Kri et al [22] based on finite element method using semiloof element in Table 1. Both results are in excellent agreement.

Table1. Critical temperature for isotropic square plates subjected to different forms of temperature distribution $\left(a/h = 100, \alpha = 2\cdot 10^{-6}, \nu = 0.3\right)$.

Temperature distribution	Analytical [21]	FEM [22]	Present
Uniform temperature rise	63.27	63.33	63.237
Linear Temperature rise	126.54	126.0	126.47

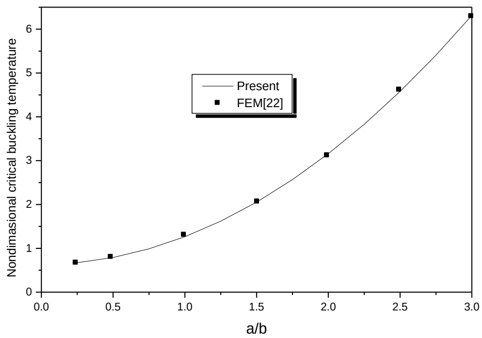

Fig. 2. Critical temperature of simply supported isotropic plates (a/h = 100).

In addition, the buckling loads for simply supported, isotropic plates under uniform temperature rise are calculated and compared in Fig 2 with finite element results obtained by Kari et al [22]. It can be seen that, for most cases the present results agree well with existing results.

b. Buckling analysis of FGM plates

Firstly, the critical buckling temperature are calculated for functionally graded plates with different volume fraction exponent under uniform temperature rise, linear and sinusoidal temperature distribution across the thickness and are plotted in Figs. 3 and 4. The plate aspect ratio is set as (a/b=1). These two figures show that the critical buckling temperature or temperature difference ΔT_{cr} increases as the relative thickness h/a increases, whatever the gradient index k is. However, the critical temperature gradient under sinusoidal temperature rise is higher than that under linear and uniform temperature rise.

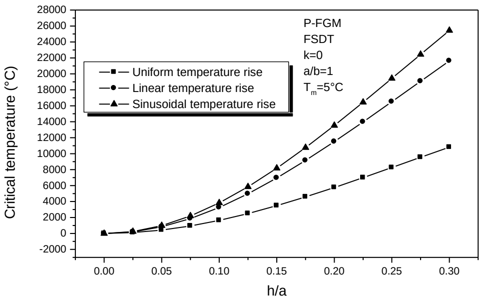

Fig. 3. Critical buckling temperature rise of a functionally graded square plate vs h/a (k = 0).

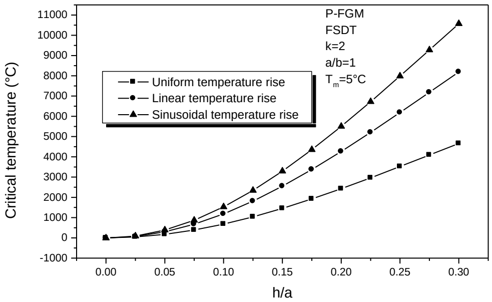

Fig. 4. Critical buckling temperature rise of a functionally graded square plate vs h/a (k = 2)

Fig. 5 shows the buckling temperature vs the material gradient exponent k for a plate with h/a=0.2, a/b =1. We can see that the critical buckling temperature for a homogeneous ceramic rectangular plate with k = 0 is considerably higher than that those for the functionally graded square plate with k ≥ 0. It is evident that the buckling temperature decreases as the material volume fraction exponent k increases monotonically. As the gradient index k changes from 0 to 1, the critical buckling temperature decreases significantly. When k changes from 1 to 2, it reduces very slowly, and as k becomes larger than 2, it will become a constant practically. It is also found that for a FGM square with small gradient index k, the difference between the buckling temperature of linear and sinusoidal temperature distributions is small.

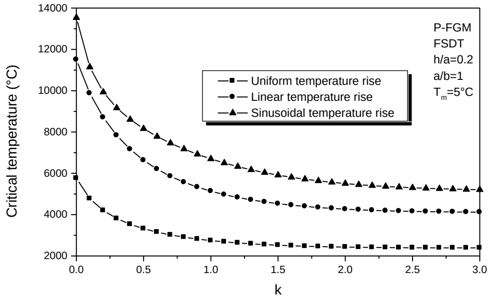

Fig.5. Critical buckling temperature rise of a functionally graded square plate vs k (a/b=1, h/a=0.2).

VI. Conclusions

Thermal buckling analyses of fully simply supported rectangular FGM plates under thermal environment are investigated by using first order shear deformation theory. The Von Karman’s nonlinear strain-displacement relation is used to account for buckling due to thermal load. The thermal load is assumed to be uniform, linear and sinusoidal temperature rise through the thickness direction. Based on the numerical results, the following conclusions are reached:

1- The critical buckling temperature for functionally graded rectangular plates are generally lower than the corresponding values for homogeneous plates. It is very important to check the strength of the functionally graded plate due to thermal buckling, although it has many advantages as a heat resistant material.

2- As the geometrical parameter h/a is increased, the critical temperature gradient increases rapidly.

3- As the volume fraction index k is increased, the critical temperature gradient decreases. This is because as volume fraction index is increased, the contained quantity of ceramic decreases.

4- As the critical temperature under sinusoidal temperature rise has the highest value in three cases, and that under linear temperature rise is higher than that under uniform temperature rise.

5- The critical buckling temperature difference for functionally graded rectangular plate calculated of power–law FGM or sigmoid FGM are identical for power law index values equals to 1

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