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Analysis of static behavior of a P-FGM Beam

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ABSTRACT

In this paper, the systematic arrangements are acquired to foresee the static behavior of the P-FGM beam and which have been gotten by Navier's solution. The numerical outcomes got by the new shear models are presented and compared with those available in the literature to see the influence of the geometry and the mixing law on the static behavior of this type of FGM beams.

1 Introduction

Functionally Graded Materials (FGM) are a class of composites that exhibit continuous variation in material properties from one surface to another without interruption and thus eliminate the stress concentration at the interface of the layers present in laminated composites.

An FGM is made from a mixture of two materials, usually metal and ceramic, as well as ceramic resists very high temperatures in thermal environments, while metal can reduce tensile stress on the surface of the ceramic in the cooling state. Functionally Graded Materials (FGM) are generally used in the fields of civil, aerospace, nuclear, military and mechanical engineering. Many researchers are interested in the behaviour of these composite materials. This is why many theories have been developed to simulate their behaviour using mathematical equations, among them are the classical beam theory (CPT), first order shear deformation beam theory (FSDBT) and Hight order shear deformation theory (HSDBT).

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The classical theory (CBT) known as the Euler-Bernoulli beam theory is based on the elementary bending theory of beams that does not take into account the effects of shear deformation.

The theory of first-order shear beam deformation (FSDBT), known as Timoshenko's beam theory, has been proposed to overcome the limits of (CBT) by taking into account the transverse shear deformation effect, but this theory requires the introduction of a correction factor.

The higher-order shear deformation theories can be developed as a function of the hypothesis of a variation of axial displacement of the higher-order through the thickness of the beam[1], [2] or of the axial displacements and transverse through the thickness of the beam (ie, use of a unified formulation) [3], [4]. It is more precise than first-order theory because it introduces a function that takes into account the phenomenon of "warping". This phenomenon occurs when the cross section of the beam loses its flatness

Due to the increased applications of FGM in engineering structures, many beam theories have been developed to predict the functionally graded beam (FG) response[5]–[15].

On the other hand, many researchers study the phenomena of instability of composite structures, for example: [16] analysed the interfacial stresses in damaged reinforced concrete beams strengthened with bonded prestressed functionally graded material plate. [17] presented a refined and simple shear deformation theory of plates and applied to the investigation of free vibration behavior of Carbon/Glass hybrid laminated composite plates. [18] discussed on a comparison of closed-form and finite-element solutions for the free vibration of hybrid crossply laminated plates. [19]studied the non- local buckling of Triple-walled carbon nanotubes (TWCNTs) embedded in an elastic medium under axial compression using Timoshenko beam model. [20]studied the free and forced vibrations of the carbon nanotubes CNTs embedded in an elastic medium including thermal and dynamic load effects based on nonlocal Euler- Bernoulli beam.

[21] presented the effect of the porosity and its distribution shape on the normal and shear interfacial stresses of the FGM beam strengthened with FRP plate subjected to a uniformly distributed load. [22] presented an original hyperbolic and parabolic shear and normal deformation theory for the bending analysis to account for the effect of thickness stretching in functionally graded sandwich plates. [23] proposed two new high-order shear deformation theory for bending analysis for a simply supported functionally graded plate with porosities resting on an elastic foundation.

[24] presented an improved theoretical solution for interfacial stress analysis for simply supported concrete beam bonded with a sandwich FGM plate. [25] examined the buckling behaviour of Carbon/Glass hybrid laminated composite plates using an accurate and simple refined higher order shear deformation theory. [26] studied the buckling analysis with stretching effect of functionally graded carbon nanotube-reinforced composite beams resting on an elastic foundation. [27] studied the free vibration behavior of antisymmetric cross-ply laminated composite plates using a refined shear deformation theory. [28] developed a new first-order shear deformation theory for dynamic behavior of functionally graded beams.

[29] presented a Numerical illustrations concern buckling behavior of FG sandwiches plates with Metal–Ceramic composition.

In this paper, various higher-order hyperbolic shear deformation beams theories for flexure of FG beams are developed as a function of the assumption of constant transverse displacement and variation in axial displacement of the higher-order through the thickness of the beam. In this paper the proposed theories satisfy the zero stress constraints on the upper and lower surfaces of the beam, so a shear adjustment factor is not required.

It is expected that the properties of the materials of the FG beam vary according to a distribution of the volume fraction of the components according to the power law. The equations of motion and the boundary conditions are gotten from the principle of virtual work. Analytical solutions for bending are obtained for a simply supported beam. Numerical cases are presented to show the validity and accuracy of current hyperbolic shear deformation theories or we may use two more recent high-order models to validate the importance and accuracy of these mathematical models. it can also be noted that this theory contains only three variables and the displacement along the axis (z) is divided into two parts: one part due to bending w_b and the other part due to shearing w_s.

The effects of the power-law index and the hyperbolic shear deformation on the flexural response of the FG beams are investigated.

2 Kinematics

The displacement field of the present theory of hyperbolic shear deformation theory are given in the following form:

$$\begin{cases} u_1(x,z,t) = u(x,t) - z\frac{dw_b}{dx} - f(z)\frac{dw_s}{dx} \\ u_2(x,z,t) = 0 \\ u_3(x,z,t) = w_b(x,t) + w_s(x,t) \end{cases} \quad (1)$$

Where:

u: is the axial displacement of a point on the median plane of the beam;

w_b and w_s are the bending and shear components of the transverse displacement of a point on the median plane of the beam, and $f(z)$ is a shape function that determines the distribution of transverse shear stress and shear stress over the thickness of the beam.

The form functions $f(z)$ are chosen to satisfy the stress boundary conditions on the top and bottom surfaces of the beam, so a shear correction factor is not required.

The present theory is given by:

$$f(z) = \begin{cases} \dfrac{2\cdot z\cdot \sinh\left(\frac{z^2}{h^2}\right)}{2\cdot \sinh\left(\frac{1}{4}\right) + \cosh\left(\frac{1}{4}\right)} & \text{Model 1 (HSDBT)[30]} \\ \dfrac{1}{10}\left(\dfrac{h \sinh\left(10\frac{z}{h}\right)}{\cosh(5)}\right) + \dfrac{h}{100} & \text{Model 2 (HSDBT)[31]} \\ z - h\sinh\left(\frac{z}{h}\right) + z\cosh\frac{1}{2} & \text{HBT according to [32]} \end{cases}$$

The strains are given by:

$$\varepsilon_x = \frac{du}{dx} - z\frac{\partial^2 w_b}{dx^2} - f\frac{\partial^2 w_s}{dx^2} \quad (2)$$

$$\gamma_{xz} = \left(1 - \frac{df}{dz}\right)\frac{dw_s}{dx} = g\frac{dw_s}{dx} \quad (3)$$

Where: $g = 1 - \frac{df}{dz}$ are the shape functions of transverse shear deformations.

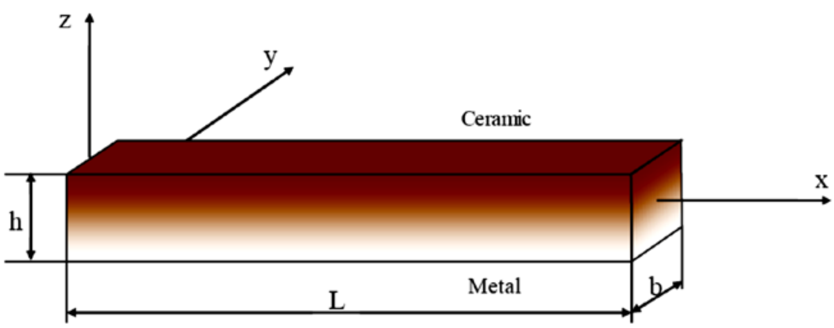

Figure1: Geometry and coordinates of the FG beam.

3 The equations of motion

The principle of virtual work is used here to derive the equations of motion. The principle can be stated in the analytical forms:

$$\int_{-h/2}^{h/2}\int_{\Omega}[\sigma_x\delta\varepsilon_x+\tau_{xz}\delta\gamma_{xz}]\,d\Omega dz-\int_{\Omega}q\delta w d\Omega=0 \tag{4}$$

The variation of the deformation energy of the beam can be indicated as:

$$\delta U=\int_0^L\int_A(\sigma_x\delta\varepsilon_x+\tau_{xz}\delta\gamma_{xz})dAdx\ =\int_0^L\left(N\frac{d\delta u}{dx}-M_b\frac{d^2\delta w_b}{dx^2}-M_s\frac{d^2\delta w_s}{dx^2}+Q\frac{d\delta w_s}{dx}\right)dx \tag{5}$$

Where N, M, and Q are the results of the constraints defined as:

$$N=\int_A\sigma_x dA \tag{6a}$$

$$M_b=\int_A z\sigma_x dA \tag{6b}$$

$$M_s=\int_A f(z)\sigma_x dA \tag{6c}$$

$$Q=\int_A g(z)\tau_{xz}dA \tag{6d}$$

The variation of the potential energy by the applied transverse load q can be written as:

$$\delta V=-\int_0^L q\delta(w_b+w_s)dx \tag{7}$$

By replacing the expressions of δU and δV by equations (5), (7) in equation (4) and integrating parts of space by collecting coefficients of δwb, and δws, we obtain the following equations of motion of the beam:

$$\boldsymbol{\delta u}: dN/dx=0 \tag{8}$$

$$\boldsymbol{\delta w_b}: \frac{d^2M_b}{dx^2}+q=0 \tag{8a}$$

$$\boldsymbol{\delta w_s}: \frac{d^2M_s}{dx^2}+\frac{dQ}{dx}+q=0 \tag{8b}$$

The boundary conditions are in the form: specify u or N

$$w_b\ or\ Q_b=\frac{dM_b}{dx} \tag{9a}$$

$$w_s\ or\ Q_s=\frac{dM_s}{dx}+Q \tag{9b}$$

$$\frac{dM_b}{dx}\ or\ M_b \tag{9c}$$

$$\frac{dM_s}{dx}\ or\ M_s \tag{9d}$$

4 Constitutive equations

FGM recommends ceramic and metal materials. The material properties of the beams FG are assumed to vary continuously in the thickness of the beam by a power law [2], [33]–[35]:

$$P(z)=P_m+(P_c-P_m)V_c.\quad V_c=\left(\frac{1}{2}+\frac{z}{h}\right)^P.\text{et}\quad V_m=1-V_c \tag{10}$$

Where P represents the effective material property such as Young's modulus E, Poisson's coefficient. The subscripts m and c represent respectively the metallic and ceramic components, and the exponent P is the power law index that governs the gradation of the volume fraction. The value of P equal to zero represents a ceramic beam, while P infinity indicates a metal beam. The variation in the combination of ceramic and metal is linear for $\mathrm{P}=1$. The linear behavioural relationships of an FG beam can be written as follows:

$$\sigma_x = Q_{11}(z)\varepsilon_x \tag{11a}$$

$$\tau_{xz} = Q_{55}(z)\gamma_{xz} \tag{11b}$$

Where:

$$Q_{11}(z) = E(z) \tag{12a}$$

$$Q_{55}\,(z) = E(z)/2[1+v(z)] \tag{12b}$$

By substituting equation (2) and (3) into equation (12) and the subsequent results in equation (6), the constituent equations for the stress results are obtained as:

$$N = A\frac{du}{dx} - B\frac{d^2w_b}{dx^2} - B_s\frac{d^2w_s}{dx^2} \tag{13a}$$

$$M_b = B\frac{du}{dx} - D\frac{d^2w_b}{dx^2} - D_s\frac{d^2w_s}{dx^2} \tag{13b}$$

$$M_s = B_s\frac{du}{dx} - D_s\frac{d^2w_b}{dx^2} - H_s\frac{d^2w_s}{dx^2} \tag{13c}$$

$$Q = A_s\frac{dw_s}{dx} \tag{13d}$$

With:

$$A = \int_A Q_{11}dA \tag{14a}$$

$$B = \int_A zQ_{11}dA \tag{14b}$$

$$B_s = \int_A f(z)Q_{11}dA \tag{14c}$$

$$D = \int_A z^2Q_{11}dA \tag{14d}$$

$$D_s = \int_A zf(z)Q_{11}dA \tag{14e}$$

$$H_s = \int_A f^2(z)Q_{11}dA \tag{14f}$$

$$A_s = \int_A g^2(z)Q_{55}dA \tag{14g}$$

5 Equations of motion in terms of displacements

By replacing the constraints resulting from equation (14) in equation (8), the equations of motion may be expressed in terms of displacements (u, w_b, w_s) as:

$$A\frac{d^2u}{dx^2} - B\frac{d^3w_b}{dx^3} - B_s\frac{d^3w_s}{dx^3} = 0 \tag{15a}$$

$$B\frac{d^3u}{dx^3} - D\frac{d^4w_b}{dx^4} - D_s\frac{d^4w_s}{dx^4} + q = 0 \tag{15b}$$

$$B\frac{d^3u}{dx^3} - D_s\frac{d^4w_b}{dx^4} - H_s\frac{d^4w_s}{dx^4} + A_s\frac{d^2w_s}{dx^2} + q = 0 \tag{15c}$$

6 Analytical solutions

The above equations of motion are solved analytically for bending problems. The Navier solution is used to determine the analytical solutions for a simply supported beam. The solution is thought to be in the form:

$$\begin{cases} u(x,t) = \sum_{n=1}^{\infty} U_n \, cos(\alpha x) \\ w_b(x,t) = \sum_{n=1}^{\infty} W_{bn} \, sin(\alpha x) \\ w_s(x,t) = \sum_{n=1}^{\infty} W_{sn} \, sin(\alpha x) \end{cases} \tag{16}$$

Where:

$\alpha = n\pi/L$, (U_n, W_{bn}, W_{sn}) Are the unknown displacement coefficients. The transverse load q is also extended in the Fourier series as:

$$q(x) = \sum_{n=1}^{\infty} Q_n sin(\alpha x) \tag{17}$$

Where

Q_n Is the load amplitude calculated from:

$$Q_n = \frac{2}{L}\int_0^L q(x) sin(\alpha x) dx \tag{18}$$

The coefficients Q_n are given below for some typical loads:

$$Q_n = \begin{cases} q_0 (n = 1) & \text{for a sinusoidal load} \\ \frac{4q_0}{n\pi} \;\; (n = 1,3,5, \dots) & \text{for a uniform load} \\ \frac{2}{L} Q_0 \sin\frac{n\pi}{2} \;\; (n = 1,2,3,..) & \text{to load, it concentrates in the middle} \end{cases} \tag{19}$$

By replacing the extensions of u, w_b, w_s and q of equations (16) and (17) in the equations of the equation of motion (15), the analytical solutions can be obtained from the following equations:

$$\begin{Bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{Bmatrix} \begin{Bmatrix} U_n \\ W_{bn} \\ W_{sn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ Q_n \\ Q_{sn} \end{Bmatrix} \tag{20}$$

Where:

$$S_{11} = Ax^2, S_{12} = Bx^3, S_{13} = B_s x^3, S_{22} = Dx^4, S_{23} = D_s x^4, S_{33} = H_s x^4 + A_s x^2,$$

7 Results and discussions

In this paper, many numerical examples are provided and discussed to verify the accuracy of the hyperbolic shear deformation theory currently being refined for the analysis of the static behaviour of a simply supported FGM beam. The properties of the materials used in this study are:

- Ceramic (P_c: Alumina, Al_2O_3): E_c=380 GPa ;υ_c=0.3.
- Metal (P_m: Aluminum, Al): E_m =70 GPa ; υ_m =0.3.

The dimensionless parameters used in this study are:

$$\bar{w} = 100\frac{E_m h^3}{q_0 L^4} w\left(\frac{L}{2}\right) \;, \qquad \bar{u} = 100\frac{E_m h^3}{q_0 L^4} u\left(0, -\frac{h}{2}\right)$$

$$\bar{\sigma}_x = \frac{h}{q_0 L}\sigma_x\left(\frac{L}{2}, \frac{h}{2}\right) \;, \qquad \bar{\sigma}_{xz} = \frac{h}{q_0 L}\sigma_{xz}(0,0)$$

8 Bending analysis results

Table 1 presents the non-dimensional numerical results of the deflection, horizontal displacements, axial and tangential stresses of the FGM beam subjected to a uniform load q0 for different values of the power law index P and at the same time by varying the ratio L/h of the beam. The results obtained are compared with other beam theories. Based on the results presented, this model gives results that are generally identical to other shear theories, as can be seen. It can be seen that the increase in the power law index P results in an increase in the deflection $\bar{w}$ through the thickness of the FGM beam under uniform load.

Figures 2, 3 and 4 show the axial displacement $\bar{u}$, the axial stresses $\bar{\sigma}_x$ and the transverse shear stresses $\bar{\sigma}_{xz}$ respectively. A comparison between the hyperbolic shear deformation theories presented by models 1 and 2 is also shown in these figures for different values of the power law index P. It can be seen that there is a good agreement between the two current higher order shear deformation models and the other models.

Figure. 5 shows the variation in axial displacement $\bar{u}$ through the thickness of the thick FGM beam $(L = 2h)$ under a uniform load and for the different values of the power law index P. the axial displacement $\bar{u}$ increases with the increase of the power law index P, then the axial displacement stabilizes.

In general, the two recent models used and the Soldatos theory have almost the same results. Figures 6 show the effect of the power law index on the bending response of an FGM beam subjected to a uniformly distributed load, the deflection $\bar{w}$ are presented in this graph.

Table 1: displacement, axial and tangential stresses of the FGM beam under uniform load.

P	Method	L/h=5				L/h=20			
		$\bar{w}$	$\bar{u}$	$\bar{\sigma}_x$	$\bar{\sigma}_{xz}$	$\bar{w}$	$\bar{u}$	$\bar{\sigma}_x$	$\bar{\sigma}_{xz}$
0	Model 01	3.1577	0.9338	3.7849	0.6294	2.8958	0.2305	15.0087	0.6350
	Model 02	3.1654	0.9396	3.8014	0.7303	2.8962	0.2306	15.0129	0.7425
	HBT	3.1654	0.9397	3.8017	0.7312	2.8962	0.2306	15.0129	0.7429
	CBT	2.8783	0.9211	3.7500	–	2.8783	0.2303	15.0000	–
0.5	Model 01	4.8189	1.6510	4.9688	0.6471	4.4638	0.4086	19.6946	0.6526
	Model 02	4.8285	1.6594	4.9916	0.7475	4.4644	0.4087	19.7003	0.7595
	HBT	4.8285	1.6595	4.9920	0.7484	4.4644	0.4087	19.7003	0.7599
	CBT	4.4401	1.6331	4.9206	–	4.4401	0.4083	19.6825	–
1	Model 01	6.2465	2.2937	5.8548	0.6294	5.8041	0.5684	23.1982	0.6350
	Model 02	6.2594	2.3035	5.8827	0.7303	5.8049	0.5685	23.2051	0.7425
	HBT	6.2594	2.3036	5.8831	0.7312	5.8049	0.5685	23.2052	0.7429
	CBT	5.7746	2.2722	5.7959	–	5.7746	0.5680	23.1834	–
2	Model 01	8.0402	3.1007	6.8443	0.5633	7.4403	0.7689	27.0896	0.5687
	Model 02	8.0674	3.1125	6.8813	0.6674	7.4420	0.7691	27.0989	0.6795
	HBT	8.0675	3.1127	6.8819	0.6685	7.4420	0.7691	27.0989	0.6802
	CBT	7.4003	3.0740	6.7676	–	7.4003	0.7685	27.0704	–
5	Model 01	9.7461	3.6898	8.0531	0.4758	8.8130	0.9130	31.7987	0.4810
	Model 02	9.8263	3.7093	8.1086	0.5869	8.8181	0.9134	31.8125	0.5988
	HBT	9.8271	3.7097	8.1095	0.5883	8.8181	0.9134	31.8127	0.5998
	CBT	8.7508	3.6496	7.9428	–	8.7508	0.9124	31.7711	–
10	Model 01	10.8613	3.8609	9.6517	0.5305	9.6856	0.9532	38.1235	0.5364
	Model 02	10.9369	3.8855	9.7102	0.6432	9.6904	0.9536	38.1382	0.6563
	HBT	10.9375	3.8859	9.7111	0.6445	9.6905	0.9536	38.1383	0.6572
	CBT	9.6072	3.8097	9.5228	–	9.6072	0.9524	38.0913	–

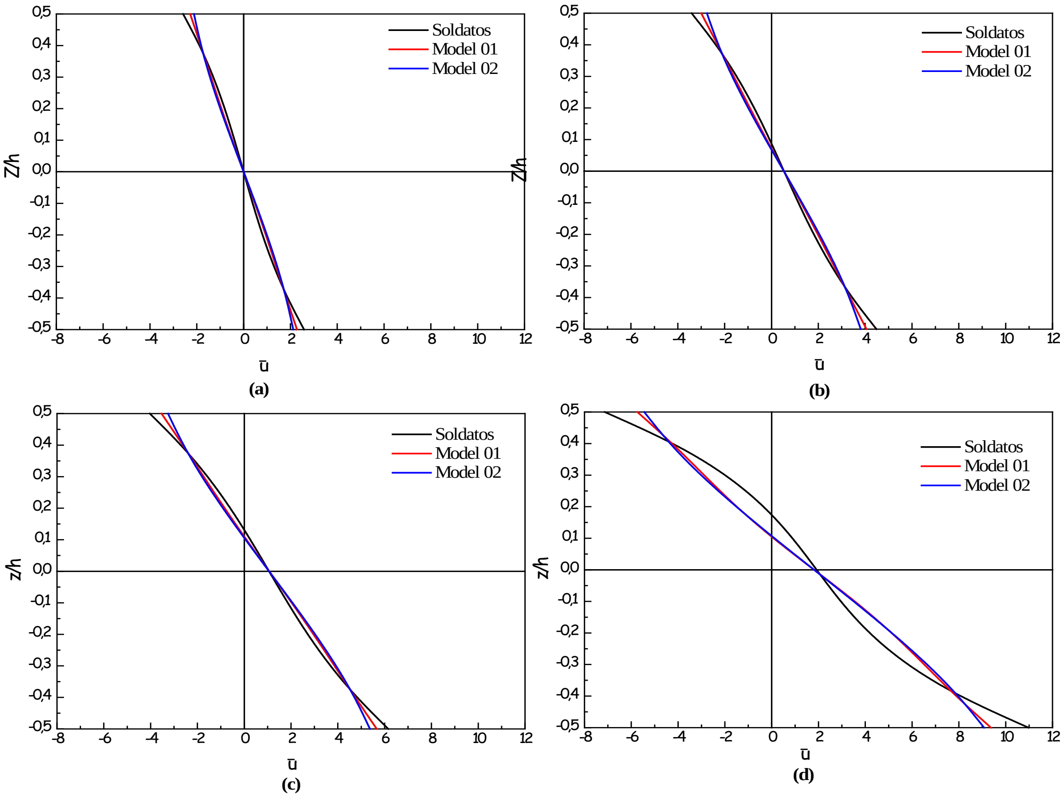

Figure 2: Dimensionless displacement variation $\bar{u}(0, z)$ through the thickness of the FGM beam under uniform load with (L=2h). (a) P =0, (b) P =0.5, (c) P =1 and (d) P =10.

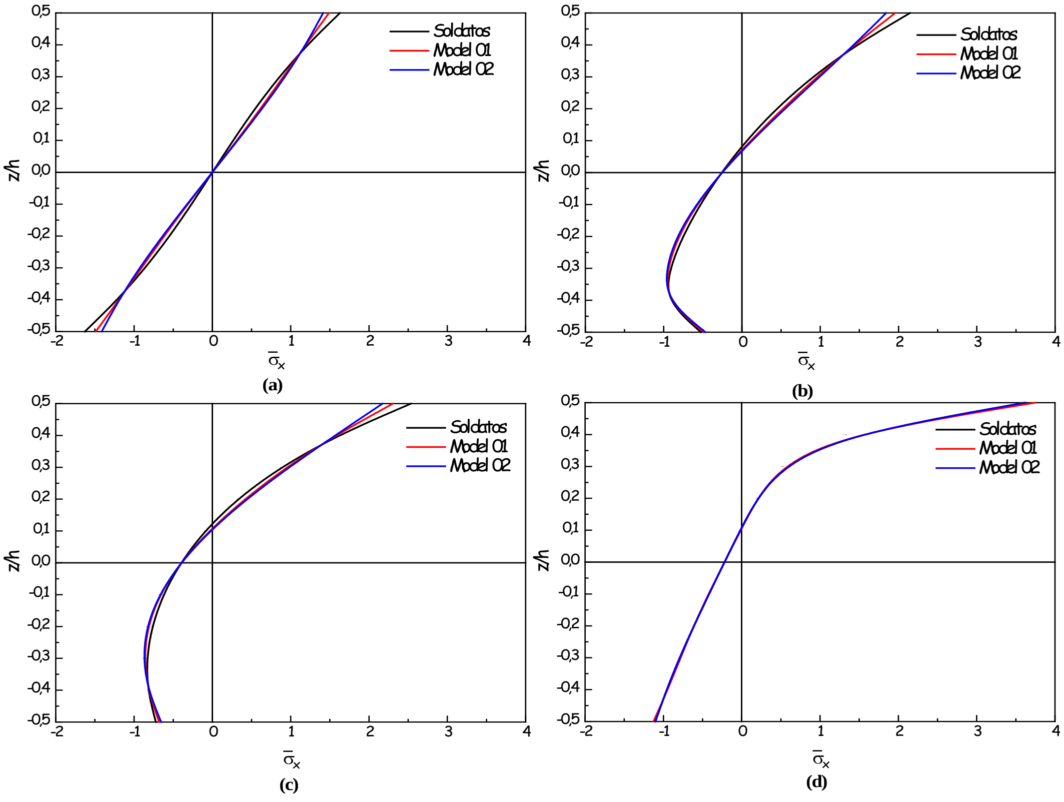

Figure 3: Dimensionless axial stress variation $\bar{\sigma}_x(L/2, x)$ through the thickness of the FGM beam under uniform load with (L = 2h). (a) P = 0, (b) P = 0.5, (c) P = 1 and (d) P = 10

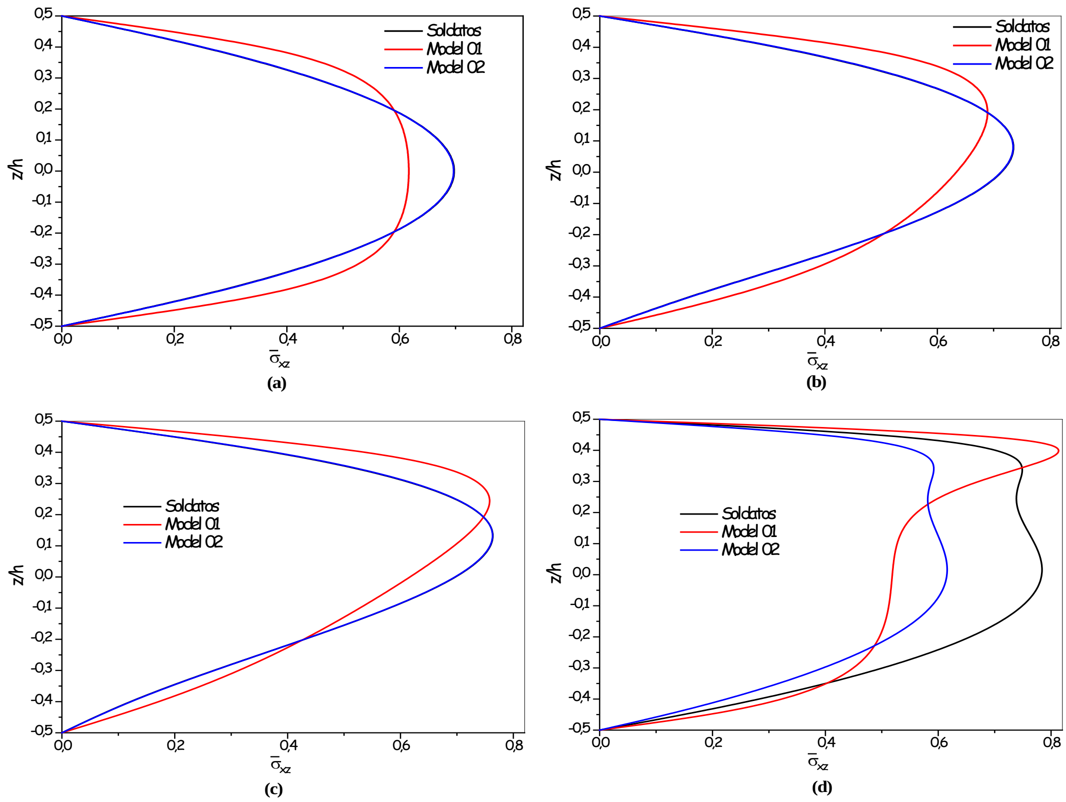

Figure 4: Dimensionless transverse shear stress variation $\bar{\sigma}_{xz}(0,z)$ through the thickness of the FGM beam under uniform load with (L =2h). (a) P =0, (b) P=0.5, (c) P =1 and (d) P=10.

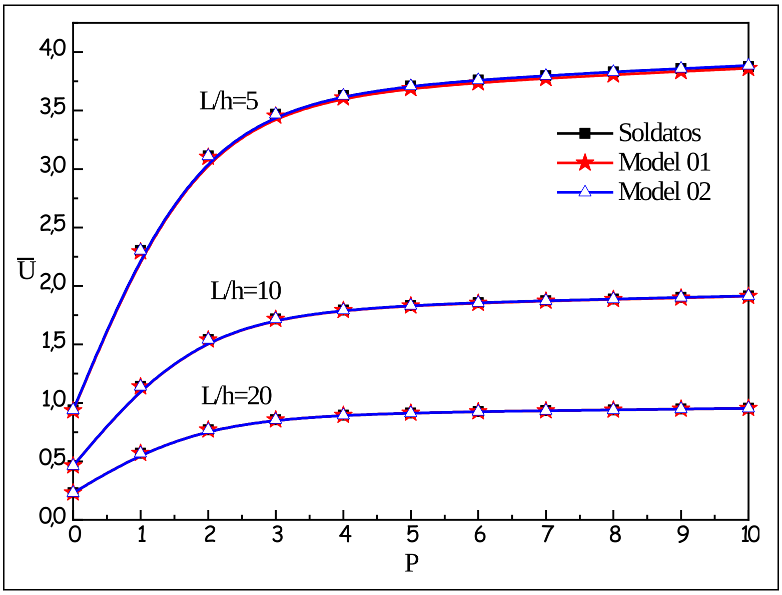

Figure 5: Variation of the non-dimensional axial displacement $\bar{u}$ as a function of the power law index p for beams FG subjected to a uniform load with different L/h ratio.

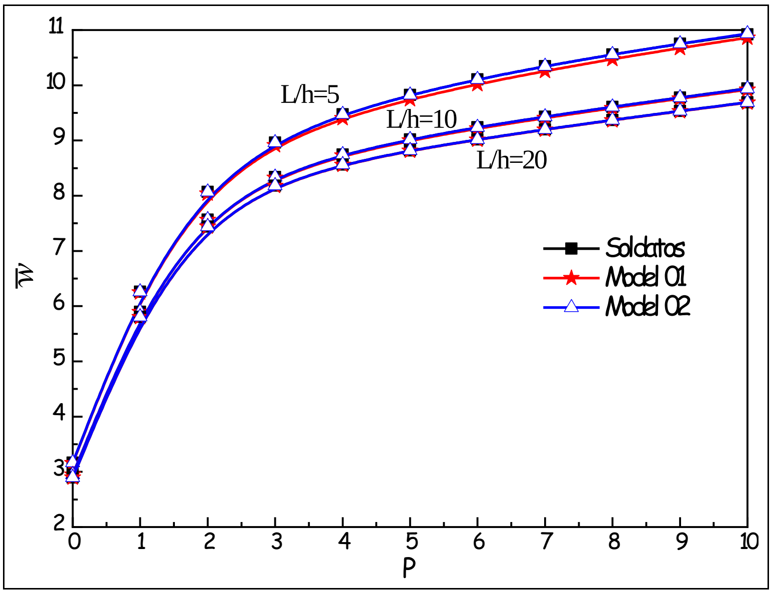

Figure 6: Dimensionless deflection variation as a function of the power law index with different L/h ratio.

9 Conclusion

In this paper, an analytical approach has been developed for the study of the stability with respect to the bending of the P-FGM beams under two cases of uniform and sinusoidal mechanical loadings. To validate this approach some examples are presented in this paper for a P-FGM simply supported beam. All the comparisons showed that the dimensionless deflexion due to the uniform or sinusoidal mechanical loading obtained using these two models and the model of Soldatos of a high order are almost identical. The proposed beam theories satisfy the unrestricted boundary conditions on the top and bottom surfaces of the beam; therefore, a shear correction factor is not required. CBT appears as a particular case of the proposed theories. The results of all the theories of the proposed beams are almost identical and agree with the existing solutions. Increasing the power-law index will reduce the rigidity of the FG beam and, as a result, will increase deflections.

The inclusion of shear deformation effects causes an increase in deflections. Different theories of higher-order shear deformation beams for bending of FG beams are developed.

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