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Research Paper

Influence parameters analysis in the gear helix correction

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ABSTRACT

The aim of the presen[...]ork is to d[...]ne the propeller correction that must be machined in the pinion of a meshe[...]ontact in[...]der to compensate the effects of the displacement of the shaft[...]o its flex[...]nd the effects of the displacement of the toothing due to its fle[...] a[...]s displacement due to the contact pressure. The objective will be achieved thro[...] the[...]tion of an analytical model of propeller correction. This m[...]ll prov[...]ults that will be compared to those obtained through the analytical [...]del d[...]ned in[...]e literature, numerical simulations using the finite element method [...]d tho[...]obtained[...]ough commercial software. Three analytical gear models will be s[...]d in[...]nt situations in order to verify the range of the model proposed. The stud[...]gears are arranged in the center (gear 1) at the end near the torque input in the pinion ([...]r 2) and opposite end of gear 2 (gear 3). The analytical model presented in this paper has results close to those of the current model of the literature and the [...]ware, with values of helix correction slightly larger, especially at the ends of the ge[...].

1 Introdu[...]on

Several methods [...]f calculating gears were studied that were implemented in Excel spreadsheets [1-4]. With this, it is intended to have a notion of the variation of the tooth-foot stresses and of the contact pressures of a gear if it is calculated in different ways.

It was also studied the gear profile displacement and the propeller angle that were also applied to the spreadsheets. Despite having rules that help in choosing the profile offset that should be used, some authors provide other methods to calculate displacements. These worksheets collect a set of input data and provide all dimensions, tooth-foot stresses and contact pressures of the gear pair.

The four main effects that can affect the propeller correction are the:

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- Twisting of the shaft;
- bending of the shaft;
- bending of the tooth of the gear;
- Movement of tooth surface of the gear due to contact pressure.

These effects can be separated into two lines: a more global one, which encompasses the displacements due to flexion and torsion of the shaft, and the other focusing on engagement, which includes dislocations of the teeth in itself, such as their flexion and displacement due to contact pressure. It is important to emphasize that all the displacements analyzed in this paper are in the elastic regime and are therefore reversible.

No reference was found that addressed all of these effects so that each one will be discussed separately in this section and then joined in section 2.

This lack of references was even one of the challenges of this work, as the correction of propeller is not widely approached in the literature. The most commonly used tools, such as those in references [illegible] and [5-[illegible]] specialized gear books such as references [1], [3-4] and even the German and American gear calculation rules [illegible] do not address the issue directly. The usual in these references is the adoption of a propeller correction factor which will be later multiplied by the load applied to the gear, in order to determine its resistance. Note that this procedure results in oversizing the applied load, increasing the size of the gears and therefore their cost.

Another commonly used approach is the adoption of a previously defined correction to study its impact on gear contact via finite elements, as represented in [8-12]. In addition, one of the works completely ignores the movement of the gears in cases where its width is much smaller than its diameter [13].

Other papers have been considered, but they are not explicitly focused on the stress analysis, but rather on issues such as "Determining the life of a gear through a computational model" addressed by [14], "Impact analysis on finite element meshing" by [15] or "Analysis of tooth contact stresses to reduce wear" addressed by [16]. However, although not the focus, these papers approach the subject superficially and can be used to have an analysis base by FEM.

It can be noted that most of the literature focuses on the finite element method to analyze corrections in gears. The second most commonly used approach is to take some correction and study its impact on gears, whether through the finite element method, analytical methods or even experiments. Other focuses of the literature are the study of noise, transmission errors (vibrations) and contact temperature in the gears.

Finally, it is important to note that the propeller correction must be calculated for the gearbox operating torque and not for the maximum torque for which it was designed. The propeller correction aims to avoid long-term effects on the engagement such as "pitting" or breaking of the root of the tooth. As the mechanism that governs these effects is fatigue damage - load concentration must be avoided for most of the life of the equipment, that is, in its daily operation condition. In this way, if there is a modification of the load to which the equipment is subjected, it is necessary to reuse the propeller correction of the gear to the new working condition.

2 Analytical method

The development of the analytical model will be divided into several stages, in which the effect affecting the contact of the gear will be studied.

2.1 Deduction of analytical formulas

The analytical model covers the four distinct effects of the gear, which are gear bending, gear torsion, gear tooth bending and gear tooth displacement due to contact pressure.

The propeller correction can be calculated and machined on both gears. However, in order to reduce manufacturing costs, the correction is only defined for the pinion, which has a lower number of teeth.

In this way, it is necessary to calculate the displacements of the crown flank in order to calculate later the propeller correction of the pinion, that is, the displacement necessary for the pinion to maintain 100% contact with the logo of the entire tooth profile when submitted to rated torque.

In addition, the method takes into account that the load is evenly distributed throughout the tooth surface. This is valid if the helix correction already exists. In this case, what is done is reverse engineering. Instead of calculating the load distribution for each helix correction on the pinion and iterating to find the correction that ensures a perfect load distribution, it is assumed that the distribution is perfect and the displacements in the gears are calculated. Thus, without any iteration, it is possible to define the correction of the helix of the pinion. If it is interesting to discover the distribution of the load on the teeth of the gear as a function of a given helix correction, reference [17] has a calculation methodology for that.

2.2 Gear line equation

In possession of the involutive equation, it is necessary to find the gear line of each section of the gears. This line contains all the points of the gear, from beginning to end. In this way, it is correct to affirm that the new point of engagement, after the displacement of the axes and the gears, will be contained in this same line.

However, this line will undergo a slight change due to the movements of the axes and gears (dxp, dyp, dxc, dyc).

This line can be determined according to the new center since it is tangent to the two basic circumferences of the pinion and the crown. To define your equation, just find these two points of tangency. The representation of the gear line can be seen in Fig. 1.

It is possible to note that there is a similarity of triangles between the lines shown in Figure 1. The detail of this similarity of the triangles is recorded in Figure 2. From Figure 2, we can determine

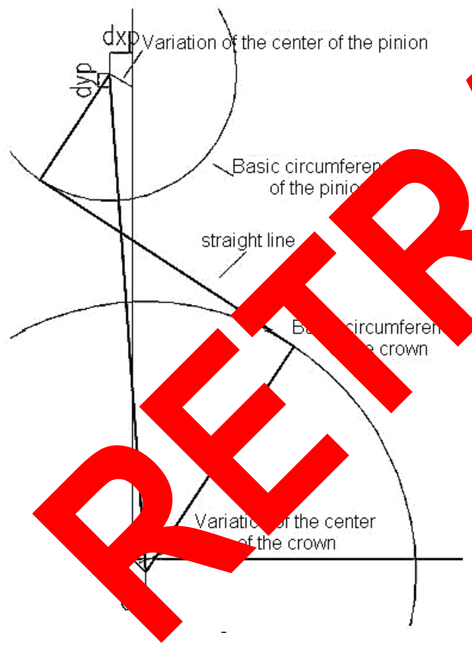


Figure 1 - Representation of the gear line

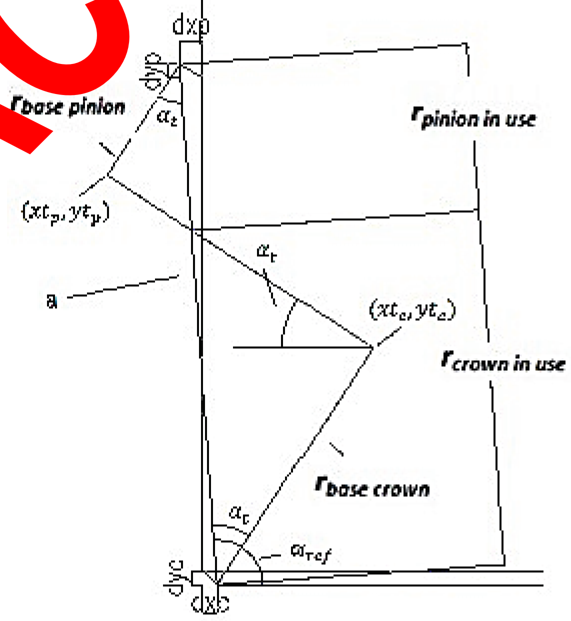


Figure 2 - Detail of Figure 1, showing the similarity of the triangles and the new working angle of the gear

$$\frac{r_{pinion\ in\ use}}{r_{base\ pinion}} = \frac{r_{crown\ in\ use}}{r_{base\ crown}} \tag{1}$$

$$r_{pinion\ in\ use} + r_{crown\ in\ use} = distance_{between\ centers} \tag{2}$$

$$\cos(\alpha_t) = \frac{r_{base\ pinion}}{r_{pinion\ in\ use}} \tag{3}$$

From (1), (2) and (3),

$$\alpha_t = acos\left(\frac{r_{base\ pinion} + r_{base\ crown}}{distance_{between\ centers}}\right) \tag{4}$$

Also, note that

$$\alpha_{ref} = \text{atan}\left(\frac{\Delta y}{\Delta x}\right) = atan\left(\frac{\left(a + dy_p\right) - dy_c}{dx_p - dx_c}\right) \tag{5}$$

With these equations, it is possible to determine the two points of tangency of the meshing line with the basic circles :

Point of contact with the crown

$$xt_c = dx_c + r_{base\ crown}.cos\left(\alpha_{ref} - \alpha_t\right) \tag{6}$$

$$yt_c = dy_c + r_{base\ crown}.sin\left(\alpha_{ref} - \alpha_t\right) \tag{7}$$

Point de contact with the pinion

$$xt_p = dx_p + r_{base\ pinion}.cos\left(\pi + \left(\alpha_{ref} - \alpha_t\right)\right) \tag{8}$$

$$yt_p = a + dy_p + r_{base\ pinion}.sin\left(\pi + \left(\alpha_{ref} - \alpha_t\right)\right) \tag{9}$$

Where xt_i and yt_i are the points of tangency of the respective gears.

It is convenient, for the moment, to make the same change of coordinates. In this way, we must

$$xt'_c = xt_c - dx_c = r_{base\ crown}.cos\left(\alpha_{ref} - \alpha_t\right) \tag{10}$$

$$xt'_p = xt_p - dx_c = dx_p - dx_c + r_{base\ pinion}.cos\left(\pi + \left(\alpha_{ref} - \alpha_t\right)\right) \tag{11}$$

$$yt'_c = yt_c - dy_c = r_{base\ crown}.sin\left(\alpha_{ref} - \alpha_t\right) \tag{12}$$

$$yt'_p = yt_p - dx_p = a + dy_p - dy_c + r_{base\ pinion}.sin\left(\pi + \left(\alpha_{ref} - \alpha_t\right)\right) \tag{13}$$

The equation of the line is

$$\frac{y - yt'_c}{yt'_p - yt'_c} = \frac{x - xt'_c}{xt'_p - xt'_c} \tag{14}$$

At this point, it is important to make a new change of coordinates: from Cartesian to Polar. The involute equation is in this second system and will be easier to solve if both equations are using the same system. In this way,

$$\frac{r.sin(\theta) - yt'_c}{yt'_p - yt'_c} = \frac{r.cos(\theta) - xt'_c}{xt'_p - xt'_c} \tag{15}$$

In simple terms, we arrived at

$$r(\theta) = \frac{yt'_c - xt'_c\left(\frac{yt'_p - yt'_c}{xt'_p - xt'_c}\right)}{\sin(\theta) - \left(\frac{yt'_p - yt'_c}{xt'_p - xt'_c}\right)\cos(\theta)} \tag{16}$$

The equation of the involution deduced above, which is

$$r(\theta) = \frac{m.z.cos(\alpha_0)}{2.cos\left(inv^{-1}\left(\theta_0 - \theta\right)\right)}$$

However, in this involute, the equation of the torsion of the crown gear, nor the displacement of its tooth due to its flexion and the contact pressure, is not being considered. In this way, the starting angle of gear must be changed in the equation above to compensate for these effects. As the torque applied by the pinion on the crown is counter clockwise and the force that the pinion exerts on the crown is positive, this angle should be discounted from the equation above. Naming this angle $\theta_{displacement\ crown}$, we have

$$\theta_{displacement\ crown} = \theta_{torsion\ pinion} + \theta_{tooth\ flexion} + \theta_{tdisplacement\ contact\ pressure} \quad (17)$$

$$r(\theta) = \frac{m.z.cos(\alpha_0)}{2.cos\left(inv^{-1}\left(\theta_0 - \theta_{displacement\ crown} - \theta\right)\right)} \quad (18)$$

Given the engagement angle (θ_{eng}) and hence the contact radius ($r_{contact}$), we can find the point of contact in the original coordinate system with the aid of equations

$$x_c = dx_c + r_{contact}.\cos(\theta_{eng}) \quad (19)$$

$$y_c = dy_c + r_{contact}.\sin(\theta_{eng}) \quad (20)$$

These are the coordinates of the new contact point of the crown. In this way, together with the coordinates of the new center of the pinion, it is possible to find the new contact radius of the pinion

$$r_{contact\ pinion} = \sqrt{\left(dx_p - x_c\right)^2 + \left(a + dy_p - y_c\right)^2} \quad (21)$$

The pinion offset angles for calculating the propeller correction can be seen in Figure 3.

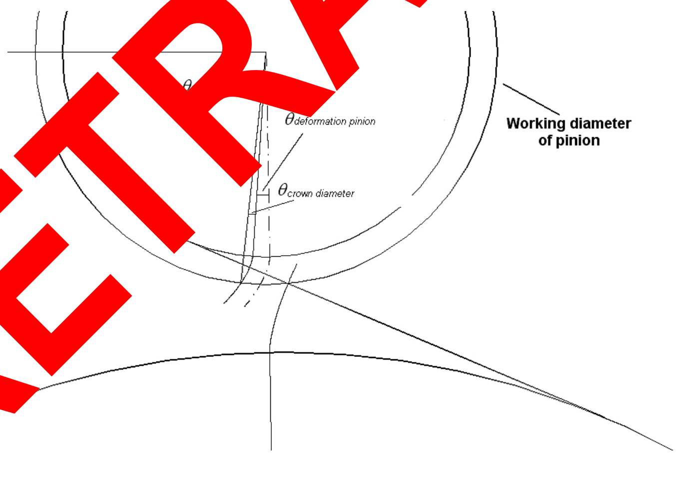


Figure 3 - Definition of the angles of displacement of the pinion

The angle of the pinion is

$$\theta_{pinion} = \pi + \theta_{0\ pinion} - \theta_{displacement\ pinion} - \theta_{contact\ diameter} \quad (22)$$

The angle $\theta_{0\ pinion}$ is determined by the equation below

$$\theta_{0\ pinion} = \frac{\pi}{2} + \frac{\pi + 4.x_{pinion\ profile}.tan(\alpha_0)}{2.z_{pinion}} \quad (23)$$

The angle $\theta_{displacement\ pinion}$ depends on the pinion twist and tooth displacement due to flexion and contact pressure. Each component will be determined later.

Finally, the $\theta_{contact\ diameter}$ is

$$r_{contact\ pinion} = \frac{m.z.cos(\alpha_0)}{2.cos\left(inv^{-1}\left(\theta_{0\ pinion} - \theta_{contact\ diameter}\right)\right)} \quad (24)$$

$$\theta_{contact\ diameter} = \theta_{0\ pinion} - inv\left(acos\left(\frac{m.z.cos(\alpha_0)}{2.r_{contact\ pinion}}\right)\right) \quad (25)$$

So that:

$$\theta_{pinion} = \pi + \theta_{0\ pinion} - \theta_{displacement\ pinion} - \theta_{0\ pinion} + inv\left(acos\left(\frac{m.z.cos(\alpha_0)}{2.r_{contact\ pinion}}\right)\right)$$

$$\theta_{pinion} = \pi - \theta_{displacement\ pinion} + inv\left(acos\left(\frac{m.z.cos(\alpha_0)}{2.r_{contact\ pinion}}\right)\right) \quad (26)$$

With this contact angle for each section of the gear, it is possible to determine the propeller correction of the gear unit taking into account the bending and twisting of the gears and the displacement of the gears due to their flexion and contact pressure.

The propeller correction may be better visualized in Figure 4 and Figure 5. The initial contact point of the gear is marked to the center in Figure 4. In possession of the new locus in which the involute of the crown is situated, and possessed of the line, it is possible to calculate the new contact point of the gear (since the propeller correction is machined in the pinion). With this point of contact, you can calculate the contact radius of the pinion and, with the new locus of the involute of the pinion; it is possible to define your new point of contact.

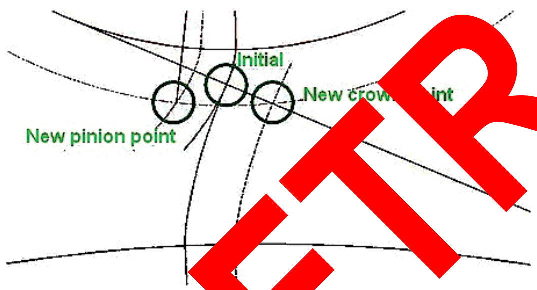


Figure 4 - Determination of the new contact points of the gear

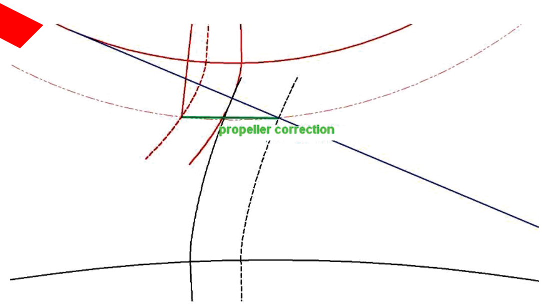


Figure 5 - Propeller correction display

The distance between the two new contact points is defined as the propeller correction of this gear and can be seen in Figure 5.

2.3 Determination of displacements due to flexion

As a next step, it is necessary to determine the displacement of the shafts and gears depending on the external loads, geometries and properties of the shafts.

The parameters used in the equations 2 and that have not yet been defined are:

- Value of new positions of gear centers;
- Value of the torsion angle for each gear.

To find these values another calculation methodology had to be developed. In this step, one must find the equation of the elastic line of the axis as well as the twisting of each section. To do so, it was necessary to divide the axes into several sections, each of which had a constant diameter.

This division can best be visualized in Figure 6.

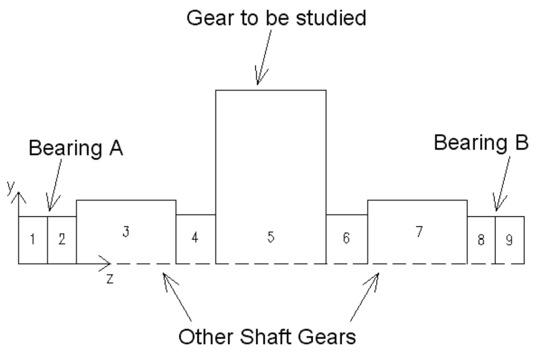


Figure 6 - Representation of the division of a generic axis into several sections of constant diameters

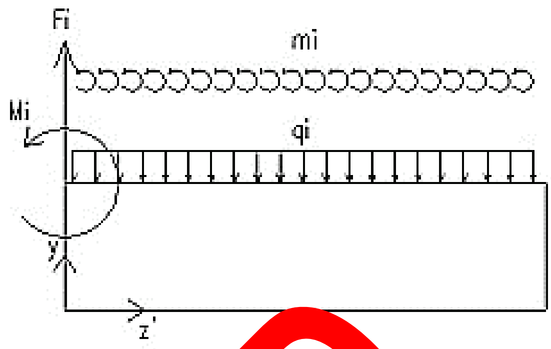


Figure 7 - Illustration of the loads defined for each axle element shown in Figure 6

The bending equation of any axis is

$$E_{(z)}I_{(z)}y''_{(z)} = M_{(z)} \tag{27}$$

Since the axis is composed of a material with uniform modulus of elasticity $E_{(z)} = E$,

$$y_{(z)} = \iint \frac{M_{(z)}}{EI_{(z)}} d^2z + K_1'.z + K_2' \tag{28}$$

However, the function $I_{(z)}$ is discrete, not easily integrable across the entire domain. In this way, it is convenient to divide the axis into regions of constant diameter as shown in Figure 6.

Thus, for each region, the variable $I_{(z)}$ is constant ($I_{(z)} = I_i$). Soon

$$y_{(z)} = \frac{1}{EI_i}\iint M_{(z)} d^2z + K_1'.z + K_2' \tag{29}$$

The constants K_i' and the moment value $M_{(z)}$ are found as a function of the boundary conditions of the problem.

In this case, the boundary conditions are the reaction forces of the two bearings, the distributed forces of the "n" gears (the model is generalized and can be used for any number of gears) and the displacements and clearance of the bearings.

However, in order to facilitate the resolution of these equations, it is easier to assume that all sections of the axis are subject to a moment and a point force at its left end and to a force distributed in the section as shown in Figure 7. In addition, a variable z' is introduced that, unlike, has zero value at the beginning of each segment of the axis.

In this way, the momentum equation for each segment of the axis is

$$M_{(z')} = M_i + (F_i + m_i) + q_i.\frac{z'^2}{2} \tag{30}$$

Thus, the equation of the displacement of the axis is

$$y_{i(z)} = \frac{M_i.\frac{z'^2}{2} + (F_i + m_i).\frac{z'^3}{6} + q_i.\frac{z'^4}{4}}{EI_i} + K_{i1}'.z' + K_{i2}' \tag{31}$$

Further, it is known that the displacement at the end of a section of the axis is equal to the displacement of the beginning of the next section, as well as the inclination angle of the elastic line. Like this

$$y_{i(z'=L_i)} = y_{i+1(z'=0)} \quad and \quad y'_{i(z'=L_i)} = y'_{i+1(z'=0)} \tag{32}$$

Thus, one can arrive at the equations of the constants of each section of the axis

$$\frac{K_{i+1_2}}{EI_{i+1}}=\frac{K_{i_2}+K_{i_1}.L_i+M_i.\frac{L_i^{\,2}}{2}+\left(F_i+m_i\right).\frac{L_i^{\,3}}{6}+q_i.\frac{L_i^{\,4}}{24}}{EI_{i+1}} \tag{33}$$

$$\frac{K_{i+1_1}}{EI_{i+1}}=\frac{K_{i_1}+M_i.L_i+\left(F_i+m_i\right).\frac{L_i^{\,2}}{2}+q_i.\frac{L_i^{\,3}}{6}}{EI_{i+1}} \tag{34}$$

After some mathematical treatment, one can deduce the following equation between generic sections (Figure 6) called '*mm*' and '*nn*', with $nn > mm$,

$$\frac{K_{nn_2}}{EI_{nn}}=\frac{K_{mm_2}}{EI_{mm}}+\frac{K_{mm_1}}{EI_{mm}}\sum_{i=mm}^{nn-1}\left(L_i\right)+\sum_{i=mm}^{nn-2}\left[\frac{\left(M_i.L_i+\left(F_i+m_i\right).\frac{L_i^{\,2}}{2}+q_i.\frac{L_i^{\,3}}{6}\right)}{EI_i}\sum_{j=i}^{nn-1}\left(L_j\right)\right]$$
$$+\sum_{i=mm}^{nn-1}\left[\frac{\left(M_i.L_i+\left(F_i+m_i\right).\frac{L_i^{\,2}}{2}+q_i.\frac{L_i^{\,3}}{6}\right)}{EI_i}\right] \tag{35}$$

$$\frac{K_{nn_1}}{EI_{nn}}=\frac{K_{mm_1}}{EI_{mm}}+\sum_{i=mm}^{nn-1}\left[\frac{\left(M_i.L_i+\left(F_i+m_i\right).\frac{L_i^{\,2}}{2}+q_i.\frac{L_i^{\,3}}{6}\right)}{EI_i}\right] \tag{36}$$

3 Results

3.1 Analytical model

After applying the analytical model discussed in Section B, using mainly equations (21), (26), (35), and (36) it was possible to calculate the bending and torsion of the axis, as well as the bending and moving of the tooth due to the contact pressure. The absolute values of these parameters for the gear 1 are shown in Figure 8.

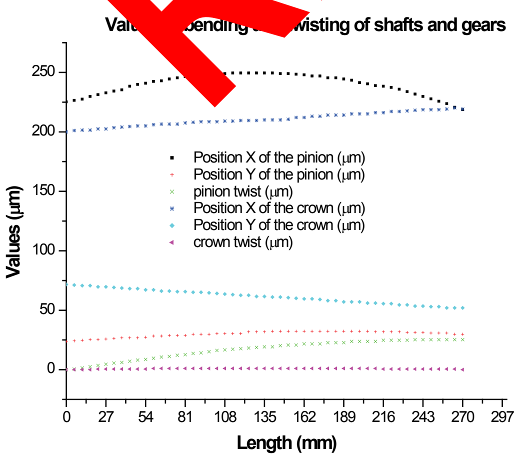


Figure 8 - Values of the flexion of the shafts and gears.

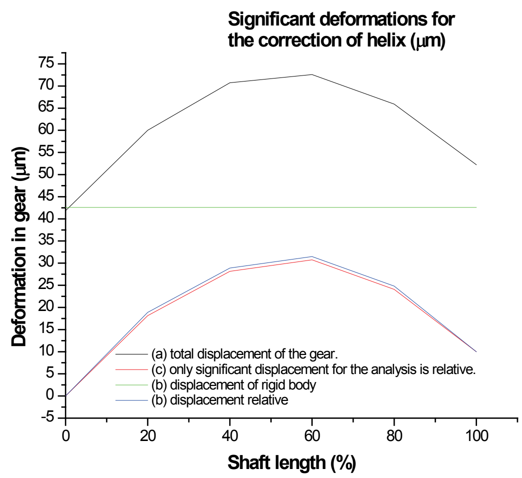


Figure 9 - Significant displacements for helix correction.

However, what is important for the helix correction value is the movements along the length of the gear, since a parameter that does not move can be fixed by a rigid rotation of the body. For example: Assuming the total displacement shown in Fig. 9a, it is possible to divide this displacement in two: that which can be corrected by rigid rotations of the body (straight and thin line in Fig. 9b) and another relative which is important for helix correction (curved and coarse line in Figure 9b). This is shown in Figure 9c.

Thus, Figure 10 shows only the relative displacement that should be taken into account for the calculation of the helix correction.

By analyzing Fig. 10, it can be seen that only the pinion is displaced during the contact. The crown, diameter 3.8 times larger than the pinion, has a moment of polar inertia 210 times greater. In this way, with respect to the same contact force, it moves much less than the pinion. What happens is that the bending of the shaft on which the crown is mounted causes it to align with the axis of the counterpart. The contribution of the bending and twisting of the crown axis in this case will be linear while the correction of the pinion has a profile close to the parabolic.

In addition, a sensitivity analysis was made for the displacement of the teeth due to its flexion and contact pressure.

The displacement of the toothing in the theoretical contact diameter has been calculated. When the contact diameter varies with the data obtained in Fig. 8, it was found that there was no significant variation in tooth movements due to their flexion and contact pressure - the order of magnitude of this variation is two or three times smaller than the variation of the other parameters. Thus, since the relative variation of these parameters is negligible, they will not be taken into account in the calculation of the helix correction of the cases analyzed in this work.

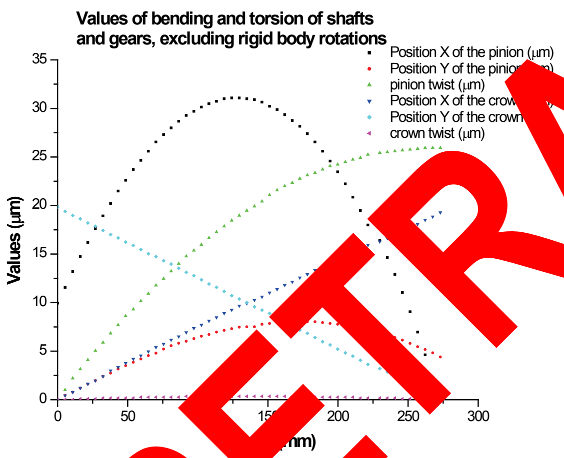


Figure 10 - Relative bending and torsion values of the shafts and gears.

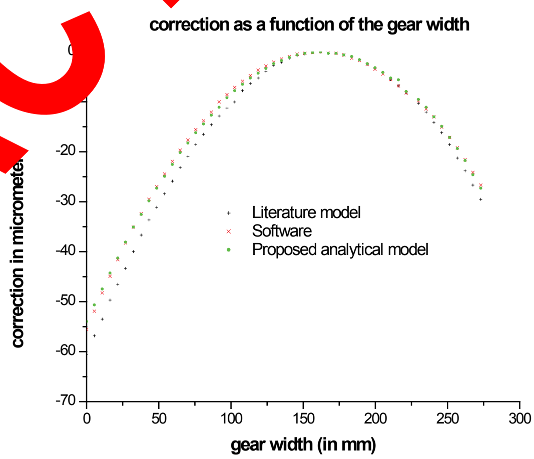


Figure 11 - Comparison of the helix correction of the three models exposed (1 gearing).

The values found in this sensitivity analysis are presented in Table 1.

Table 1 - Sensitivity analysis of bending and tooth movement due to contact pressure.

	Theoretical Diameter	Corrected Diameter	Error (%)
Bending of the teeth of the pinion (μm)	5.11	5.13	0.39 %
Bending of the teeth of the crown (μm)	18.32	18.41	0.49 %
Deformation due to contact pressure (μm)	13.937	13.938	0.01 %

The graphs shown in Fig. 11 show the thickness of material to be removed from the gear tooth along its width for the gear 1. Thus, greater correction implies greater mass shrinkage.

The sign "×" shows the helix correction obtained with the commercial software. The initial value of the correction is 55 µm, the zero value occurs in a width of approximately 160 mm and the final value of the correction, at the end of the tooth, is 28 µm.

The "+" sign shows the correction of the helix obtained with the analytical model proposed in the literature. The initial value of the correction is 54 µm, the zero value appears on a width of approximately 160 mm and the final value of the correction, at the end of the toothing, is 28 µm.

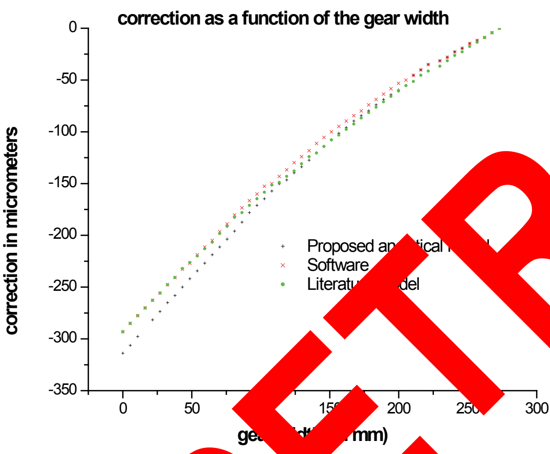
Finally, the signal "•" shows the correction of the helix obtained with the analytical model proposed in this paper. The initial value of the correction is 60 µm, the zero value appears in a width of approximately 160 mm and the final value of the correction at the end of the toothing is 30 µm.

The graphs shown in Fig. 12 show the thickness of the material to be removed from the gear tooth along its width to the gear 2.

The sign "×" shows the helix correction obtained with the commercial software. The initial value of the correction is 292 µm, the value "zero" appears at the end of the toothing.

The "+" sign shows the correction of the helix obtained with the analytical model proposed in the literature. The initial value of the correction is 293 µm, the value "zero" appears at the end of the toothing.

Finally, the signal "•" shows the correction of the helix obtained with the analytical model proposed in this paper. The initial value of the correction is 315 µm, the value "zero" appears at the end of the toothing.



Figures 12 – Comparison of the helix correction of the three models exposed (gearing 2).

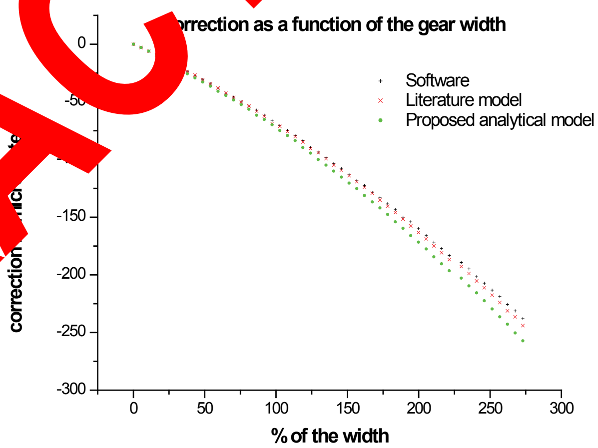


Figure 13 - Comparison of the helix correction of the three models exposed (gearing 3).

The graphs shown in Figure 13 show the thickness of the material to be removed from the gear tooth along its width to the gear 3.

The sign "×" shows the helix correction obtained with the commercial software. The initial value of the correction is zero and is 238 µm at the end of the toothing.

The "+" sign shows the correction of the helix obtained with the analytical model proposed in the literature. The initial value of the correction is zero and is 244 µm at the end of the toothing.

Finally, the signal "•" shows the correction of the helix obtained with the analytical model proposed in this paper. The initial value of the correction is zero and is 257 µm at the end of the toothing. By analyzing Figure 11, Figure 12 and Figure 13, it is possible to visualize that the helix correction obtained with the commercial software is very similar to that obtained with the model proposed in the literature. The values of the extremes of the corrections are similar and the maximum values occur in the same length of the gear. This is an indication that the model used in commercial software may be the same as in the literature, which considers only the normal displacements to the contact plane.

However, considering the involute profile of the tooth, the model proposed in this paper, the helix correction increases. As the bending of the gears is more pronounced at the ends of the teeth, the distance between the gears will also be greater in this region. With this greater distance, the tangential displacements towards the contact become important, since the separation between the gears increases. This factor is responsible for increasing the helix correction on these sites. The percentage difference between models is about 8%.

This factor can be seen in Figure 14. Since the tangential displacement to the contact plane is neglected, it is assumed that the profile of the tooth is a straight line. However, as we move away from the initial contact area, there is a separation to consider in the helix correction. He is responsible for the difference between the analytical model of the literature and that proposed in this paper.

It should be mentioned that the shape of the helix correction that must be provided to the grinding of the teeth should be a linear equation (right) or a second degree (parabola). All the corrections proposed in this thesis can be approximated by a second degree equation. The correlations of the profiles with the equation of a parabola are presented in Table 2. Thus, although the effects governing the helix correction are not linear, the final result can be approximated by a parabola, which guarantees its manufacturing capacity.

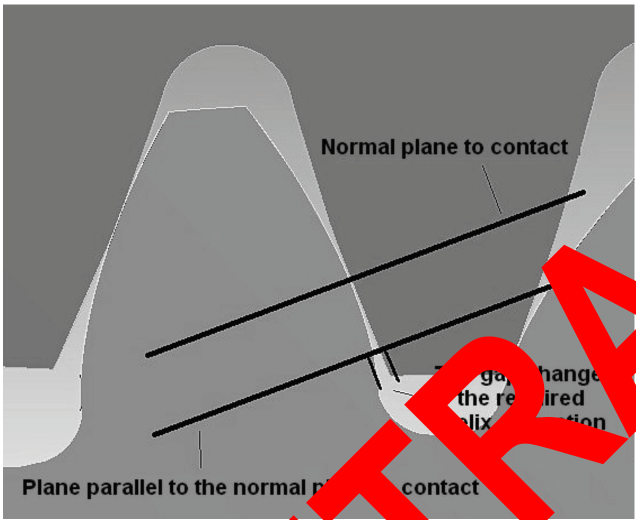


Figure 14 - Gap responsible for the difference between the analytical models proposed by the literature and this paper.

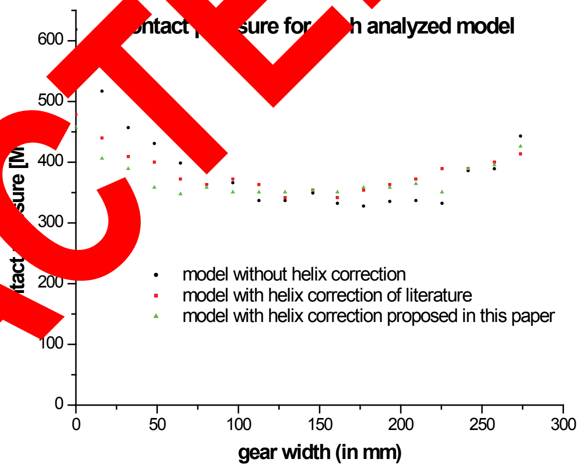


Figure 15 - Contact pressure along the width.

Table 2 – Correlation of the profiles studied with a parabola equation.

Fashion Model	Correlation with second-order equation (R^2)
Software	1000
Literature	0.9990
Proposed	0.9988

3.2 Finite element model

Figure 15 contains a graph with the maximum values obtained with the finite elements for each section of the toothing. It can be noticed that for the uncorrected model, the pressure distribution along the denture is not constant, being higher at the beginning of the gear, decreasing along the same and increasing again at the end. This is in line with the helix corrections found for gear 1, which state that relief should be given to the toothing at the ends, which relief must be greater in the region of the gear teeth through which the torque is transmitted to the gear and less region.

With the helix correction proposed in the literature, it is possible to notice that there is a better distribution of load along the tooth face as compared to the simulated model without any correction. However, at the ends of the toothing there is still a stress concentration (lower than that of the uncorrected model). By analyzing the helix corrections proposed by the literature and by this paper (Figure 11) it is possible to visualize that the model proposed by this paper has slightly larger corrections than the literature.

With the helix correction proposed in this paper, it is possible to visualize that the distribution of the load, although not perfect, is better than the one found by the helix correction proposed in the literature.

3.3 Comparison between analytical model and FEM

The discrepancies between the results found with the analytical model (3.1) and with the finite elements (3.2) may come from several factors. Despite all the care to maintain the same boundary conditions between analytical model and finite element methods, the analytical model is two-dimensional while the finite element method is three-dimensional. In this way, it is expected that there will be some differences between the results. For example, the analytical method considers that the entire gears are requested (the entire cross-section of them is requested) whereas in the finite elements this is not the behavior displayed when analyzing the results obtained.

Another point of divergence between the models lies in the bearings. In the analytical model they are punctual, whereas in the three-dimensional model they have an area of contact with the axis. This contact area increases the moment of inertia of this region so that the displacement of the axis, when using the FEM, is lower than the calculated in the analytical model.

By analyzing the finite element data found in section 3.2 one can calculate the load concentrator of each of the helix correction calculation methods. The value of this concentrator is found by dividing the maximum contact pressure by the mean pressure value in the section. The values found are shown in Table 3.

Table 3 - Values of stress concentrator found by the finite element

Model	Concentrator	Diff. relation to uncorrected
No correction	1.[illegible]	-
Correction literature	1.25	57 %
Correction of this paper	1.23	60 %

Analyzing Table 3, it can be concluded that the model formulated in this dissertation, when compared with the others by means of the finite element method, improves the pressure distribution along the denture by 3%. Using the calculation methodology discussed in [illegible], a reduction of 3% in the load concentration on the gear teeth allows an approximate reduction of 3% in the width of the gears. This reduction, when applied to all gears of a gearbox, reduces its cost by 1%. Note that this cost reduction is only achieved by changing the methodology of calculating helix correction, without the need for investment in machine tools, software, material change, etc.

The difficulties of [illegible], however, are great. The literature model [18] is linear and can be solved without any knowledge of the geometry and properties of the gears that are in contact. It is only necessary to know the gear ratio, the diameters of the gears and the torque transmitted between the axles.

With the proposed model it is necessary to have a deep knowledge of the gears in contact, and several gear parameters are necessary in order to calculate the helix correction. However, the manufacturer of the gear reducer has these data, therefore only more time is needed to calculate the correction.

4 Conclusion

In the course of this work, an analytical model was developed to calculate the helix correction in a cogged geared pair of spurs. This model was compared with the literature model and the software.

By observing the Figures in section 3.1, it is possible to affirm that the analytical model presented in this paper has results close to those of the current model of the literature and the software, with values of helix correction slightly larger, especially at the ends of the gear.

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Analyzing Table 3, it is possible to visualize that the stress concentration in the denture is 3% smaller in the model developed in this paper if compared with the literature model. This gain is responsible for a reduction in the cost of the reducer by approximately of 1%.

In this way, it is possible to conclude that considering the effect of the involute profile of the gear in the calculation of the helix correction improves the distribution of pressure in the contact, reducing the cost of the reducer and increasing the competitiveness of the equipment in the market.

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