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Cylindrical bending of orthotropic plate strip based on *n*th-order plate theory

Atteshamuddin S. Sayyad*, Shantaram M. Ghumare, Sachin T. Sasane

Department of Civil Engineering, SRES's College of Engineering, University of Pune, Kopargaon-423601, Ahmednagar, Maharashtra, India

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ABSTRACT

In this paper, cylindrical bending of orthotropic plates is presented using *n*th-order plate theory. Classical plate theory and parabolic shear deformation theory of Reddy can be considered as special cases of present theory. The theory accounts for realistic variation of the transverse shear stress through the thickness of plate and satisfy the traction free conditions at top and bottom surfaces of the plate. The number of unknown variables in the present theory is same as that of first order shear deformation theory. The theory is variationally consistent. The use of shear correction factors which are problem dependent and are normally associated with first order shear deformation theory is avoided in the present theory. The governing equations and associated boundary conditions are derived by the principle of virtual work. Navier solution technique is employed for the simply supported plates. The program has been developed in FORTRAN. The displacement and stresses of a simply supported plate infinitely long in y-direction under sinusoidally distributed load are calculated to demonstrate the accuracy and efficiency of the present theory.

1 Introduction

With the increasing use of composite materials, the need for advanced methods of analysis became obvious. In case of composite materials, transverse stresses and strains strongly influence the bending behaviour. In particular, the transverse shear stress effects are more pronounced. The classical plate theory (CPT) which is not formulated to account for the effect of these stresses is not satisfactorily applicable to orthotropic plate analysis. Therefore, over the years, researchers developed many theories, which took into account transverse shear effects, such as; first order shear deformation theory [1], parabolic shear deformation theories [2-4], trigonometric shear deformation theory [5], hyperbolic shear deformation theory of Soldatos [6], exponential shear deformation theory [7], hyperbolic shear deformation theory of Akavci [8] and many more. However, these theories are mainly applied to bidirectional bending of plate but their applications for the one dimensional analysis of plate are not fully explored.

Pagano [9] has presented the exact solution for cylindrical bending of laminated composite plates. Author has investigated limitations of classical plate theory comparing with the solutions of several specific boundary value problems

* *Corresponding author. Tel.: +91 9763567881.*
E-mail address: attu_sayyad@yahoo.co.in

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to the corresponding 3D elasticity solutions for cross-ply laminates. Wan [10] presented sixth and twelfth order plate theories for cylindrical bending plate problems considering effect of transverse normal stress and demonstrate the efficiency of these theories for thick plate.

Jalali and Taheri [11] examined the bending response of cross-ply laminated plates under cylindrical bending using semi-exact method which is based on the assumption of inextensibility of the plate through the thickness. The deflections and stresses are obtained for symmetric and anti-symmetric laminated plates under half sine load and concentrated load.A new stress analysis method that was developed by Soldatos and Watson [12] for the cylindrical bending of cross-ply laminated plates has been successfully extended by Shu and Soldatos [13] towards the accurate determination of the stress distributions in angle-ply laminated plates subjected to cylindrical bending. The stress distribution of simply supported and clamped anti-symmetric angle-ply laminated plates subjected to sinusoidal loading is obtained.

Vel and Batra [14, 15] studied the cylindrical bending of linear piezoelectric laminated plates. Authors have used Eshelby–Stroh formalism to obtain analytical solutions for displacements, stresses, and electric fields for laminated plates with piezoelectric actuators and sensors. The results are obtained for clamped-clamped and cantilever laminates. The results are validated with finite element coding. Perel and Palazotta [16] developed new plate theory for the cylindrical bending of sandwich plate simplifying the assumptions regarding distribution of transverse strain components in the thickness direction. The plate has thick homogenous isotropic face sheet and isotropic core made up of foam. The finite element formulation based on this plate theory is presented for simply supported transversely compressible sandwich plates subjected to uniform load.

Khdeir [17] presented free and force vibration analysis of anti-symmetric angle-ply laminated composite plates using classical plate theory and first order shear deformation theory. The plate is of arbitrary boundary conditions and loading conditions. Exact natural frequencies are determined for various boundary conditions using the state space concept.Park and Lee [18] presented a new exponential theory for the cylindrical bending of symmetric/anti-symmetric cross-ply, angle-ply and unsymmetrically laminated plates. In this theory, in plane displacements are assumed to vary exponentially across the plate thickness. The results obtained for displacements and stresses of simply supported laminated plates are compared with those obtained from the first-order shear deformation plate theory and the classical laminated plate theory.

Chen and Lee [19] developed an elasticity method to study the bending and free vibration response of simply-supported angle-ply laminated cylindrical panels in cylindrical bending using method of state-space. Authors have obtained exact solution for the static bending problem using variable substitution technique whereasapproximate analytical solution is derived for dynamic problem by employing the layerwise method. Lu *et al.* [20] obtained elasticity solutions for free vibration of angle-ply laminates subjected to cylindrical bending using semi-analytical approach. The formulation is carried out using the transfer matrix method based on the state space concept.Effects of variation of ply angle on the vibration properties of laminates are investigated. Numerical results are obtained for simply supported and fully clamped thick laminates.

Cylindrical bending of an elastic rectangular sandwich plate having rigid filler and resting on an elastic foundation has been presented by Starovoytov et al. [21].Flexural analysis of thick orthotropic plate strip under cylindrical bending using trigonometric shear deformation theory taking into account transverse shear deformation effect as well as transverse normal strain effect has been presented by Ghugal and Sayyad [22]. Authors have applied this theory for the simply supported boundary condition and obtained displacements and stresses for various modular ratios.

Xiang and Kang [23] developed *n*th-order plate theory for the bending analysis of functionally graded plates. However, the theory is applied to bidirectional bending only. Therefore, in this paper attempt is made to check the efficiency of this theory for the cylindrical bending of orthotropic plates. Governing equations and boundary conditions are obtained using analytical form of principle of virtual work. A simply supported orthotropic plate strip under sinusoidally distributed load is considered for the numerical study. The displacements and stresses are obtained for various aspect ratios and modular ratios are compared with exact solutions provided by Pagano [9].

1.1 Orthotropic plate under consideration

Consider an orthotropic plate of length '*a*' and overall thickness '*h*' as shown in Fig. 1. It is assumed that the plate is of an infinite extent in the *y* direction while it is simply supported at its edges $x = 0$ and $x = a$. '*U*' is the displacement in *x*-direction while '*W*' is the displacement in *z*-direction. A plate is made up of linearly elastic and orthotropic material. A plate is thin, moderately thick or thick and obeys hook's law of plane strain problem. A load is applied at the top surface of the plate i.e. $z = -h/2$.

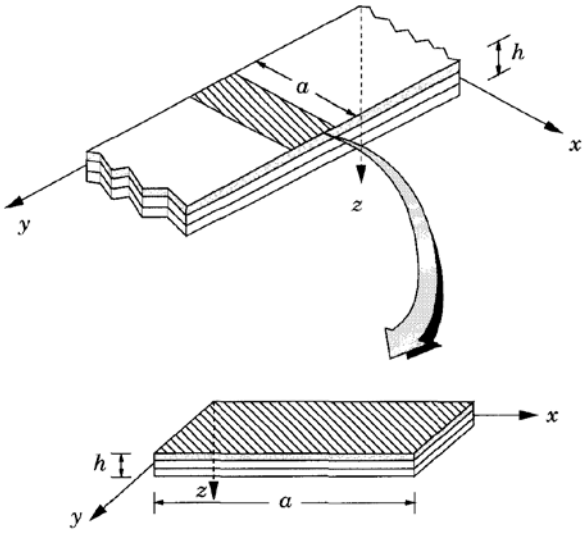

Fig. 1 – Plate geometry and co-ordinate system

1.2 The displacement field, strains, stresses and stress resultants associated with present theory

The displacement field of the *n*th-order plate theory [23] for the plate under consideration is given as below:

$$U = z\phi(x) - \frac{1}{n}\left(\frac{2}{h}\right)^{n-1} z^n \left(\phi(x) + \frac{dw(x)}{dx}\right) \qquad W = w(x) \tag{1}$$

where '*U*' and '*W*' are the axial and transverse displacement of the plate centre line. ϕ and *w* are the unknown functions to be determined. Classical plate theory and third order parabolic shear deformation theory of Reddy [5] are the modified form of present theory at $n = 1$ and $n = 3$ respectively. With the definition of strains from theory of elasticity, the linear strain-displacement relationships associated with the displacement field can now be obtained as follows:

$$\varepsilon_x = \frac{dU}{dx} = z\frac{d\phi}{dx} - C_1 z^n \left(\frac{d\phi}{dx} + \frac{d^2w}{dx^2}\right) \tag{2}$$

$$\gamma_{zx} = \frac{dU}{dz} + \frac{dW}{dx} = \left[1 - C_2 z^{n-1}\right]\left(\phi + \frac{dw}{dx}\right) \tag{3}$$

The stress strain relationship for the orthotropic plate strip can be written as:

$$\sigma_x = Q_{11}\varepsilon_x = Q_{11}\left[z\frac{d\phi}{dx} - C_1 z^n \left(\frac{d\phi}{dx} + \frac{d^2w}{dx^2}\right)\right] \tag{4}$$

$$\tau_{zx} = Q_{55}\gamma_{zx} = Q_{55}\left[1 - C_2 z^{n-1}\right]\left(\phi + \frac{dw}{dx}\right) \tag{5}$$

where,

$$C_1 = \frac{1}{n}\left(\frac{2}{h}\right)^{n-1}, C_2 = \left(\frac{2}{h}\right)^{n-1}, \quad Q_{11} = \frac{E_1}{1 - \mu_{12}\mu_{21}} \quad \text{and} \quad Q_{55} = G_{31} \tag{6}$$

where, E_1 is elastic moduli, G_{31} is shear moduli and μ_{12}, μ_{21} are Poisson's ratios. Here subscripts 1, 2, 3 correspond to *x*, *y*, *z* directions of Cartesian coordinate system respectively (These notations are taken from Jones [24]). Using equations (2) through (6), the moment resultants (M_x, P_x) and shear force resultants (R_x, Q_x) of present theory can be expressed in-terms of displacement variables as follows:

$$M_x = \int_{-h/2}^{+h/2} \sigma_x \, z \, dz = D_{11} \frac{d\phi}{dx} - C_1 E_{11} \left(\frac{d\phi}{dx} + \frac{d^2 w}{dx^2} \right) \tag{7}$$

$$P_x = \int_{-h/2}^{+h/2} \sigma_x \, z^n \, dz = E_{11} \frac{d\phi}{dx} - C_1 F_{11} \left(\frac{d\phi}{dx} + \frac{d^2 w}{dx^2} \right) \tag{8}$$

$$R_x = \int_{-h/2}^{+h/2} \tau_{xz} \, z^{n-1} \, dz = \left(E_{55} - C_2 F_{55} \right) \left(\phi + \frac{dw}{dx} \right) \tag{9}$$

$$Q_x = \int_{-h/2}^{+h/2} \tau_{xz} \, dz = \left(A_{55} - C_2 E_{55} \right) \left(\phi + \frac{dw}{dx} \right) \tag{10}$$

1.3 Governing equations and boundary conditions of present theory

The governing equations and associated boundary conditions for the orthotropic plate strip under cylindrical bending can be obtained using following principle of virtual work.

$$\int_{-h/2}^{h/2} \int_0^a \left[\sigma_x \, \delta\varepsilon_x + \tau_{zx} \, \delta\gamma_{zx} \right] dx \, dz - \int_0^a q(x) \, \delta w \, dx = 0 \tag{11}$$

Substituting strains and stresses from Eqs. (2) - (6) into the Eq. (11) we get,

$$\begin{aligned} &\int_{-h/2}^{h/2} \int_0^a Q_{11} \left[z \frac{d\phi}{dx} - C_1 z^n \left(\frac{d\phi}{dx} + \frac{d^2 w}{dx^2} \right) \right] \left[z \frac{d\delta\phi}{dx} - C_1 z^n \left(\frac{d\delta\phi}{dx} + \frac{d^2 \delta w}{dx^2} \right) \right] dx \, dz \\ &\int_{-h/2}^{h/2} \int_0^a Q_{55} \left[1 - C_2 z^{n-1} \right]^2 \left(\phi + \frac{dw}{dx} \right) \left(\delta\phi + \frac{d\delta w}{dx} \right) dx \, dz \\ &- \int_0^a q(x) \, \delta w \, dx = 0 \end{aligned} \tag{12}$$

After carrying out integrations with respect to *z*, the Eq. (12) leads to the following form:

$$\begin{aligned} &C_1^2 F_{11} \int_0^a \frac{d^2 w}{dx^2} \frac{d^2 \delta w}{dx^2} \, dx + \left(C_1^2 F_{11} - C_1 E_{11} \right) \int_0^a \frac{d\phi}{dx} \frac{d^2 \delta w}{dx^2} \, dx \\ &+ \left(A_{55} - 2C_2 E_{55} + C_2^2 F_{55} \right) \left[\int_0^a \frac{dw}{dx} \frac{d\delta w}{dx} \, dx + \int_0^a \phi \frac{d\delta w}{dx} \, dx \right] \\ &+ \left(C_1^2 F_{11} - C_1 E_{11} \right) \int_0^a \frac{d^2 w}{dx^2} \frac{d\delta\phi}{dx} \, dx + \left(D_{11} - 2C_1 E_{11} + C_1^2 F_{11} \right) \int_0^a \frac{d\phi}{dx} \frac{d\delta\phi}{dx} \, dx \\ &+ \left(A_{55} - 2C_2 E_{55} + C_2^2 F_{55} \right) \left[\int_0^a \phi \, \delta\phi \, dx + \int_0^a \frac{dw}{dx} \delta\phi \, dx \right] - \int_0^a q(x) \, \delta w \, dx = 0 \end{aligned} \tag{13}$$

Integrating the Eq. (13) by parts and collecting the coefficients of δw and $\delta\phi$ to obtain governing equations and boundary conditions. The governing equations and associated boundary conditions in-terms of stress resultants are as follows:

The governing equations:

$$\begin{aligned} &\delta w: \quad C_2 \frac{dR_x}{dx} - \frac{dQ_x}{dx} - C_1 \frac{d^2 P_x}{dx^2} = q \\ &\delta\phi: \quad \frac{dM_x}{dx} - C_1 \frac{dP_x}{dx} - Q_x + C_2 R_x = 0 \end{aligned} \tag{14}$$

The boundary conditions at edges $x = 0$ and $x = a$:

$$\begin{aligned}
&\text{Either} \quad C_2 R_x - Q_x + C_1 \frac{dP_x}{dx} = 0 \quad \text{or} \quad w = 0 \\
&\text{Either} \quad P_x = 0 \quad \text{or} \quad \frac{dw}{dx} = 0 \\
&\text{Either} \quad M_x - C_1 P_x = 0 \quad \text{or} \quad \phi = 0
\end{aligned} \tag{15}$$

Substitution of stress resultants from equations (7)-(10) into a set of governing equations (14) and boundary conditions (15) gives the following governing equations and boundary conditions in-terms of displacement variables.

The governing equations in-terms of displacement variables are as follows:

$$\delta w: \quad C_1^2 F_{11} \frac{d^4 w}{dx^4} - \left(A_{55} - 2C_2 E_{55} + C_2^2 F_{55}\right) \frac{d^2 w}{dx^2} - \left(A_{55} - 2C_2 E_{55} + C_2^2 F_{55}\right) \frac{d\phi}{dx} + \left(C_1^2 F_{11} - C_1 E_{11}\right) \frac{d^3 \phi}{dx^3} = q \tag{16}$$

$$\delta\phi: \quad \left(C_1 E_{11} - C_1^2 F_{11}\right) \frac{d^3 w}{dx^3} + \left(A_{55} - 2C_2 E_{55} + C_2^2 F_{55}\right) \frac{dw}{dx} - \left(D_{11} - 2C_1 E_{11} + C_1^2 F_{11}\right) \frac{d^2 \phi}{dx^2} + \left(A_{55} - 2C_2 E_{55} + C_2^2 F_{55}\right) \phi = 0 \tag{17}$$

The boundary conditions in-terms of displacement variables at edges $x = 0$ and $x = a$ areas follows:

$$\begin{aligned}
&\text{Either} \quad C_1^2 F_{11} \frac{d^3 w}{dx^3} - \left(C_2^2 F_{55} - 2C_2 E_{55} + A_{55}\right)\left(\frac{dw}{dx} + \phi\right) + \left(C_1^2 F_{11} - C_1 E_{11}\right) \frac{d^2 \phi}{dx^2} = 0 \quad \text{or} \quad w = 0 \\
&\text{Either} \quad \left(C_1 F_{11} - E_{11}\right) \frac{d\phi}{dx} + C_1 F_{11} \frac{d^2 w}{dx^2} = 0 = 0 \quad \text{or} \quad \frac{dw}{dx} = 0 \\
&\text{Either} \quad \left(C_1^2 F_{11} - C_1 E_{11}\right)\left(\frac{d^2 w}{dx^2} + \frac{d\phi}{dx}\right) + \left(D_{11} - C_1 E_{11}\right) \frac{d\phi}{dx} = 0 \quad \text{or} \quad \phi = 0
\end{aligned} \tag{18}$$

Stiffness coefficients (A_{11}, F_{11}, D_{11}, E_{11}, A_{11}, A_{55}, E_{55}, F_{55}) appeared in the governing equations and boundary conditions are as follows:

$$\begin{aligned}
&\left(A_{11}, A_{55}\right) = \left(Q_{11}, Q_{55}\right) \int_{-h/2}^{h/2} dz; \quad F_{11} = Q_{11} \int_{-h/2}^{h/2} z^{2n} dz; \quad D_{11} = Q_{11} \int_{-h/2}^{h/2} z^2 dz; \\
&E_{11} = Q_{11} \int_{-h/2}^{h/2} z^{n+1} dz; \quad E_{55} = Q_{55} \int_{-h/2}^{h/2} z^{n-1} dz; \quad F_{55} = Q_{55} \int_{-h/2}^{h/2} z^{2n-2} dz
\end{aligned} \tag{19}$$

2 Cylindrical bending of orthotropic plate

An orthotropic plate as shown in Fig. 1 is considered for the detail numerical study.The plate is subjected to transverse sinusoidal load;$q(x)$ acts normally and downwards on its top lateral plane ($z = -h/2$) but is independent of the y co-ordinate (see Fig. 2).

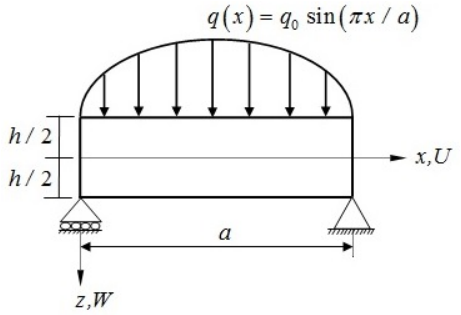

Fig. 2 – Simply supported plate subjected to sinusoidal load at top surface

Due to the symmetries involved in both the geometrical and loading characteristics, both the displacement components w, ϕ are independent of the y co-ordinate. The boundary conditions of simply supported orthotropic plates are as follows:

$$\text{at } x=0,\ x=a: \quad w=0, \quad P_x=0, \quad M_x - C_1 P_x = 0 \tag{20}$$

The following is the form assumed for $q(x)$, $w(x)$ and $\phi(x)$ satisfying the boundary conditions perfectly for a plate with all edges simply supported:

$$q(x)=q_0 \sin\frac{\pi x}{a}, \quad w(x)=w_1 \sin\frac{\pi x}{a} \quad \text{and} \quad \phi(x)=\phi_1 \cos\frac{\pi x}{a} \tag{21}$$

where q_0 is the maximum intensity of sinusoidally distributed load and w_1 and ϕ_1 are the unknown coefficients to be determine. Substitution of Eq.(21)into governing equations (16) and (17) leads to the following matrix form:

$$\begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{Bmatrix} w_1 \\ \phi_1 \end{Bmatrix} = \begin{Bmatrix} q_0 \\ 0 \end{Bmatrix} \tag{22}$$

where,

$$\begin{aligned} K_{11} &= C_1^2 F_{11} \frac{\pi^4}{a^4} + \left(A_{55} - 2C_2 E_{55} + C_2^2 F_{55}\right)\frac{\pi^2}{a^2} \\ K_{12} = K_{21} &= \left(C_1^2 F_{11} - C_1 E_{11}\right)\frac{\pi^3}{a^3} + \left(A_{55} - 2C_2 E_{55} + C_2^2 F_{55}\right)\frac{\pi}{a} \\ K_{22} &= \left(D_{11} - 2C_1 E_{11} + C_1^2 F_{11}\right)\frac{\pi^2}{a^2} + \left(A_{55} - 2C_2 E_{55} + C_2^2 F_{55}\right) \end{aligned} \tag{23}$$

Values of w_1 and ϕ_1 are obtained by solving Eq. (22). Having obtained the values of w_1 and ϕ_1 one can then calculate all the displacement and stress components within the plate strip.

3 Numerical results and discussion

3.1 Numerical results

The results are obtained for different modular ratios (E_1/E_2) and 'h/a' ratios of plate. The results obtained are presented in Tables 1 through 4. The results provided by Pagano [9] available in the literature are used as a basis for comparison of results obtained by present theory. The results present by Ghugal and Sayyad [22] also used for the comparison. The results obtained for displacements and stresses are presented in the following normalized form.

$$\bar{u} = \frac{u\, E_2}{q\, h}; \quad \bar{w} = \frac{w100\, h^3\, E_2}{q\, a^4}; \quad \bar{\sigma}_x = \frac{\sigma_x\, h^2}{q a^2}; \quad \bar{\tau}_{zx} = \frac{\tau_{zx}\, h}{qa} \tag{24}$$

The following material properties given by Pagano [9] are used to obtain the numerical results.

$$E_1 = \text{open}, \quad E_2 = 1\times10^6 \text{ psi}\left(7\,\text{GPa}\right), \quad G_{31} = 0.5\,E_2, \quad \mu_{12} = 0.25 \quad \text{and} \quad \mu_{21} = \frac{E_2\,\mu_{12}}{E_1} \tag{25}$$

The percentage error in results obtained by present theory with respect to the corresponding results obtained by exact theory of elasticity solution is calculated as follows:

$$\%\ \text{Error} = \frac{\text{Present Theroy-Exact Solution}}{\text{Exact Solution}} \times 100 \tag{26}$$

Table 1-Comparison of axial displacement $\bar{u}$ at (x = 0, z = $\pm$ h/2) for orthotropic plate strip

h/a	Theory	Model	Modular Ratios (E_1 / E_2)					
			2	5	10	25	40	50
0.15	Present	n = 1	27.7721	11.3239	5.6978	2.2877	1.4312	1.1453
	Present	n = 3	28.1961	11.7471	6.1199	2.7066	1.8468	1.5588
	Present	n = 5	28.0839	11.6354	6.0090	2.5981	1.7407	1.4542
	Present	n = 7	28.0154	11.5670	5.9408	2.5305	1.6736	1.3875
	Present	n = 9	27.9708	11.5226	5.8964	2.4862	1.6295	1.3436
	Ref. [22]	TSDT	27.9988	11.6977	6.1124	2.7218	1.8667	1.5799
	Ref. [1]	FSDT	27.7721	11.3239	5.6978	2.2877	1.4312	1.1453
0.25	Present	n = 1	5.9988	2.4460	1.2307	0.4941	0.3091	0.2474
	Present	n = 3	6.2527	2.6987	1.4817	0.7398	0.5496	0.4846
	Present	n = 5	6.1857	2.6326	1.4168	0.6788	0.4924	0.4298
	Present	n = 7	6.1447	2.5917	1.3763	0.6393	0.4537	0.3916
	Present	n = 9	6.1180	2.5651	1.3498	0.6130	0.4278	0.3659
	Ref. [22]	TSDT	6.1464	2.6752	1.4821	0.7523	0.5638	0.4988
	Ref. [1]	FSDT	5.9988	2.4460	1.2307	0.4941	0.3091	0.2474
0.35	Present	n = 1	2.1861	0.8914	0.4485	0.1801	0.1127	0.0902
	Present	n = 3	2.3670	1.0707	0.6253	0.3497	0.2757	0.2491
	Present	n = 5	2.3195	1.0243	0.5808	0.3104	0.2411	0.2174
	Present	n = 7	2.2903	0.9954	0.5523	0.2831	0.2150	0.1921
	Present	n = 9	2.2713	0.9764	0.5335	0.2647	0.1970	0.1743
	Ref. [22]	TSDT	2.2982	1.0595	0.6304	0.3618	0.2877	0.2603
	Ref. [1]	FSDT	2.1861	0.8914	0.4485	0.1801	0.1127	0.0902

Table 2-Comparison of transverse displacement $\bar{w}$ at (x = a/ 2, z = 0) for orthotropic plate

h/a	Theory	Model	Modular Ratios(E_1 / E_2)											
			2	% Error	5	% Error	10	% Error	25	% Error	40	% Error	50	% Error
0.15	Present	n = 1	5.9671	-7.91	2.4330	-17.9	1.2242	-30.4	0.4915	-52.1	0.3075	-63.4	0.2461	-68.3
	Present	n = 3	6.5135	0.52	2.9787	0.49	1.7684	0.45	1.0315	0.35	0.8433	0.20	0.7791	0.06
	Present	n = 5	6.5027	0.35	2.9684	0.14	1.7592	-0.07	1.0250	-0.28	0.8395	-0.25	0.7771	-0.19
	Present	n = 7	6.4896	0.15	2.9554	-0.30	1.7464	-0.80	1.0131	-1.44	0.8284	-1.57	0.7665	-1.55
	Present	n = 9	6.4794	-0.01	2.9453	-0.64	1.7364	-1.37	1.0034	-2.38	0.8190	-2.69	0.7574	-2.72
	Ref.[22]	TSDT	6.4799	0.00	2.9650	0.03	1.7601	-0.02	1.0253	-0.25	0.8369	-0.56	0.7723	-0.81
	Ref. [1]	FSDT	6.4230	-0.88	2.8890	-2.54	1.6802	-4.56	0.9475	-7.82	0.7634	-9.29	0.7020	-9.84
	Ref. [9]	Exact	6.4799	0.00	2.9642	0.00	1.7605	0.00	1.0279	0.00	0.8416	0.00	0.7786	0.00
0.25	Present	n = 1	5.9671	-19.1	2.4330	-37.5	1.2242	-54.5	0.4915	-74.6	0.3075	-82.4	0.2461	-85.3
	Present	n = 3	7.4823	1.38	3.9416	1.23	2.7219	1.06	1.9574	0.79	1.7429	-0.46	1.6618	-1.16
	Present	n = 5	7.4543	1.00	3.9180	0.62	2.7053	0.45	1.9614	1.00	1.7662	0.87	1.6975	0.96
	Present	n = 7	7.4183	0.51	3.8832	-0.27	2.6727	-0.76	1.9349	-0.37	1.7458	-0.29	1.6811	-0.01
	Present	n = 9	7.3903	0.13	3.8557	-0.98	2.6459	-1.76	1.9106	-1.62	1.7238	-1.55	1.6606	-1.23
	Ref.[22]	TSDT	7.3851	0.06	3.8943	0.02	2.6869	-0.24	1.9233	-0.96	1.7040	-2.68	1.6194	-3.68
	Ref. [1]	FSDT	7.2336	-1.99	3.6996	-4.98	2.4907	-7.52	1.7580	-9.47	1.5740	-10.1	1.5126	-10.0
	Ref. [9]	Exact	7.3805	0.00	3.8937	0.00	2.6933	0.00	1.9420	0.00	1.7509	0.00	1.6813	0.00
0.35	Present	n = 1	5.9675	-31.4	2.4332	-53.6	1.2243	-69.7	0.4916	-84.9	0.3075	-89.9	0.2461	-91.7
	Present	n = 3	8.9289	2.57	5.3694	2.21	4.1194	1.74	3.2701	-0.11	2.9785	-2.48	2.8499	-4.18
	Present	n = 5	8.8796	2.01	5.3366	1.58	4.1132	1.59	3.3378	1.96	3.1123	1.90	3.0239	1.67
	Present	n = 7	8.8107	1.22	5.2725	0.36	4.0570	0.20	3.3049	0.95	3.1017	1.56	3.0277	1.80
	Present	n = 9	8.7563	0.59	5.2200	-0.64	4.0076	-1.02	3.2646	-0.28	3.0703	0.53	3.0021	0.94
	Ref.[22]	TSDT	8.7147	0.11	5.2459	-0.14	4.0142	-0.86	3.1529	-3.69	2.8419	-6.95	2.7013	-9.18
	Ref. [1]	FSDT	8.4495	-2.93	4.9154	-6.43	3.7066	-8.45	2.9739	-9.16	2.7899	-8.65	2.7284	-8.26
	Ref. [9]	Exact	8.7049	0.00	5.2534	0.00	4.0489	0.00	3.2737	0.00	3.0542	0.00	2.9742	0.00

Table 3-Comparison of in-plane normal stress $\bar{\sigma}_x$ at ($x = a/2, z = \pm h/2$) for orthotropic plate

h/a	Theory	Model	Modular Ratios(E_1/E_2)											
			2	% Error	5	% Error	10	% Error	25	% Error	40	% Error	50	% Error
0.15	Present	n = 1	27.0190	-1.35	27.0190	-3.53	27.0190	-6.88	27.0190	-15.1	27.0190	-21.64	27.0190	-25.2
	Present	n = 3	27.4314	0.16	28.0289	0.07	29.0209	0.02	31.9660	0.34	34.8653	1.12	36.7733	1.79
	Present	n = 5	27.3223	-0.24	27.7623	-0.88	28.4949	-1.79	30.6846	-3.68	32.8623	-4.69	34.3075	-5.03
	Present	n = 7	27.2556	-0.48	27.5992	-1.46	28.1716	-2.90	29.8861	-6.19	31.5962	-8.37	32.7339	-9.39
	Present	n = 9	27.2123	-0.64	27.4930	-1.84	27.9609	-3.63	29.3634	-7.83	30.7640	-10.78	31.6966	-12.2
	Ref. [22]	TSDT	27.6213	0.85	28.1296	0.43	29.1603	0.50	32.2959	1.37	35.3851	2.62	37.4132	3.57
	Ref. [1]	FSDT	27.0190	-1.35	27.0190	-3.53	27.0190	-6.88	27.0190	-15.1	27.0190	-21.64	27.0190	-25.2
	Ref. [9]	Exact	27.3877	0.00	28.0079	0.00	29.0142	0.00	31.8585	0.00	34.4807	0.00	36.1252	0.00
0.25	Present	n = 1	9.7268	-3.98	9.7268	-9.77	9.7268	-17.4	9.7268	-32.5	9.7268	-41.46	9.7268	-45.7
	Present	n = 3	10.1385	0.08	10.7320	-0.44	11.7101	-0.67	14.5614	1.06	17.2941	4.09	19.0536	6.28
	Present	n = 5	10.0299	-0.99	10.4689	-2.88	11.1978	-5.01	13.3626	-7.26	15.4946	-6.74	16.8981	-5.74
	Present	n = 7	9.9634	-1.65	10.3066	-4.39	10.8777	-7.73	12.5831	-12.6	14.2766	-14.07	15.3991	-14.1
	Present	n = 9	9.9201	-2.08	10.2007	-5.37	10.6680	-9.51	12.0665	-16.2	13.4597	-18.99	14.3856	-19.7
	Ref. [22]	TSDT	10.2064	0.75	10.8100	0.28	11.8656	0.65	14.9471	3.73	17.8727	7.57	19.7404	10.1
	Ref. [1]	FSDT	9.7268	-3.98	9.7268	-9.77	9.7268	-17.4	9.7268	-32.5	9.7268	-41.46	9.7268	-45.7
	Ref. [9]	Exact	10.1305	0.00	10.7799	0.00	11.7885	0.00	14.4091	0.00	16.6149	0.00	17.9270	0.00
0.35	Present	n = 1	4.9627	-8.37	4.9627	-18.6	4.9627	-30.1	4.9627	-47.7	4.9627	-56.35	4.9627	-60.1
	Present	n = 3	5.3731	-0.79	5.9608	-2.28	6.9186	-2.58	9.6379	1.53	12.1467	6.83	13.7140	10.1
	Present	n = 5	5.2654	-2.78	5.7029	-6.51	6.4264	-9.51	8.5545	-9.88	10.6206	-6.60	11.9650	-3.94
	Present	n = 7	5.1991	-4.00	5.5418	-9.15	6.1109	-13.9	7.8030	-17.8	9.4720	-16.70	10.5721	-15.1
	Present	n = 9	5.1559	-4.80	5.4362	-10.8	5.9027	-16.8	7.2953	-23.1	8.6776	-23.68	9.5935	-22.9
	Ref. [22]	TSDT	5.4181	0.04	6.0565	-0.71	7.1197	0.25	10.1023	6.42	12.7942	12.52	14.4469	15.9
	Ref. [1]	FSDT	4.9627	-8.37	4.9627	-18.6	4.9627	-30.2	4.9627	-47.7	4.9627	-56.35	4.9627	-60.1
	Ref. [9]	Exact	5.4160	0.00	6.0998	0.00	7.1020	0.00	9.4925	0.00	11.3705	0.00	12.4553	0.00

Table 4-Comparison of transverse stress $\bar{\tau}_{zx}$ at ($x = 0, z = 0$) for orthotropic plate

h/a	Theory	Model	Modular Ratios(E_1/E_2)											
			2	% Error	5	% Error	10	% Error	25	% Error	40	% Error	50	% Error
0.15	Present	n = 1	0.0000	-100	0.0000	-100	0.0000	-100	0.0000	-100	0.0000	-100	0.0000	-100
	Present	n = 3	3.1796	0.21	3.1746	0.61	3.1663	1.22	3.1415	2.86	3.1171	4.22	3.1010	4.99
	Present	n = 5	2.7273	-14.04	2.7258	-13.62	2.7233	-12.94	2.7158	-11.08	2.7083	-9.45	2.7033	-8.47
	Present	n = 7	2.5539	-19.51	2.5532	-19.09	2.5522	-18.41	2.5489	-16.54	2.5457	-14.89	2.5435	-13.88
	Present	n = 9	2.4594	-22.48	2.4591	-22.07	2.4586	-21.40	2.4569	-19.56	2.4552	-17.91	2.4541	-16.91
	Ref. [22]	TSDT	3.2791	3.35	3.2739	3.75	3.2636	4.33	3.2321	5.82	3.2010	7.02	3.1806	7.69
	Ref. [1]	FSDT	2.1221	-33.12	2.1221	-32.75	2.1221	-32.16	2.1221	-30.52	2.1221	-29.05	2.1221	-28.15
	Ref. [9]	Exact	3.1728	0.00	3.1555	0.00	3.1281	0.00	3.0542	0.00	2.9910	0.00	2.9536	0.00
0.25	Present	n = 1	0.0000	-100	0.0000	-100	0.0000	-100	0.0000	-100	0.0000	-100	0.0000	-100
	Present	n = 3	1.9041	0.61	1.8958	1.65	1.8820	3.15	1.8421	6.39	1.8037	8.23	1.7791	8.93
	Present	n = 5	1.6353	-13.60	1.6328	-12.46	1.6286	-10.74	1.6162	-6.65	1.6039	-3.76	1.5959	-2.28
	Present	n = 7	1.5319	-19.06	1.5308	-17.92	1.5290	-16.20	1.5236	-12.00	1.5183	-8.89	1.5147	-7.26
	Present	n = 9	1.4754	-22.04	1.4749	-20.92	1.4739	-19.22	1.4711	-15.03	1.4684	-11.89	1.4665	-10.21
	Ref. [22]	TSDT	1.9630	3.72	1.9529	4.71	1.9352	6.07	1.8836	8.79	1.8346	10.09	1.8033	10.42
	Ref. [1]	FSDT	1.2732	-32.73	1.2732	-31.74	1.2732	-30.22	1.2732	-26.46	1.2732	-23.60	1.2732	-22.04
	Ref. [9]	Exact	1.8926	0.00	1.8651	0.00	1.8245	0.00	1.7314	0.00	1.6665	0.00	1.6332	0.00
0.35	Present	n = 1	0.0000	-100	0.0000	-100	0.0000	-100	0.0000	-100	0.0000	-100	0.0000	-100
	Present	n = 3	1.3561	1.22	1.3446	3.14	1.3258	5.59	1.2724	8.68	1.2231	9.99	1.1924	9.56
	Present	n = 5	1.1669	-12.90	1.1634	-10.76	1.1575	-7.81	1.1405	-2.59	1.1239	1.07	1.1131	2.27
	Present	n = 7	1.0937	-18.37	1.0922	-16.22	1.0896	-13.22	1.0822	-7.57	1.0748	-3.35	1.0700	-1.69
	Present	n = 9	1.0536	-21.36	1.0528	-19.25	1.0515	-16.26	1.0476	-10.52	1.0438	-6.13	1.0412	-4.34
	Ref. [22]	TSDT	1.3972	4.28	1.3822	6.02	1.3573	8.10	1.2874	9.96	1.2242	10.09	1.1854	8.91
	Ref. [1]	FSDT	0.9095	-32.12	0.9095	-30.24	0.9095	-27.56	0.9095	-22.32	0.9095	-18.21	0.9095	-16.44
	Ref. [9]	Exact	1.3398	0.00	1.3037	0.00	1.2556	0.00	1.1708	0.00	1.1120	0.00	1.0884	0.00

3.2 Discussion of results

Table 1 show the comparison of in-plane displacement for simply supported plate infinitely long in *y*-direction under sinusoidally distributed load. Since exact elasticity solution is not available in the literature, results obtained by present theory for in-plane displacement are compared with those presented by Ghugal and Sayyad [22] and found agree well for n = 3, 5, 7 and 9 for all aspect ratios (h/a) and modular ratios(E_1/E_2). At n = 1, present theory underestimates the value of in-plane displacement. Fig. 3 shows the through thickness distribution of in-plane displacement ($\bar{u}$) for modular ratio 25 and aspect ratio 4.

In Table 2 the non-dimensional value of transverse displacement ($\bar{w}$) is presented for different values of aspect ratios and

modular ratios. From the results it is observed that, at n = 7 and n = 9, the present theory provides accurate values of transverse displacement whereas at n =1, it underestimates the values of transverse displacement for all'h/a' and 'E_1/E_2'.The examination of Table 2 also reveals that, transverse displacement decreases with respect to increases in modular ratios.Due to consideration of effect of transverse normal strain/stress, the transverse displacement predicted by trigonometric shear deformation theory Ghugal and Sayyad [22] is more accurate for all aspect ratios and modular ratios. First order shear deformation theory (FSDT)of Mindlin [1] underestimates the value of transverse displacement for all aspect ratios and modular ratios. Maximum transverse displacement predicted by present theory at n = 1 is independent of aspect ratios.

Table 3 shows comparison of in-plane normal stress ($\bar{\sigma}_x$) for several values of modular ratios and aspect ratios. The examination of Table 3 reveals that, in-plane normal stress predicted by present theory for n =3 and n =5 is in good agreement with that of exact solution for all aspect ratios and modular ratios whereas values predicted by present theory at n = 1 are on lower side. In-plane normal stress predicted by FSDT and CPT (n = 1) are identical. Fig.4 shows the through thickness distribution of in-plane normal stress for modular ratios 25 and aspect ratio 4.Table 4 shows comparison of transverse shear stress obtained by present theory and other theories reported in the literature. It is observed from Table 4 that, Present theory predicts the zero transverse shear stress at n = 1 whereas values obtained at n = 3 are in close agreement with exact solution. The present theory under predict the transverse shear stress at n = 5, n = 7 and n = 9. Fig. 5 shows through thickness distribution of transverse shear stress for modular ratios 25 and aspect ratio 4.

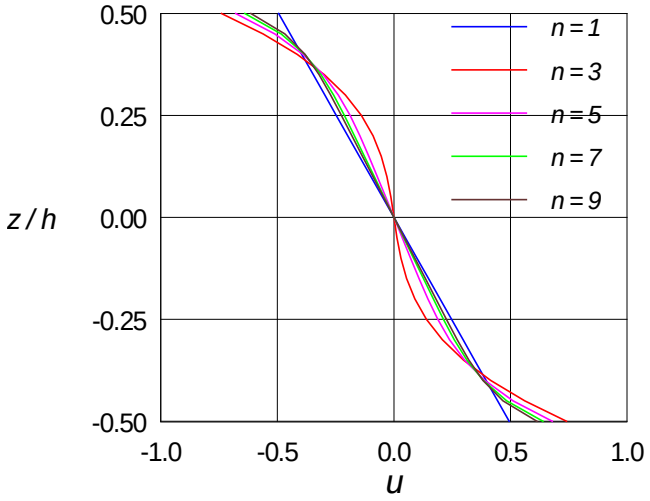

Fig. 3 – Through thickness distribution of in-plane displacement ($\bar{u}$) (E_1/E_2= 25 and h/a = 0.25)

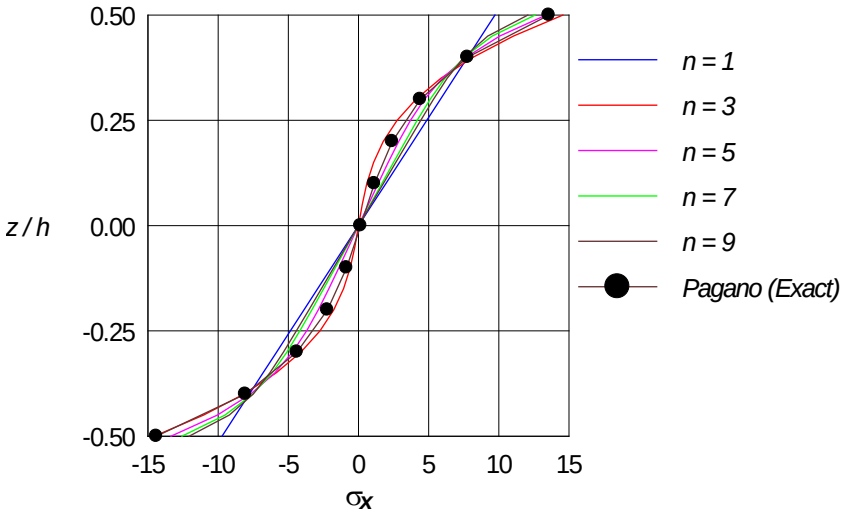

Fig. 4 – Through thickness distribution of in-plane normal stress ($\bar{\sigma}_x$) (E_1/E_2= 25 and h/a = 0.25)

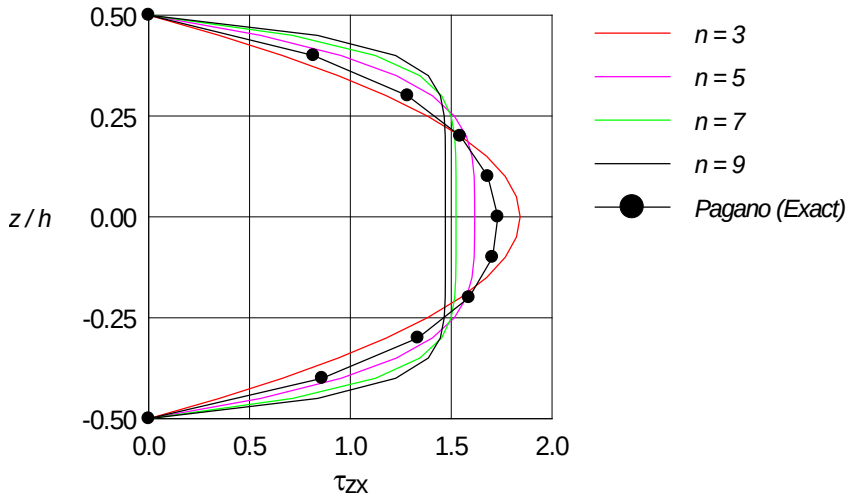

Fig. 5 – Through thickness distribution of transverse shear stress ($\bar{\tau}_{zx}$)(E_1/E_2= 25 and h/a = 0.25)

4 Conclusions

In this paper, *n*th-order plate theory developed by Xiang and Kang [23] is applied for the one dimensional analysis of orthotropic plates.Classical plate theory (n = 1) and parabolic shear deformation theory of Reddy (n = 3) can be considered as special cases of present theory. The governing differential equations and associated boundary conditions associated with present theory are variationally consistent and obviate the need of shear correction factor. From the numerical results and discussion following conclusions are drawn.

a. Results of other refined shear deformation theories for the in-plane displacement are in tune with the results of the present theory at n = 3, 5, 7 and 9. At n = 1, present theory underestimate the value of in-plane displacement.
b. The use of present theory at n = 3, 5 and 7 gives excellent results in respect of transverse displacements. The transverse displacement reduces with respect to increase in modular ratios and '*h*/*a*' ratios.
c. The results of in-plane normal stress obtained by the present theory at n = 3 and 5are in good agreement with those of exact solution and those of other refind theories.
d. As the value of n increases, the error in the results of transverse shear stress increases. At n = 3 and 5, present theory predicts excellent values of transverse shear stress whereas at n = 1 it predicts zero transverse shear stress.

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