Singular Spectrum Analysis of Global Mean Sea Level Variations

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ملخص : إن موضوع هذا المقال يتمحور حول تحليل السلاسل الزمنية للإختلافات المتوسطة لإرتفاع سطح البحر(MSLA) للمرفاعية الفضائية بإستخدام الطيف المفرد (SSA).
إن الهدف من هذه الدراسة هو تمييز وتحديد كمية التغير على المدى البعيد لمستوى سطح البحر، المعطيات المستعملة أثناء المعالجة مكونة من سلسلة أسبوعية من التغيرات المتوسطة لإرتفاع سطح البحر الناتجة عن الأقمار الصناعية للمرفاعية الفضائية، ما بين 1993 إلى 2009.
أظهرت النتائج المتحصل عليها أن SAA تتكيف بشكل جيد للإستخلاص المتزامن مع الإتجاه الغير خطي و الإشارات الفصلية المحتواة داخل السلاسل الزمنية (MSLA). الإتجاه و الإشارة السنوية و النصف سنوية يمثلون بالترتيب 93.36 %، 6.01 % و 0.34 % من الإشارة الإجمالية. إن الإتجاه المقدر بـ 2.88 مم/ سنة من المستوى العام لسطح البحر هو مشابه للنتائج الحديثة التي قام بنشرها AVISO المرفاعية على إتجاه المستوى العام لسطح البحر و الذي يقدر بحوالي 2.92 مم/سنة.

الكلمات الأساسية : المستوى المتوسط العام لسطح البحر، تحليل السلاسل الزمنية، تحليل الطيف المفرد (SSA)، جهاز قياس الفترات.

Résumé : Le thème développé à travers cet article porte sur l'analyse des séries temporelles d'anomalies moyennes de hauteur de la mer (MSLA) d'altimétrie spatiale par le spectre singulier (SSA). Le but de cette étude est de caractériser et de quantifier la variabilité à long terme du niveau de la mer. Les données utilisées lors du traitement sont constituées d'une série hebdomadaire d'anomalies moyennes de hauteur de la mer issues des satellites d'altimétrie spatiale, conduite entre 1993 et 2009. Les résultats obtenus ont montré que la SAA est bien adaptée pour l'extraction simultanée de la tendance non linéaire et des signaux saisonniers contenus dans les séries temporelles MSLA. La tendance, le signal annuel et semi annuel représentent respectivement 93.36%, 6.01% et 0.34% du signal total. La tendance estimée de 2.88 mm/ans de niveau global de la mer est similaire aux récents résultats publiés par AVISO Altimétrie sur la tendance de niveau global de la mer qui est d'environs 2.92 mm/ans.

Mots-clés : Le niveau moyen global de la mer, l'analyse des séries chronologiques, l'analyse du spectre singulier (SSA), périodogramme.

Abstract : The theme developed through this article focuses on the analysis of time series of Mean Sea Level Anomalies (MSLA) of space altimetry using the singular spectrum Analysis (SSA). The aim of this study is to characterize and quantify long-term variability of the sea level. The data used are the weekly series of MSLA, provided by space altimetry satellites from1993 to 2009. The obtained results show that the SAA is well adapted for the simultaneous extraction of the nonlinear trend and the seasonal signals contained in the MSLA time series. The trend, the annual and semi signals represent respectively 93.36%, 6.01% and 0.34% of the total signal. The global sea level trend estimate of 2.88 mm/yr is similar to recently published results of global sea-level rise by AVISO Altimetry which is about 2.92 mm/yr.

Keywords : Global mean sea level, Time series analysis, Singular Spectrum Analysis (SSA), Periodogram.

1. Introduction

Mean sea level is a fundamental geophysical parameter in meteorological and oceanographic studies, particularly long-term sea level variation and its relation to global climate changes. Long-term sea level variations are primarily measured by two techniques: tide gauges and altimetry satellites. Precise monitoring of changes in the mean level of the oceans, particularly through the use of altimetry satellites (Fu and Chelton, 2001; Picaut and Busalacchi, 2001), is vitally important, for understanding not just the climate but also the socioeconomic consequences of any rise in sea level. Thanks to the global, continuous and repetitive set of altimetric observations allowed by the successive launches of ERS-2, Topex/Poseidon, Jason-1, Envisat, GFO and Jason-2. These multiple altimeter missions have led to vast improvements of accuracy of altimeter measurements by cross calibration and validation between them, as well as of the mapping capability for mesoscale variability and ocean circulation (Zhang and Chen, 2006; Le Traon and Dibarboure, 2004; Leuliette et al., 2003).

The Sea level Anomaly (SLA) is generally used as precious and main indicator for development of scientific applications which aims to study the ocean variability (mesoscale circulation, seasonal variation, El Niño…). AVISO Altimetry (AVISO web site) estimates that the sea level has been rising at an average rate of about 2.92 mm per year. The majority of this rise can be attributed to the increase in temperature of the sea and the resulting thermal expansion of sea water. Additional contributions come from water sources on land such as melting snow and glaciers.

Historically, the analysis of sea-level trends has been undertaken using simple linear or polynomial least squares fitting, or low-pass fi ltering of data followed by extension to data set boundaries by some method (*Mann*, 2004), but more recently nonlinear analysis has been introduced (Barbosa et al., 2008; Jevrejeva et al., 2006; Grinsted et al., 2004; Torrence and Compo, 1998). This paper is a contribution to these methodological developments. The purpose of this paper is to assess the nonlinear trend and the seasonal components contained in the MSLA time series using SSA technique in order to estimate the long-term mean sea level change.

The SSA technique is used to extract information from time series without prior knowledge of the dynamics affecting the time series (Vautard and Ghil, 1989). It is essentially a principal components analysis in the time domain that makes a decomposition of the original series into the sum of a small number of independent and interpretable components such as a slowly varying trend, oscillatory components and a structureless noise. It can be used for solving the following problems (Hassani, 2007): 1) finding trends of different resolution; 2) smoothing; 3) extraction of seasonality components; 4) simultaneous extraction of cycles with small and large periods; 5) extraction of periodicities with varying amplitudes; 6) simultaneous extraction of complex trends and periodicities; 7) finding structure in short time series; and 8) change-point detection. Geophysicists have used SSA to analyze a wide variety of time series such as solar oscillations (Varadi et al., 1999), precipitation (Prieto et al., 1999; Wang et al. 1996), streamflow and sea-surface temperature (Robertson and Mechoso, 1998), chemical constituents of ice cores (Yiou et al., 1997), global temperature (Vautard et al., 1992 ; Allen and Smith, 1994), magnetosphere dynamics (Sharma et al., 1993), and suspended sediment concentration in an estuary (Schoellhamer, 1996).

In the first part of this study, we start with a brief description of the methodology of SSA, and in the second part, we apply this technique to the weekly global maps of Delayed-Time Sea Level Anomalies averaging week by week from January 1993 to November 2009.

2. Singular Spectrum Analysis (SSA)

The SSA method allows to extract significant components from time series (trends, periodic signals and noise) (Vautard et al., 1992; Ghil et al., 2002; Broomhead and King, 1986). The method is based on the computation of the eigenvalues and the eigenvectors of a covariance matrix C formed from the time series $\{X_t, t=1\ldots, N\}$ and the reconstruction of this time series based on a number of selected eigenvectors associated with the significant eigenvalues of the covariance matrix C. The algorithm of SSA includes the following four steps :

Step (1) : *Choice of the embedding dimension M*

The time series $\{X_t, t=1\ldots, N\}$ is embedded into a vector space of dimension M. The embedding dimension M must be sufficiently long to include the assumed periodicity of the time series without exceeding half of the length of the time series.

Step (2) : *Computation of the MxM covariance matrix C given by:*

$$C = \frac{1}{N'} D^t D$$

$$D = \begin{pmatrix} X_1 & X_2 & \cdots & X_M \\ X_2 & X_3 & \cdots & X_{M+1} \\ \vdots & \vdots & \ddots & \vdots \\ X_{N-M+1} & X_{N-M+2} & \cdots & X_N \end{pmatrix} \qquad (1)$$

Where: N is the length of the time series and N' = N – M + 1.

Step (3) : *Study of the eigenvalues of the covariance matrix C*

The M eigenvalues of the covariance matrix C once determined are ordered by decreasing value. Each eigenvalue λ_k gives the Partial Variance (PV), given by Eq. (2), of the original time series in the direction specified by the corresponding eigenvectors E_k.

$$PV(k) = \frac{\lambda_k}{\sum_{k=1}^{M} \lambda_k} \qquad (2)$$

If we arrange and plot the ordered eigenvalues, one can often distinguish an initial steep slope, representing the true signal, and a (more or less) "flat floor" representing the noise level (Vautard and Ghil, 1989). Thus the theory of the SSA allows to conclude that the:
- signal has a trend if the diagram contains an isolated eigenvalues,
- signal is periodic if there are two close eigenvalues that have the same dominant frequency,
- small eigenvalues constitute the noise of the signal.

Step (4): *Projection of the original time series onto the k-th eigenvectors and its reconstruction*

By the eigenvectors $\{E_k, 1 \le k \le M\}$, called Empirical Orthogonal Functions EOFs, we can construct the time series of length N' (N'= Nm+1) given by:

$$A_k(t) = \sum_{j=1}^{M} X(t+j-1)E_{k,j} \quad t=1, \ldots, N' \qquad (3)$$

Where: $E_{k,j}$ represents the value of the jth line corresponding to k^{th} eigenvector.
$A_k(t)$ called the k-th principal component (PC). It represents the projection of the original time series onto the k-th EOF (with $1 \le k \le M$). The sum of the power spectral of the PCs is identical to the power spectral of the time series X(t) (Vautard et al., 1992) and therefore, we can study separately the spectral contribution of the various components.
The PCs, however, have length N', not N, and do not contain phase information. In order to extend the time series to length N, it is necessary to choose a subset of $\mathcal{K}$ EOFs on which the reconstruction is based, the associated PCs are combined to form the partial reconstruction $R_\kappa(t)$ of the original time series X(t).

$$R_\kappa(t) = \begin{cases} \frac{1}{t}\sum_{k\in\kappa}\sum_{j=1}^{t} A_k(t-j+1)E_{k,j}, & 1 \le t \le M-1 \\ \frac{1}{M}\sum_{k\in\kappa}\sum_{j=1}^{M} A_k(t-j+1)E_{k,j}, & M \le t \le N' \\ \frac{1}{N-t+1}\sum_{k\in\kappa}\sum_{j=t-N+M}^{M} A_k(t-j+1)E_{k,j}, & N'+1 \le t \le N \end{cases} \qquad (4)$$

These series $R_\kappa(t)$ of length N are called the reconstructed components (RCs). They have the important property of preserving the phase of the time series; therefore X(t) and $R_\kappa(t)$ can be superimposed. No information is lost in the reconstruction process, since the sum of all individual RCs gives back the original time series.

3. Data used

For this study, we have used the weekly global maps of Delayed-Time (DT) Sea Level Anomalies (SLA) averaging week by week from January 06, 1993 to November 04, 2009 (see figure 1). These measurements obtained from different satellites/instruments (ERS-1, ERS-2, Envisat, Topex/Poseidon, GFO, Jason-1 and Jason-2) are considered through the use of the merged products from AVISO Altimetry.
AVISO global DT SLA solutions are used with 1/8 x 1/8 degrees of latitude and longitude resolutions, and are available through the AVISO site:
ftp://ftp.aviso.oceanobs.com/pub/oceano/AVISO/SSH/duacs/global/dt/upd/msla/merged/h/.
All of the standard corrections to the altimeter data were applied including removal of ocean tides and an inverted barometer correction.

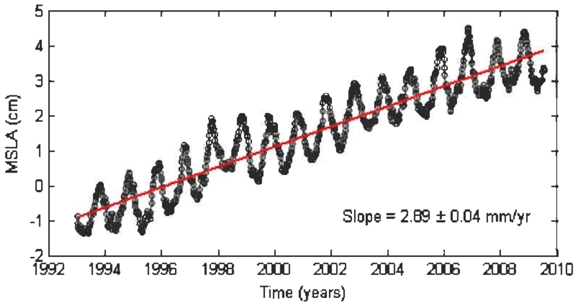

Fig. 1 MSLA time series and their linear trend obtained by least square fit.

4. Results and discussions

4.1 Trend and periodic signals

The SSA application requires a priori the choice of the embedding dimension M which depends on the periodicity of the signal. The figure 1 shows that the MSLA time series contains an evident annual signal. Therefore, in our SSA application, we have taken M=52 weeks which corresponds to one year.
The SSA of MSLA time series shows that for the embedding dimension M=52 weeks, the diagram of the eigenvalues (figure 2) shows that the first eigenvalue which is well separated from the others remaining values, indicates a signature of a dominant trend (Reconstructed Component RC 1) in the signal (figure 3) with a partial variance of 93.36%.
While, the two following eigenvalues, which are clearly detached from the 50 remaining ones, indicate a dominant annual signal RC 2-3 (figure 3) with a partial variance of 6.01%.
However, the reconstructed components RC 4-5, based on the four and the five EOFs, correspond to a semi annual signal with a partial variance of 0.34%.

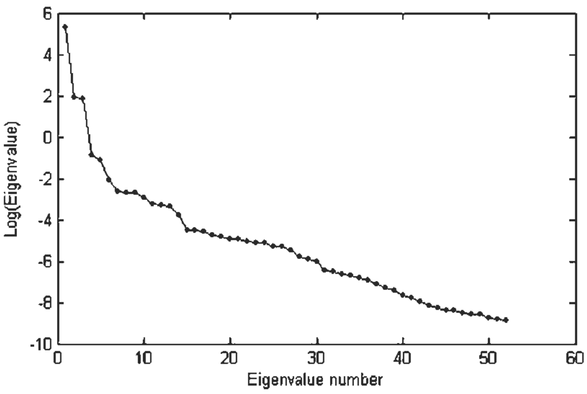

Fig. 2 Logarithms of the 52 eigenvalues.

The figure 3 shows that the amplitude of the annual signal over 2003-2004 and 2007-2008 is about 61mm which is weak relatively to the other years when the amplitude exceeds 70 mm. it reaches the minimum in April-May and the maximum in October-November.However, the semi annual signal and the remaining periodicity contained in the signal are not significant relatively to the annual signal.

Figure 4 depicts the periodogram of the reconstructed components (RC2-3 and RC 4-5). The most important peaks on the periodograms indicate the periodicities values of the annual and semi annual signals, which are about 50.82 weeks and 26.18 weeks, respectively.

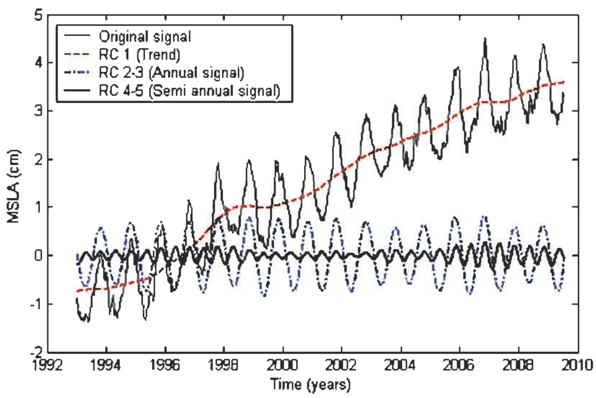

Fig. 3 Superposition of the original MSLA time series with their reconstructed components.

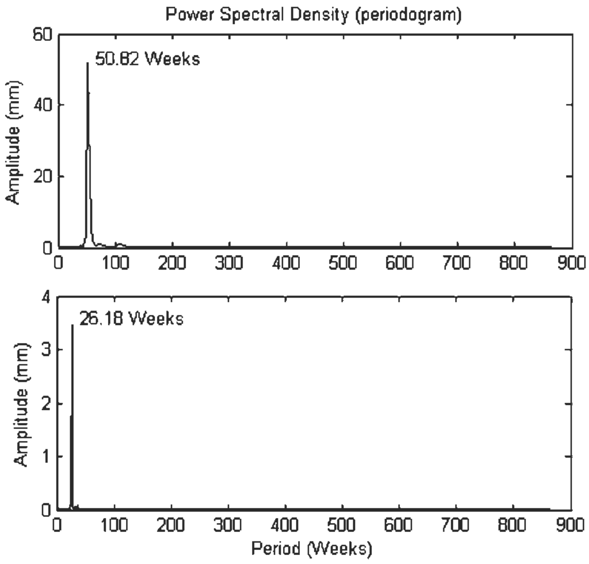

Fig. 4 Periodograms of the reconstructed components (RC 2-3 and RC 4-5).

Figure 5 illustrates the nonlinear trend (RC 1) and the MSLA time series after have been removed the seasonal signals (annual and semi annual signals). The global trend observed for the period 1993 to 2009 is about 2.88 mm/yr which is comparable with the 2.92 mm/yr sea level rise (no GIA (Glacial Isostatic Adjustment) correction, seasonal signal removed) obtained from AVISO Altimetry (AVISO web site). Between 1999 and 2007, we observe stability in the sea level evolution due to the warming of the ocean which seems to pause; its contribution to sea level rise becomes smaller. This pause occurring in the ocean warming has slowed the rise in global ocean and is partly behind the trough observed on the figure in late 2007.

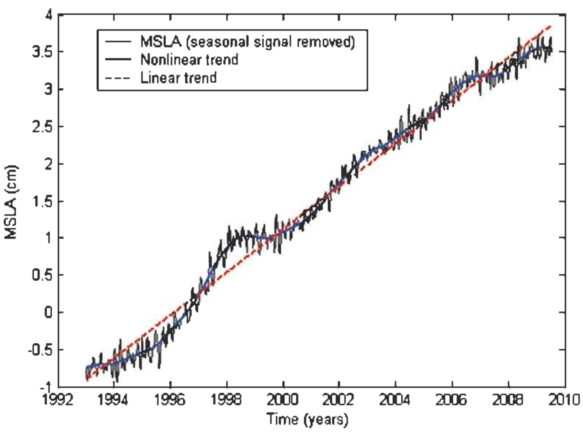

Fig. 5 MSLA time series after removed the seasonal signals (annual and semi annual signals) and the nonlinear trend (RC 1).

4.2 Noise

As we mentioned previously, the SSA allows to assess the noise affecting the time series by extracting from the initial series, the reconstructed components. The noise is characterized by much lower values that form a flat floor or a mild slope (Vautard and Ghil, 1989; Kumaresan and Tufts, 1980; Pike et al., 1984). In our case, we have taken the largest eigenvalues which correspond to trends and various oscillatory components.
The following figure 6 represents the original MSLA time series, their reconstructed components based on the eight first EOFs, and their noise. The noise represents 0.17% of the total signal, its amplitude is about 7 mm.

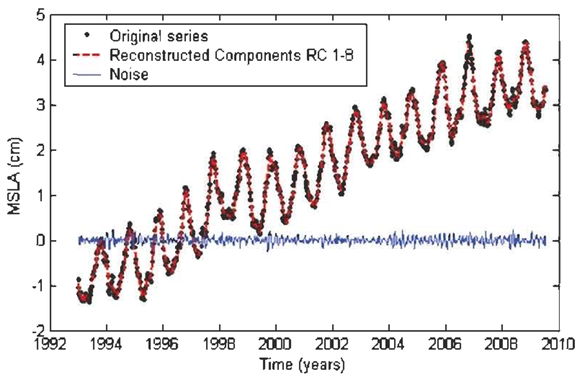

Fig. 6 Superposition of the original MSLA time series with their reconstructed components and noise.

5. Conclusion

The main purpose of this paper is to apply the SSA technique into the analysis of the MSLA time series from 1993 to 2009, in order to extract maximum information on its signal (nonlinear trend and seasonal signals) which allowed to apprehend the variability in global sea level.
The application of the SSA technique to MSLA time series permits to better extract their trend and the seasonal components. The annual and the semi annual signals represent 6.01% and 0.34% of the total signal, respectively. While the trend represents over 93% of the total signal. The obtained trend is about 2.88 mm/yr which is comparable with the 2.92 mm/yr sea level rise (no GIA correction, seasonal signal removed) obtained from AVISO Altimetry. For a signal noise separation (de-noising), we have assessed the noise by extracting the reconstructed components from the initial series. The obtained results show that the noise level is very small (negligible), it represents 0.17% of the total signal and its amplitude is about 7 mm.

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