

# Determination and analysis of stations coordinates based on Starlette and Lageos-1 &-2 satellites laser ranging data

B. Gourine<sup>1</sup>, S. Kahlouche<sup>1</sup> and M.F. Belbachir<sup>2</sup>

(1): Centre des Techniques Spatiales (CTS)–Division de Géodésie Spatiale, BP 13, 31200, Arzew – Algérie

(2): Université des Sciences et de la Technologie (USTO), Faculté du Génie Electrique, Département d'Électronique, Oran - Algérie

**ملخص:** يعالج هذا العمل حساب شبكة المصلحة الدولية لقياس البعد الليزري (ILRS) القائمة ليس فقط على أرساد القمر الصناعي LAGEOS لكن أيضا على الأقمار الصناعية ذات مدار أرضي منخفض (LEO)، مثل Starlette. يمكن التحدي في الحصول على نوعية جيدة من إحدثيات المحطات بدمج بين القمر الصناعي ذات معطيات عليا و منخفضة للأقمار الصناعية. أنجزت إستعادة مدار الأقمار الصناعية المختلفة بالبرنامج GINS (GRGS, France) و معالجة المعطيات الليزرية بالبرنامج MATLO (فرنسا، OCA & IGN)، لفترة طويلة نسبيا قدرت بـ 14 سنة (بين أكتوبر 1993 إلى فيفري 2007). تم تقديم ومناقشة النتائج الأولية لتحليل دمج معطيات SLR لـ 14 سنة من أجل دراسة معالم المرجع الأرضي (TRF).

**الكلمات الأساسية:** إستعادة المدار، دمج بين الأقمار الصناعية، تقنية SLR، تحليل طيفي، دراسة التشويش.

**Résumé :** Le présent travail traite le calcul du réseau du Service International de Télémétrie Laser (ILRS) non seulement basé sur les observations des deux satellites LAGEOS mais aussi sur ceux des satellites d'Orbite terrestre basse (LEO), tel que Starlette. Le défi est d'obtenir une bonne qualité des coordonnées des stations par la combinaison inter satellite de Hautes et Basses données satellitaires. La restitution d'orbite de différents satellites est effectuée avec le logiciel GINS (GRGS, France) et le traitement des données laser est réalisé avec le logiciel MATLO (OCA & IGN, France), pour une période relativement longue de 14 années (entre octobre 1993 et février 2007). Les résultats préliminaires de l'analyse de la combinaison de données SLR de 14 années, pour étudier les repères de Référence Terrestre (TRF) sont présentés et discutés.

**Mots-clés :** restitution d'Orbite, combinaison Inter satellite, technique SLR, analyse Spectrale, étude du Bruit.

**Abstract :** The present work deals with the calculation of International Laser Ranging Service (ILRS) network not only based on observations of both LAGEOS satellites but also on those of Low Earth Orbit (LEO) satellites, such as Starlette. The challenge is to achieve good quality on stations coordinates by inter-satellite combination of High and Low satellites data. The orbit restitution of the different satellites is carried out by GINS software (GRGS, France) and the laser data processing is performed with MATLO software (OCA & IGN, France), for a relatively long period of 14 years

(between October 1993 and February 2007). The preliminary results about the analysis of 14 years SLR data combination, in order to study the Terrestrial Reference Frames (TRF) are presented and discussed.

**Key words:** Orbit restitution, Inter-satellite combination, SLR technique, Spectral analysis, Noise study.

## 1. Introduction

Satellite Laser Ranging (SLR) is one of the main spacegeodesy techniques for the establishment and the maintenance of the International Terrestrial Reference Frame (ITRF), as Very Long Baseline Interferometry (VLBI), Global Positioning System (GPS) and Doppler Orbitography Radiopositioning Integrated by Satellite (DORIS). It contributes to the frame determination by providing time series of terrestrial stations positions, i.e., Terrestrial Reference Frame (TRF), and Earth Orientation Parameters (EOPs). Generally, for such determination, only the measurements on high altitude satellites (LAGEOS-1 & LAGEOS-2, 6000 km) are used. However, computation of the laser ranging station's coordinates on the basis of data other than those from LAGEOS-1&-2 observations is desirable for the following reasons: (1) significantly increases the number of observations used for determination of the station's coordinates and EOPs, (2) permit verification of results obtained from the LAGEOS-1 &-2 data, (3) permit determination of coordinates of the stations that cannot observe LAGEOS satellites. The aim of the study is to check if the laser ranging observations of low altitude satellites such as Starlette (altitude of 800 km) can be used for a precise determination of the laser ranging stations' coordinates and to investigate the contribution of the Starlette data on the geodynamic study, during relatively a long period. Hence, the work concerns the re-computation of the network of International Laser Ranging Service (ILRS) based on both LAGEOS satellites measurements with those of Starlette over 14 years period (from October 1993 to February 2007), according to three data combination solutions, namely LAGEOS-1 (LA-1), LAGEOS-1&-2 (LA-1&-2) and LAGEOS-1 & Starlette (LA-1 & STAR). The methodology adopted, in this paper, comprises three main steps:



- a. The orbit restitution of different tracked satellites is performed by the GINS (Géodésie par Intégration Numérique Simultanée, Geodesy by Simultaneous Numerical Integration, in English) software (GRGS, France), based on purely dynamical approach, see section 2.
- b. The estimation of stations coordinates updates and of Earth orientation parameters is performed using the MATLO (MATHématiques pour la Localisation et l'Orbitographie, MAThematics for Localization and Orbitography, in English) software (IGN, Grasse)[6], which can also determine orbit correction via simply kinematical assumptions, see sections 3 and 4. This estimation provides weekly time series of stations positions, in which their analysis permits to make in evidence the geophysical phenomena effects on the station vertical coordinate (Up component).
- c. Analysis of coordinate time series based on (i) frequency analysis by FAMOUS software (*Frequency Analysis Mapping On Unusual Sampling*) [9], and (ii) noise estimation (type and level noise) by Allan variance method.

Finally, the preliminary results of 14 years combined SLR data analysis of different satellites used, namely, LAGEOS-1 & -2 and Starlette, for the study the TRF are presented and discussed in section 4.

## 2. Determination of Orbital Arcs

The orbit restitution of different satellites used (LAGEOS-1, LAGEOS-2 and Starlette) is performed with the GINS software, from a subset of SLR fixed stations well distributed on the Earth as reference frame for the orbitography. For example, the dynamical models and the reference frame used are: *Fes-2004* ocean tides model, *Dtm-94* atmospheric density model, *Ecmwf* atmospheric pressure model, *EOPC04* conventional pole model etc. and *ITRF2005* solution [5].

We have used the Grim5-s1 for LAGEOS satellites and Eigen-Grace03s for Starlette, in order to reach more precision in the orbit estimation [7].

The satellites arcs were calculated using the laser ranging data from precise ILRS stations.

Great quantities of observations were collected during a period of 14 years (from 2nd October 1993 to 24th February 2007). The quality of the positioning is directly linked to the accuracy of the orbits used (in addition to the data accuracy it self). For this reason high altitude geodetic satellites (LAGEOS-1 and LAGEOS-2) are used primarily by geodesists to the SLR network computation (EOPs and stations coordinates). Indeed, these satellites have the advantage of being less sensitive to remaining uncertainties in the dynamical models than low altitude satellites like Starlette. It concerns gravitational and non gravitational effects. But since few years, global Earth gravity field models have greatly improved the accuracy of their coefficients notably thanks to the recent GRACE mission [10]. As a consequence, empirical coefficients can be estimated along the orbit with more consistency than before; their role is to compensate part of the unknown non gravitational forces (constant and periodic). In this fact, we have used Eigen-Grace03s gravity field model for Starlette [7].

The last ITRF realization has shown that a scale difference between SLR and VLBI exists and has revealed a bias in scale factor between both solutions of about 1.0 ppb (drift of 0.08 ppb/year) at epoch 2000.0. The ITRF2005 scale is defined by VLBI technique. Consequently, it was decided to make available to SLR users an SLR solution extracted from the ITRF2005 and re-scaled back by the aforementioned scale and scale rate [4]. So, this ITRF rescaled version is considered in our all computations. The figure (1) illustrates the ILRS network considered in the determination of the ITRF2005. The table (1) gives some statistics resulting from calculations of different satellites orbits. One can retain that the WRMS of orbit residuals are at the centimetre level but with more precision for LAGEOS satellites orbits, because they have low sensitivity to gravitational and non gravitational forces effects. However Starlette data are slightly dominating the measurements set and the average contribution of normal points per satellite is about 36.8 % of all data.

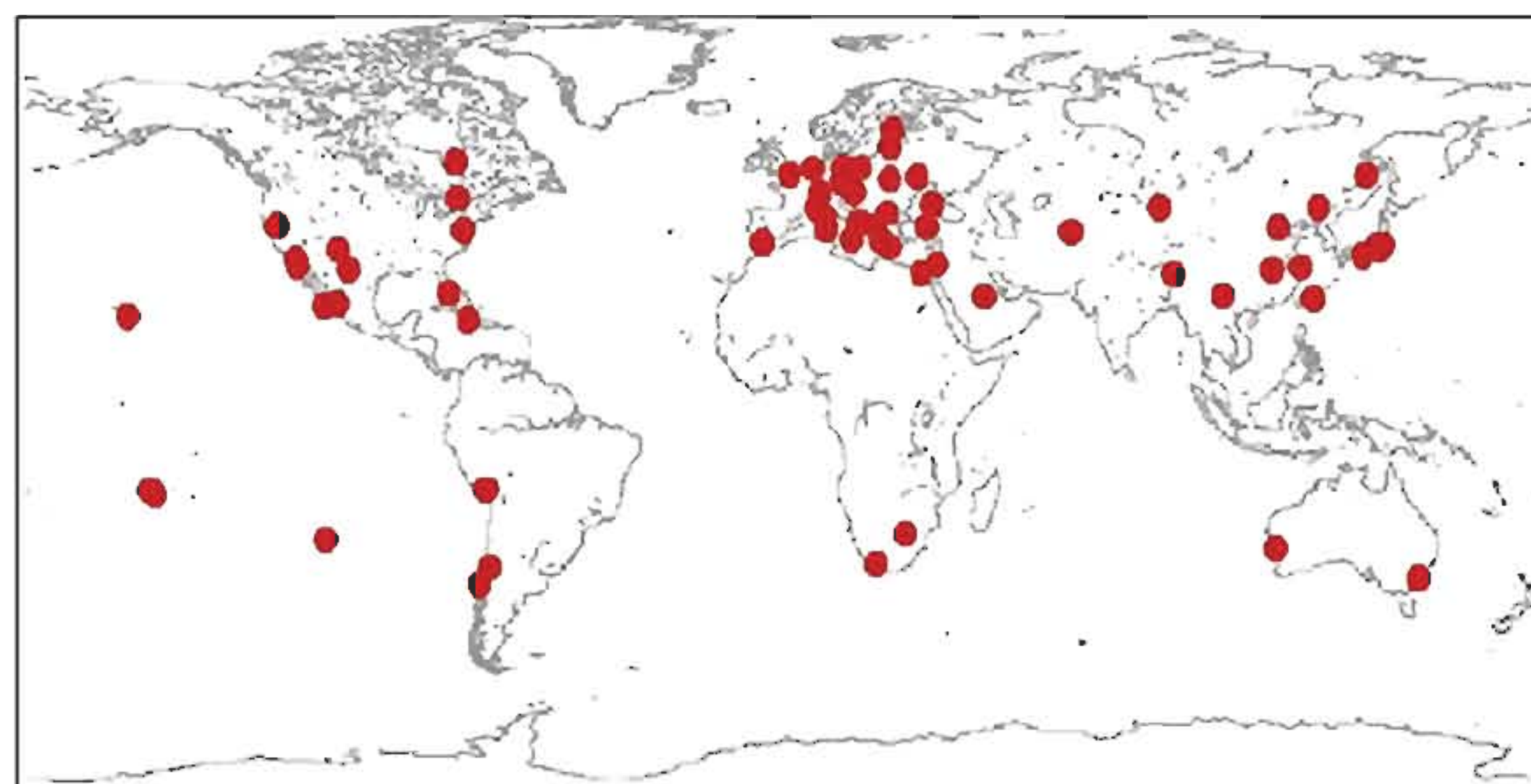


Fig. 1 ITRF2005: ILRS network, source: [3].



**Table 1.** Statistics of the orbit residuals (in mm). RMS: Root Mean Square of orbit residuals & WRMS: Weighted Root Mean Square of orbit residuals.

Satellite	Normal points Number & (%)	Mean residual RMS	Mean residual WRMS
LAGEOS-1	543969 (32.2%)	12.7	9.8
LAGEOS-2	521734 (30.9%)	12.2	8.5
Starlette	621408 (36.8)	18.3	13.9

### 3. Computation of SLR Stations Coordinates

The determination of SLR stations coordinates was carried out by MATLO software [6]. In this study, the computation comprises two principal phases. The first relates to the use of the *minimal constraints* for the resolution of the weekly normal equations systems of the network which are initially singular due the rank defect corresponding to three rotations, in case of the laser ranging technique. In order to define the datum of the network in question, we have applied the following values of constraints:  $\pm 1$ mm (3.3 mas) for rotations ( $R_x$ ,  $R_y$  and  $R_z$ ) and  $\pm 1$ cm for range bias per station and per satellite.

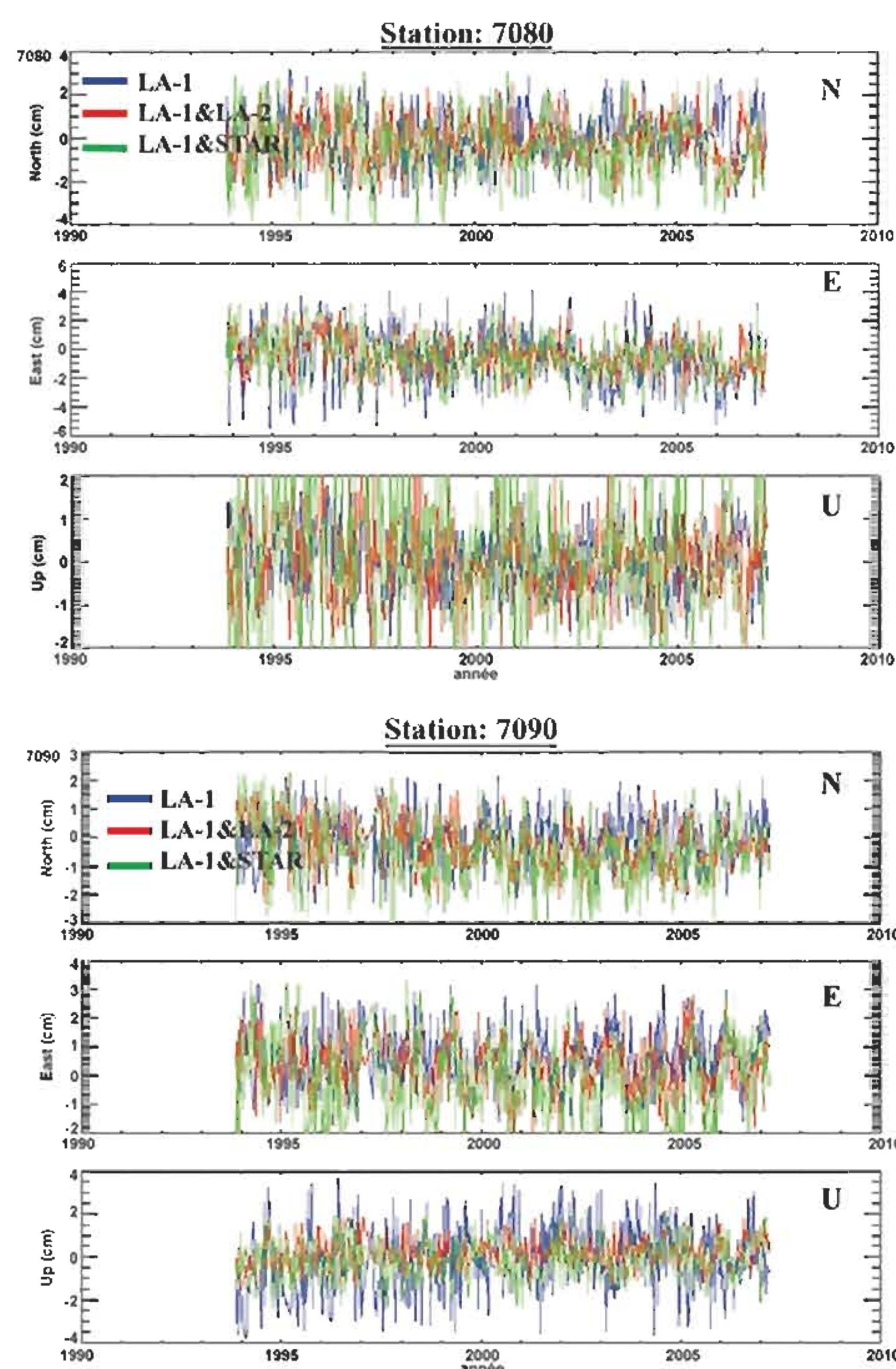
The reference frame of the network was defined by the ILRS stations among the best ones as (7090, 7840, 7080, 7110, 7105, 7810, 7839, 7237).

The results obtained are in terms of time series of coordinate updates which are considered as individual solutions. Each solution generates its proper terrestrial reference frame.

However, the second phase consists in applying a seven parameters Helmert transformation (3 translations, 1 scale factor and 3 rotations), by choosing a set of stations of good quality, on weekly solutions of the stations coordinate updates. This transformation permits to project the individual solutions according to a combined and homogeneous terrestrial reference frame. The results of the processing carried out, illustrated hereafter, are expressed according to topocentric coordinate updates of laser tracking stations (North component:  $N$ , East component:  $E$  and Up component:  $U$ ).

### 4. Results and Analysis

The station position time series are estimated with respect to the ITRF2005 mean position corrected from plate tectonics (ITRF2005 velocities), Earth solid tides, pole tide and oceanic loading effects. These time series must consequently evidence the atmospheric and hydrologic loading effects. The figure 2 gives examples of series of topocentric coordinate updates ( $N$ ,  $E$ ,  $U$ ) for two stations 7080 and 7090, according to different combination solutions.



**Fig. 2** Topocentric coordinate updates of SLR stations : 7080 and 7090. The series corresponding to the satellites combinations are in red for LA-1, in green for LA-1&2, and in blue for LA-1 & STAR.



Table (2) exhibits the statistics of results obtained on coordinates updates of some ILRS stations. The contribution of the Starlette data on the determination of the estimated parameters of these stations is as follows:

- Increase of solutions number compared to those obtained from the LA-1 and LA-1&LA-2 solutions;
- Improvement, of some mm, the quality of few series; for example UP series of station 7090 and East series of station 7105; by adopting the solution LA-1&STAR compared to that of LA-1. However, we notice in general, a quite degradation of quality of coordinate updates series of about 3–5 mm which is mainly due to noise induced by low altitude orbit of Starlette;
- Statistically, the results obtained by the Starlette observations without and with weighting are slightly identical with a margin of some millimetres.

The interest of the time series of stations coordinates calculated in a homogeneous reference frame is to enable us to highlight residual signals compared to the a priori signals used in modelisation (geophysical signals). In this framework, one carried out a frequency analysis on vertical component series of some stations by FAMOUS software. We have focused on this component because it is important for the geodynamical studies since it holds amplitude  $\frac{2}{3}$  of signals acting on the station motion [6].

The results in terms of amplitude and phase of the signal in each UP series of station and according to combination solutions (LA-1, LA-1&LA-2 and LA-1&STAR) are given by table 3. One could detect annual and semi-annual signals, in vertical time

series of stations (7080, 7840, 8834, 7110 and 7839), with amplitudes between 1 to 8 mm. Since, the effects of ocean loading were considered in the model a priori of restitution, the signals detected are probably related to residual loading effects as atmospheric and hydrologic loading effects, which typically have amplitudes of a few mm, (see figure 3).

**Table 2.** RMS and number solutions of the topocentric coordinates of some stations, according to different satellite combinations. The letters a and b correspond to non weighted observations and weighted observations of Starlette satellite, respectively.

Station	Combination	North (cm)	East (cm)	Up (cm)
7090	LA-1	±0.90 527	±1.02 605	±1.52 594
	LA-1&LA-2	±0.84 590	±0.96 602	±0.69 580
	LA-1&STAR (a)	±1.14 599	±1.39 600	±0.96 587
	LA-1&STAR (b)	±1.22 613	±1.79 617	±1.06 600
7105	LA-1	±1.20 459	±1.88 489	±0.62 513
	LA-1&LA-2	±0.90 521	±0.99 531	±0.87 528
	LA-1&STAR (a)	±1.43 568	±1.49 556	±1.34 561
	LA-1&STAR (b)	±1.20 546	±1.39 557	±1.33 552
7403	LA-1	±2.68 230	±1.60 246	±2.22 261
	LA-1&LA-2	±1.41 304	±1.46 302	±1.81 290
	LA-1&STAR (a)	±3.00 368	±3.16 369	±2.81 341
	LA-1&STAR (b)	±2.48 361	±2.52 356	±2.57 341
8834	LA-1	±1.46 451	±1.57 461	±1.30 470
	LA-1&LA-2	±1.12 489	±0.87 468	±1.31 488
	LA-1&STAR (a)	±1.40 507	±1.36 510	±1.68 501
	LA-1&STAR (b)	±1.34 508	±1.26 498	±1.87 515
7840	LA-1	±1.15 549	±1.18 536	±0.90 576
	LA-1&LA-2	±0.76 556	±0.76 578	±0.94 578
	LA-1&STAR (a)	±1.16 596	±1.05 590	±1.30 585
	LA-1&STAR (b)	±1.12 596	±0.97 576	±1.21 591
7839	LA-1	±1.26 512	±1.17 497	±0.96 517
	LA-1&LA-2	±0.74 516	±0.78 555	±1.18 576
	LA-1&STAR (a)	±1.05 547	±1.04 549	±1.36 560
	LA-1&STAR (b)	±1.09 572	±0.98 563	±1.32 568

**Table 3.** Annual and semi-annual terms of vertical coordinate (Up component) of some stations, according to combinations (LA-1, LA-1&-2 and LA-1&STAR). Amplitude (A in mm) and Phase ( $\varphi$  in degrees).

Station	Period	LA-1		LA-1&LA-2		LA-1&STAR	
		A ± $\sigma A$	$\varphi \pm \sigma\varphi$	A ± $\sigma A$	$\varphi \pm \sigma\varphi$	A ± $\sigma A$	$\varphi \pm \sigma\varphi$
7110	1 yr	1.2 ± 0.6	215.2 ± 30.7	3.7 ± 0.7	241.8 ± 12.5	4.1 ± 0.7	263.9 ± 18.8
7090	1 yr	6.6 ± 1.3	43.5 ± 11.4	-	-	2.1 ± 0.8	53.4 ± 23.2
	½ yr	-	-	1.4 ± 0.4	177.3 ± 33.7	-	-
7080	1 yr	-	-	2.9 ± 0.9	220.9 ± 17.4	3.6 ± 1.3	235.4 ± 21.9
	½ yr	2.4 ± 0.7	139.9 ± 17.4	-	-	-	-
7105	1 yr	-	-	2.2 ± 0.8	56.8 ± 22.6	4.2 ± 0.7	88.1 ± 20.4
	½ yr	1.6 ± 0.6	220.8 ± 22.3	-	-	-	-
8834	1 yr	-	-	2.9 ± 0.9	350.5 ± 30.4	4.8 ± 1.1	98.9 ± 23.0
	½ yr	-	-	-	-	4.4 ± 1.5	59.0 ± 22.3



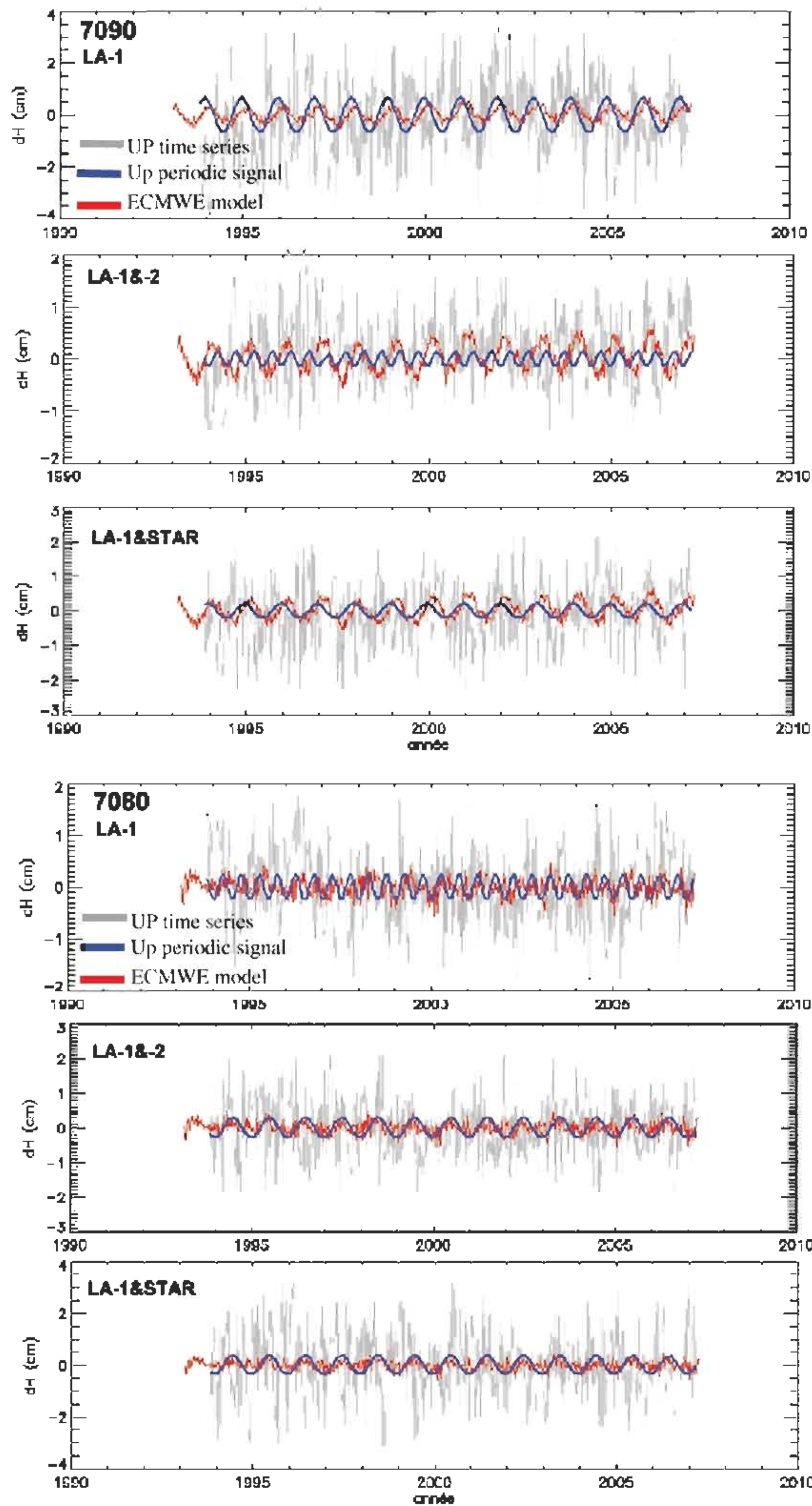


Fig. 3 Comparison between seasonal signals of the Up components and atmospheric loads model (ECMWF) for 7090 and 7080 stations.

The figure (3) gives an example of comparison between the UP periodic signals according to the different combinations and the atmospheric loads model (ECMWF series), for 7080 and 7090 laser stations. The correlation between the two signals is about 40-50%, in the case of the LA1&STAR combination, which explains that the UP variations of the two stations (7080 and 7090) are due to atmospheric loads. The remain variations are mainly come from noise.

It is interesting to exhibit which type of noise affecting the time series of coordinates in order to determine the possible sources of errors in the estimation of these parameters. In this stage,

we have used the Allan variance [1],[2]. By definition the Allan variance of position residuals, for a given time interval, is computed by averaging position the residuals over that interval and computing the variance of differences between adjacent averaged values. The Allan variance analysis was developed and is widely used for estimating the frequency stability of atomic clocks. This tool was extended to geodetic data. It allows one to characterize the type of noise and, in particular, to identify white noise (spectral density  $S$  independent of frequency  $f$ ), flicker noise ( $S$  proportional to  $1/f$ ), and random walk noise ( $S$  proportional to  $1/f^2$ ).

The dependence of the Allan variance of a time series on the sampling time  $\tau$  can be interpreted in terms of its error spectrum by means of the Allan diagram, which gives Allan variances for increasing values of  $\tau$  [8].

In logarithmic scales, slopes  $-1,0$  and  $+1$  correspond to white noise, flicker noise and random noise, respectively.

In the context of this study, a white noise signature in the position residuals would point to random errors (Gaussian errors) affecting the measurements, while a flicker noise signature would point to perturbations that may have different origins, like local tectonics, instrument defects, analysis consistency, etc.

The noise type is measured by the slope of the Allan graph, which describes the log-log relationship of the Allan variance of the time series.

The noise level measured by the Allan deviation for a one-year sampling time of the non-linear, non-seasonal position time-series.

The table 4 provides the bias and slope of the Allan graph of some stations UP components. According to figure 4, and as first interpretation, we notice that the white noise is the dominant noise in these series according to the different solutions (LA-1, LA-1 &LA-2 and LA-1&STAR).

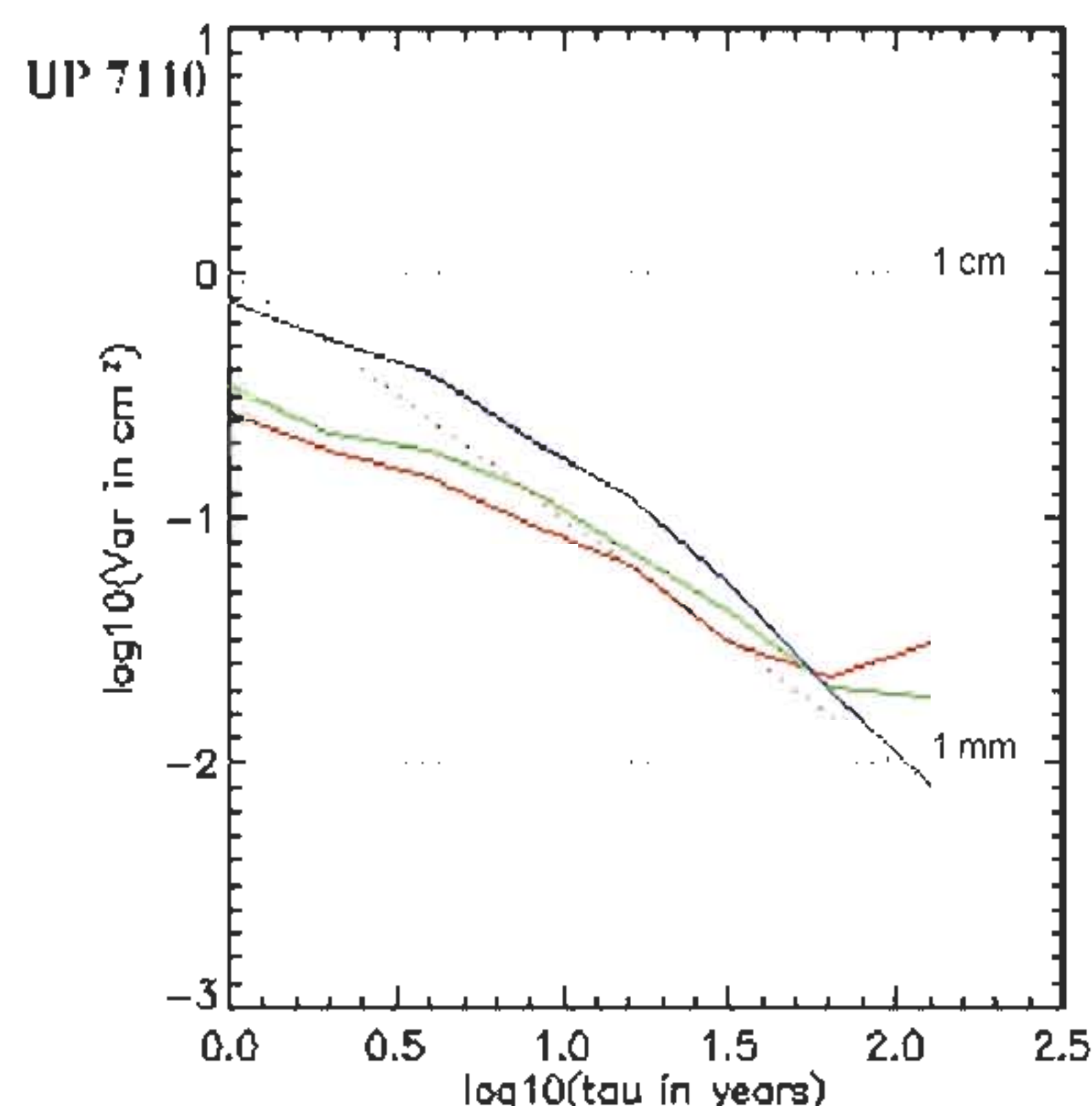
While a weak flicker noise combined with white noise is observed in the Up component of the different stations, according to the LA-1 and LA-1&LA-2 solutions. In addition, periodic signal was detected in UP series of 7835 station.

According to the table 4, the noise level is about of 3 – 7 mm.

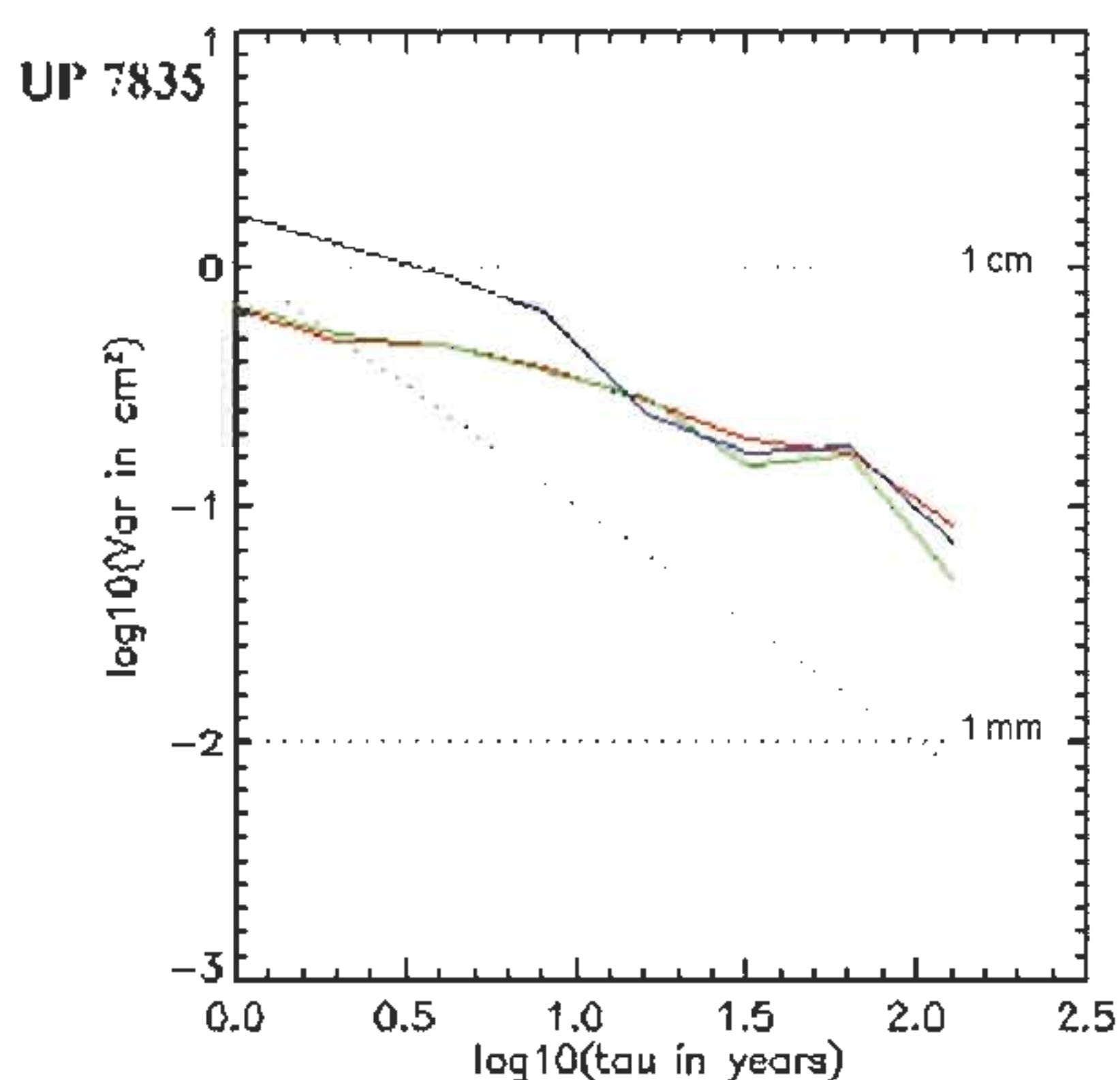
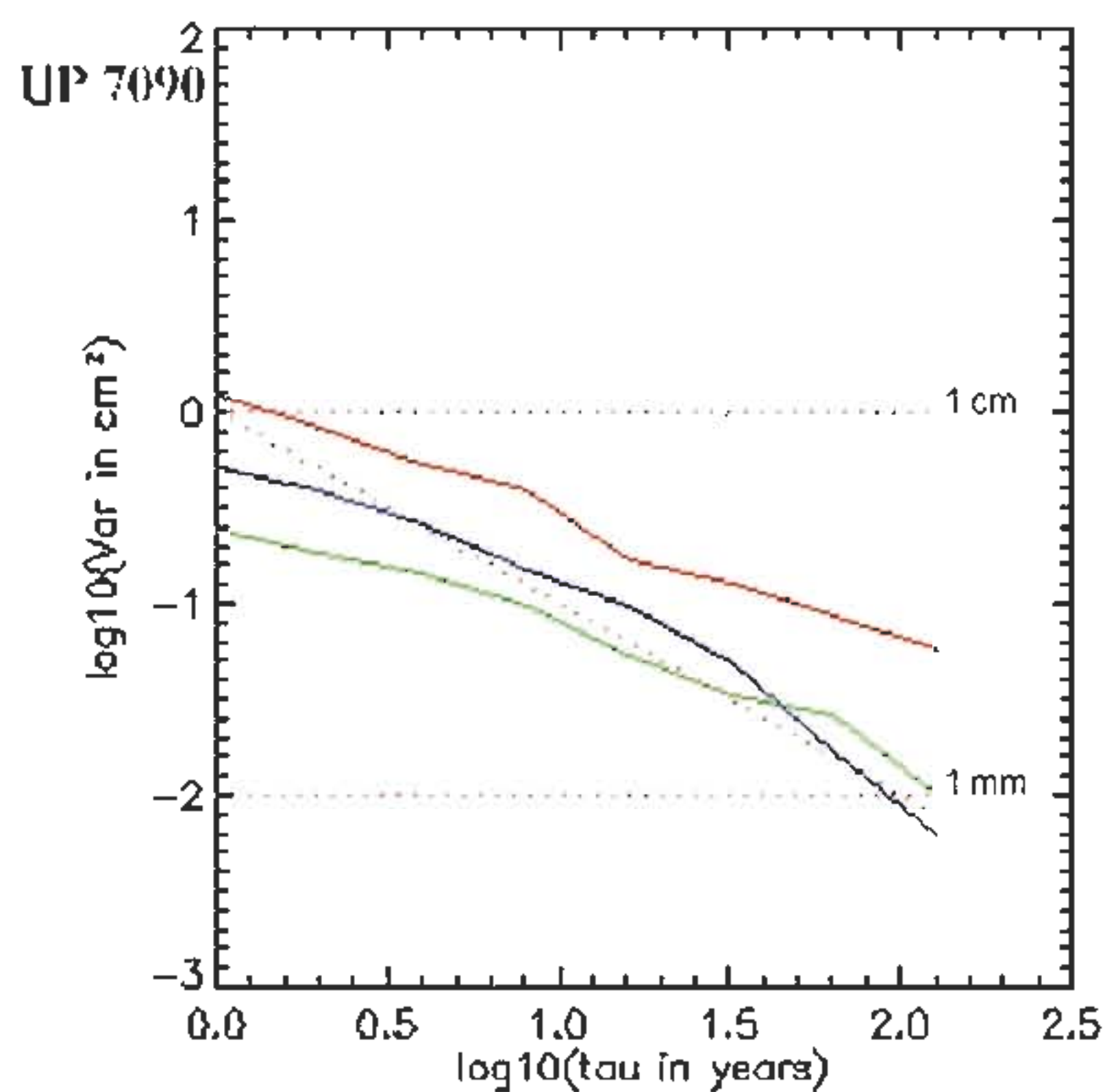
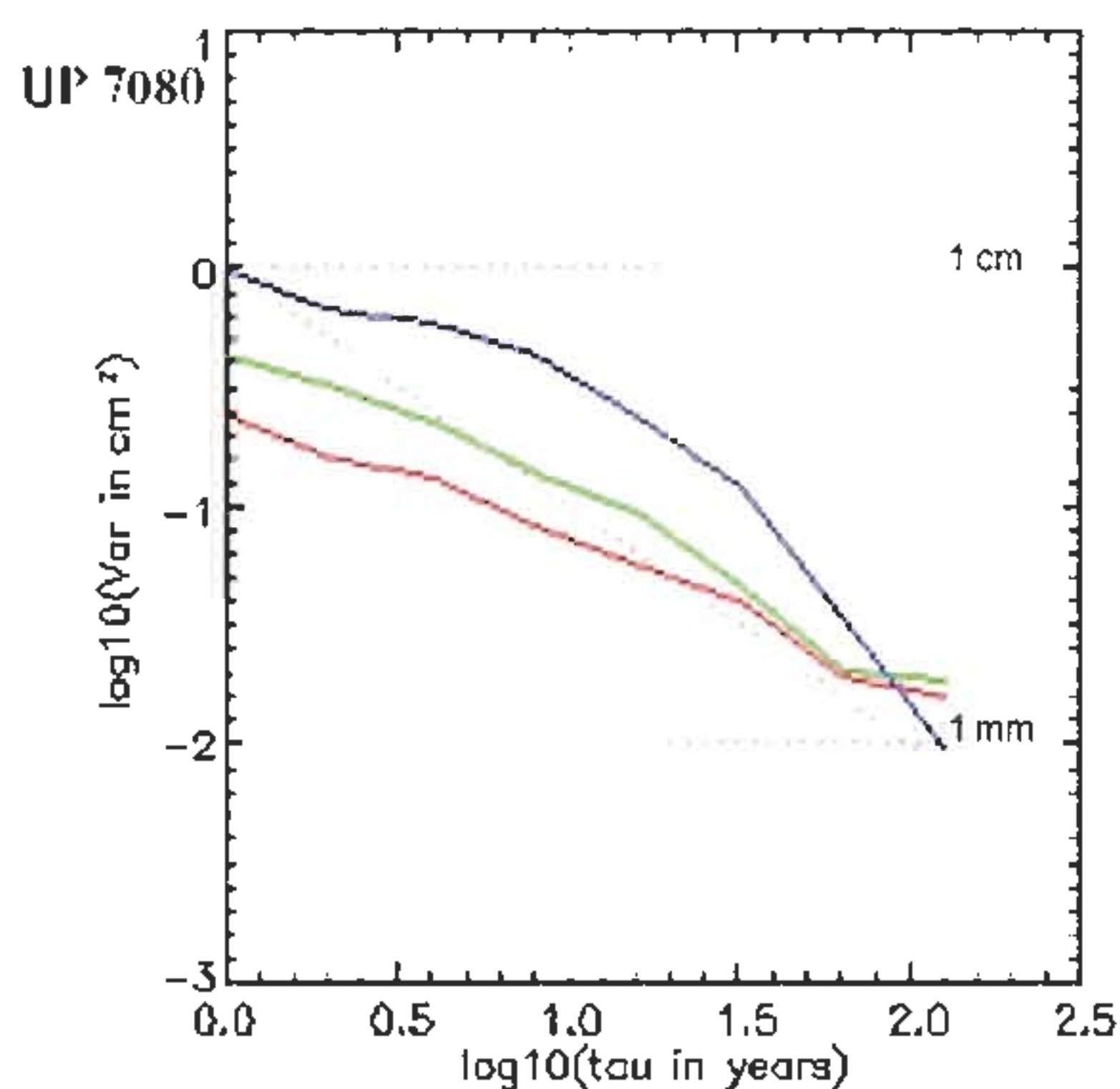


**Table 4.** Slope and bias of noise affecting the Up component signals of some SLR stations, according to the LA-1, LA-1&-2 and LA-1&STAR solutions. These values are computed from Allan variance for each series. The noise type, (Wh.) or Flicker noise (Fl.) is listed when available. Ps. means periodic signal.

Station	Solution	Slope (cm/yr)	Noise level (cm)	Noise type
7080	LA-1	-0.6	0.3	Wh.+ weak Fl.
	LA-1&-2	-0.7	0.4	Wh.+ weak Fl.
	LA-1&STAR	-0.9	0.5	Wh.
7090	LA-1	-0.7	0.6	Wh.+ weak Fl.
	LA-1&-2	-0.6	0.3	Wh.+ weak Fl.
	LA-1&STAR	-0.9	0.4	Wh.
7835	LA-1	-0.4	0.6	Wh.+Fl.+ ps.
	LA-1&-2	-0.5	0.6	Wh.+Fl.+ ps.
	LA-1&STAR	-0.7	0.7	Wh.+ ps.
7110	LA-1	-0.5	0.3	Wh.+Fl.
	LA-1&-2	-0.6	0.3	Wh.+Fl.
	LA-1&STAR	-0.9	0.4	Wh.



**Fig. 4** Allan variance graph of Up component updates of stations 7080, 7090, 7835 and 7110. The series corresponding to the satellites combinations are in red for LA-1, in green for LA-1&-2, and in blue for LA-1 & STAR.



### 5. Conclusion

The results obtained revealed that the contribution of the Starlette data is acceptable, in general, to study the Terrestrial Reference Frame and to analyse the quality of the related parameters. In spite of the significant number of the normal points on this satellite and the recent dynamic models used, in particular the Eigen-Grace03s gravity field model, in the calculation of its orbit, the impact of Starlette remain not very important. Nevertheless, we demonstrate that the use of low orbit altitude such Starlette, in geodynamic field, is reliable in order to supply the study of behaviour of stations position motion.

In order to improve these results, it is necessary to develop and to investigate the following points:

- method of rigorous weighting of measurements SLR by satellite and station;
- comparison of the coordinate time series with geophysical signals.

As prospects, it will be interesting to generate and to study, for a long period (34 years: 1976 – 2009), the time series of TRF and Earth Orientation Parameters (EOPs), based on low satellites such Stella, Topex/Poseidon, Jason-1, in addition to Starlette and LAGEOS satellites.

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