Optimal Portfolio and Performance-Risk Metric: A Study on Saudi Stock Market

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Abstract:
The aim of this study is to estimate the optimal portfolio weights for the mean-variance efficient optimal portfolio using linear programming technique for single and multi-period using rebalance technique, then compare the performances and risk metrics with the equal-weighted portfolio, we used in performance metrics Sharpe and Treynor ratios, CAPM Alpha and Beta, and for the risk metrics VaR and CVaR. We found that the MV optimal portfolio performed better and less risky than the equal-weighted portfolio for both single and multi-periods.
***Key Words*:** Mean-Variance optimization, Rebalance technique, Performance metric, Risk metric, Value at Risk, Conditional Value at Risk.

JEL Classification : G11, G32

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Introduction:

The portfolio selection issue was firstly developed by "Markowitz" in 1952. This issue is based on linear programming methods to find the optimal weight portfolio. Mean-Variance efficient portfolios can be determined by solving a quadratic programming (QP) model with minimization of variance as its objective and expected return as a constraint. The efficient frontier can be determined either by solving this QP model for different values of expected return or by using a parametric QP model with risk adjusted expected return, that is a function of variance of return, expected return and a risk-return trade-off parameter, as its objective function (Markowitz H. , 1952)

the optimal portfolio should be also rebalanced regularly to take into account of changes in the market. In rebalancing a portfolio, the initial portfolio, the change in funding and the transaction costs associated with buying and selling securities. (Glen, 2011)

In our study, we discuss the vast literature on the portfolio theory and empirically examine the performance of Markowitz's MV model of optimal portfolio versus the 1/N equal-weighted portfolio. We considered the single-asset case of risky-securities, i.e. the equities traded on the Saudi Stock Exchange for constructing the mean-variance efficient portfolio using the portfolio rebalancing technique for the

single-period optimization and multi-period optimization scenarios, in this study we assumed that the transaction costs equal to zero in rebalancing the optimal portfolio. The stocks traded on the Saudi Stock Exchange are considered for the optimal portfolio as the top 20 stocks in the market base on the traded volume (Tadawul, 2020) . The optimal weights for weight allocation on the equities were estimated using the linear programming technique. The comparison was based on performance metrics and risk metrics, the Treynor and Sharpe ratios were used as the performance metrics, we also used CAPM Alpha and Beta.
There are many measures and methods in measuring the portfolio risk , the standard deviation or variance of expected returns is the traditional one; the advanced methods are VaR and CVaR (Conditional Value at Risk, (Artzner, P., Delbaen, F., Eber, J. M., & Heath, D, 1999) used in the computation of portfolio risk and returns. As the portfolio risk is a function of variance and co-variances of asset returns in the portfolio, an optimal portfolio for investors should have lower correlation/ co-variance in asset returns for various combination of assets for earnings the maximum expected returns.
Research' problematic : Based on the above, we can formulate the problematic of this research in the following questions: "What is the best performing portfolio between the optimal and the equal-weighted in both single and multi-period? And What is the less risky portfolio?"
Data and methodology: In order to answer the research problematic we used a descriptive methodology in theoretical background to demonstrates the main theoretical concepts of the Mean-variance method and performance-risk metric, and comparative method in analyzing the results. We used the Daily closing price data of the Saudi listed stocks, equities traded for the period between Nov 2014 and Nov 2020. The data is referred from Tadawul website.
Literature review:
The works by **(Tobin, 1958)**, **(Sharpe, 1963)**, **(Lintner, 1965)**, **Merton (1969, 1973)**, and **Black (1972)** studied the mean-variance (MV) efficient portfolio frontier theory of Markowitz (1952) and found the results consistent with the concept of investments of expected utility maximization and risk-minimization by the risk-averse investors which the investors maximized their expected portfolio returns by minimizing the portfolio variance.
Chan, Karceski, and Lakonishok (1999) used a factor model to generate a global minimum variance portfolio to obtain factor loadings for the estimation of covariance matrices of asset returns. However, they failed to accurately forecast the asset return covariance matrix for the optimal portfolio.
De Miguel, Garlappi, and Uppal (2009) performed a comparison of 14 different estimation models for MV optimal portfolio for the US stocks and found that no model performed better than the 1/N heuristic model in terms of Sharpe ratio, and the optimal diversification failed due to estimation errors of asset return moments.
Vahn (2011) used a CVaR (conditional Value at Risk) as a measure of risk for estimating conditional VaR for portfolio optimisation. They suggested that the CVaR was weak due to the estimation errors for portfolio returns.

Ahmed Marhfor (2016) has discussed the weaknesses and distinguish between traditional portfolio performance measures and more recent conditional performance measures. His study showed that the conditional approach addresses one major short-coming of the traditional approach (risk stability assumption). Conditional measures allow expected returns and risk to vary with the state of the economy. We also propose new avenues for future research and some improvements to the existing measures.

Moulya, Mohammadi and Mallikarjunappa (2019) used the linear programming technique to estimate the optimal portfolio weights for the mean-variance efficient optimal portfolio using rebalanced and non-rebalanced portfolios and compared the performances against the 1/N heuristic portfolio. They found that the minimum-variance optimal portfolio performed better than the 1/N heuristic portfolio.

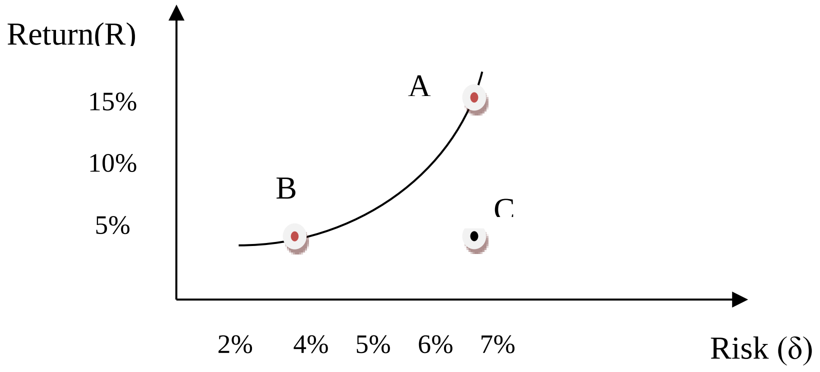
I. Mv Framework:

The portfolio selection issue was firstly developed by "Markowitz" in 1952. This issue is based on linear programming methods to find the optimal weight portfolio, which can maximize the portfolio return and minimize the portfolio risk at the same time.

1. Definition of the optimal portfolio:

Markowitz presented in his research on the modern portfolio theory (MPT) definition of the optimal portfolio as "a portfolio that maximizes the return at a certain level of risk or minimizes risk to an acceptable level of return", Markowitz has shown that the optimal portfolio for the investor is located on the mean- variance efficient frontier for any given expected return, there is no other portfolio with lower variance and for any given variance there is no other portfolio with higher expected return (Howard Howan, 2011, p. 69),this is shown in the following figure**:**

Figure (01): «The investor's behaviour to determine the optimal portfolio »



Source: By the authors

Through Figure (01), we note that portfolios A and C have the same risks and different returns, so if the investor is rational, he chooses A (the highest return). When we compare portfolio B and C note that they have the same return and different risks, if the investor is rational, he chooses b (the lowest risk).

2. The Markowitz Model 1952 (Quadratic modeling):

The financial portfolio has witnessed a successive development starting with the research presented by Harry Markowitz on the basic principles of forming a general portfolio where Markowitz made new additions on investment decisions in 1952 using the most important models of modern quantitative techniques based on the Quadratic modeling model in choosing the investment portfolio, So that he relied on mathematical and statistical methods to determine the variation in rates of return as well as on determining the correlation coefficient between the returns of the tools formed for the portfolio. Markowitz also relied on the correlation to determine efficient diversification between assets.

The Markowitz model is based on several Hypotheses (Bruna & Zrinka, 2012, p. 237):

- Returns follow the normal distribution;
- Investor rationality**:** the investor prefers the highest return at the same level of risk or the lowest risk at the same level of return;
- Investors maximize expected returns for one period on the basis of interpreting the approved curves with diminishing marginal benefit of wealth;
- Perfect competition with no commissions.

Based on the assumptions presented, the optimal investment portfolio at Markowitz is based on a basic idea that the investor's benefit is explained by the function of the independent variables, the expected return and the variance (or standard deviation) provided that the investor prefers the highest return with the lowest standard deviation. So, the variance of the portfolio return can be calculated as follows:

$$\boldsymbol{\delta^2_{R_p} = \sum_{i=1}^{N} W_i^2\, \delta^2_{R_i} + 2\sum_{i=1}^{N} \sum_{\substack{i\neq j \\ i=j+1}}^{n} W_i W_j\ cov\left(R_i R_j\right)}$$

With:

- $\delta^2_{R_p}$: Portfolio variance
- W: Weight of assets
- $cov\left(R_i R_j\right)$: Common variation between portfolio returns

Return variance can also be calculated as follows (Evstigneev & others, 2015, p. 12):

$$\boldsymbol{\delta^2_{R_p} = E\left[\left(\sum_{i=1}^{n} R_i x_i - \sum_{i=1}^{n} m_i x_i\right)^2\right]}$$

$$\boldsymbol{\delta^2_{R_p} = E\left[\left(\sum_{i=1}^{n} (R_i - M_i)\, x_i\right)\left(\sum_{j=1}^{n} \left(R_j - m_j\right)\right) x_j\right]}$$

$$\boldsymbol{\delta^2_{R_p} = \left[\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j (R_i - ER_i)\left(R_j - ER_j\right)\right]}$$

$$\boldsymbol{\delta^2_{R_P} = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j\, cov_{ij}}$$

The MV framework (Markowitz, 1952) proposes diversification of assets, maximization of expected utility for investors under the full-investment and long-only constraints, and minimization of portfolio variance.

The MV portfolio is proposed under the two constraints: (V. Harshitha Moulya, Abuzar Mohammadi, T. Mallikarjunappa, 2019)

1) full-investment constraint, where, all the investible fund is used for the asset-investment. The summation of allocation weights should be equal to 1.

$$W_{n=} \sum_{i}^{n} = 1 \ , Wi = 1$$

2) The long-only constraint, where, all the allocation weights are positive, i.e. no short-selling is admissible.

$$Wi \geq 0$$

Achieving an optimal portfolio corresponds to quadratic modelling which is considered a type of nonlinear mathematical modelling so that it is formulated in the form of the goal function, so that it represents in the goal to be achieved by decision makers in the form of a second-degree (quadratic) mathematical picture, its goal of minimizing or maximizing in terms of significance variants under a set of restrictions. (Paul & Jonathan, 2011), So that the general form of quadratic programming is as follows (Markowitz, 2007):

$$Max\left(\delta_p^2\right) = \sum_{i=1}^{n} \frac{r_{\iota t}}{d_t} * w_i$$

$$Min\left(\delta_p^2\right) = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \, cov_{ij}$$

$$\sum_{i=1}^{n} w_i = 1$$

3. Portfolio optimization scenarios:

We carried out portfolio optimization under two-scenario:

a) Single-period optimization,

b) Multiple-period optimization.

Under the single-period case, optimal weights are estimated at the beginning of the portfolio formation period for one time. Under the multiple-period case, the MV portfolio is re-balanced by altering the allocation weights on the assets at the beginning of every quarter.

II. Performance metrics:

The evaluation of the performance of the investment portfolio is based on the principle of comparison depending on several methods, the most prominent of which is Benchmark Comparison, Style Comparison, and Risk-Adjusted Methods.

The Risk-Adjusted Methods is the most popular method based on several measures, such as the Treynor ratio (T_R), Sharpe ratio (S_R), Jensen's alpha (α)….

1. Treynor ratio (T_R):

"In 1965, Treynor's was the first researcher who computed measure of the portfolio

performance". The Treynor ratio calculates the risk premium for each unit of the systematic risk.
Symbolically, the Treynor ratio (T_R) is presented as (Bernd & Russ, 2012, p. 66):

$$T_R = \frac{r_p - r_f}{\beta_p}$$

$$\beta_p = \sum_{i=1}^{n} \beta_i w_i$$

$$\beta_i = \frac{cov(A, M)}{\delta_M^2}$$

With:

- T_R: Treynor ratio; r_p: Return of the portfolio; r_f: Risk-free rate; β_p: Portfolio beta,

$cov(A, M)$: The covariance between the returns of an asset and market returns; δ_M^2:The variance of market returns.
When a Treynor ratio is greater, the performance of the portfolio is efficient

2. Sharpe ratio (S_R):

Sharpe Ratio (Sharpe, 1966) reflects the risk premium (excess return) of the investment portfolio for each a unit of the total risk of the portfolio. (Lalith & Tanweer, 2005, p. 05)
So, this indicator measures the efficiency of a portfolio's performance in achieving an excessive return on risk that represents the interest rate on short-term government instruments or the interest rate on deposits.
The sharp ration can be calculated as follows (Lalith & Tanweer, 2005, pp. 05,06):

$$s_R = \frac{r_p - r_f}{\delta_p}$$

With:
s_R: Sharpe Ratio or reward-to-variability; r_p: Return of the portfolio; r_f**:** Risk-free rate; δ_p**:** Standard deviation, the use of the standard deviation in this case means that this measure takes into account the total risks (systematic and unsystematic risks)
The higher the Sharpe ratio indicates a better performance because each unit of total risk is rewarded with greater excess return.

3. Jensen's alpha (α) and Beta (β):

Jensen's alpha (1968) and Beta are based on the capital asset pricing model, as it can be formulated on the following model:

$$R_p = R_f + \beta_p (R_m - R_f)$$

With:

- $R_m - R_f$ is the expected excess rate of return of the market portfolio

The alpha value is the deviation of the average portfolio return from the expected return on capital assets, so the Jensen measure equation can be formulated as follows:

$$\alpha_p = R_p - \left[R_f + \beta_p\left(R_m - R_f\right)\right]$$

The beta can be calculated by dividing the product of the covariance of the security's returns and the market's returns by the variance of the market's returns over a specified period, the equation can be formulated as follows:

$$\textbf{Beta coefficient}(\boldsymbol{\beta}) = \frac{\textbf{Covariance}(\boldsymbol{Re},\boldsymbol{Rm})}{\textbf{Variance}(\boldsymbol{Rm})}$$

where**:**
Re=the return on an individual stock
Rm=the return on the overall market
Covariance=how changes in a stock's returns arerelated to changes in the market's returns
Variance=how far the market's data points spreadout from their average value
α_p should be zero: it means that the stock has performed exactly same as the market expected based on its systematic risk.
if β equal to 1, it means that the price of stock is strongly correlated with the market, if it is less than 1 means that the security is less volatile than the market, and if it is greater than 1 it means that the security is more volatile than the market.

III. Risk metrics:

Defining and measuring the risk is one of the most important things for portfolio management, and one of the most useful tools for measuring portfolio risks are Value at Risk and Conditional Value at Risk, they are defined as follow:

1. Value at-risk (VaR):

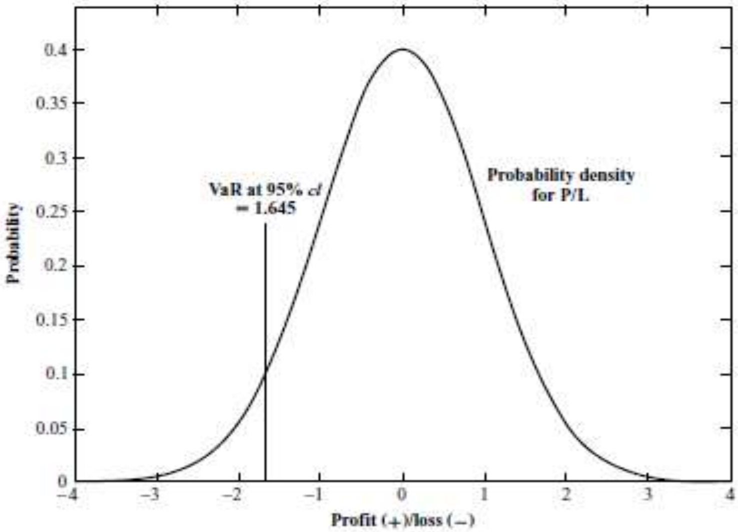
Value at-risk is a model for measuring risks and determining the optimal portfolio. This model was spread and achieved success by JP Morgan in 1990 through the risk metric system (Miller, 2019, p. 51), and also spread by the Basel Committee in 1996 through the use of VaR banks to meet adequacy requirements. Capital where VaR has become one of the most important models for risk measurement (Moohwan, 2011, p. 12)

1.1 Definition of value at risk:

- Starck defines value at Risk as the maximum estimated loss that cannot be exceeded during the retention period and at a certain level of confidence. (Starck, 2008, p. 20)
- " The VaR on a portfolio is the maximum loss we might expect over a given holding or horizon period, at a given level of confidence.4 Hence, the VaR is defined contingent on two arbitrarily chosen parameters a holding or horizon period, which is the period of time over which we measure our portfolio profit or loss, and which might be daily, weekly, monthly, or whatever; and a confidence level, which indicates the likelihood that we will get an outcome

no worse than our VaR, and which might be 50%, 90%, 95%, 99% or indeed any fraction between 0 and 1 ". (Kevin, 2002, p. 22)

Figure (02): «Value at risk at 95% confidence level»



Source: (Kevin, D. (2002). *Measuring market risk.* England: JOHN WILEY & SONS, p.22)

Figure (02) shows the common profit / loss density function (P / L) during the chosen retention period. For VaR, we must choose the confidence level (cl). If this was 95%, the point VaR value is given by the point on the x-axis that cuts above 95% of P / L notes from below 5% of the tail notes. In this case, the relevant *x*-axis −1.645, so the VaR value is 1.645. The negative profit / loss value corresponds to the positive VaR, noting that the worst result at this level of confidence is the loss of 1.645.

- In order to measure the performance of the portfolio with the value at risk, the standard deviation is replaced by the VaR, based on Sharp measure, and this is through the following formula (Sourd, 2007, p. 14):

$$\frac{\boldsymbol{R_p - R_f}}{\frac{\boldsymbol{VaR_p}}{\boldsymbol{V_p^0}}}$$

With:

- $\boldsymbol{R_p}$: The return on the portfolio;
- $\boldsymbol{VaR_p}$: The VaR of the portfolio;
- $\boldsymbol{V_p^0}$**:** The initial value of the portfolio.

The Non-parametric approach requires no distributional assumption and it estimates the VaR as the quantile of the empirical distribution of historical returns. For this approach, CVaR can be estimated as the mean of the returns that exceeds the VaR estimation. (Taylor, 2008)

One of the most used method in non-parametric approach is the historical simulation

(HS), it is a very simple method in simulating Value at Risk, (Miller M. B., 2019) mentioned in his book (Quantitative financial risk management) as follow: *"In this approach we calculate VaR directly from past returns. For example, suppose we want to calculate the one-day 95% VaR for an equity using 100 days of data. The 95 th percentile would correspond to the least worst of the worst 5% of returns. In this case, because we are using 100days of data, the VaR simply corresponds to the fifth worst day."* He completes saying: *"…The historical approach is non-parametric. We have not made any assumptions about the distribution of historical returns. There are advantages and disadvantages to both approaches. The historical approach easily reproduces all the quirks that we see in historical data: changing standard deviation, skewness, kurtosis, jumps, etc.*

Developing a parametric model that reproduces all of the observed features of financial markets can be very difficult. At the same time, models based on distributions often make it easier to draw general conclusions. In the case of the historical approach, it is difficult to say if the data used for the model are unusual because the model does not define usual."

2. Conditional Value-at-Risk:

Although VaR has become the standard measure of market risk, it has been criticized for reporting only a quantile, and cannot report outcomes beyond the quantile. On the other hands, VaR sub additive risk measure, this means that the total risk on a portfolio should not be greater than the sum of the risks of the part of the portfolio. (Taylor, 2008) VaR does not tell us anything about the tail distribution, two portfolios could have the exact same 95 % VaR but very different distributions beyond the 95% confidence level. (Miller M. B., 2019)

The conditional value at risk is a risk measure that overcomes these weaknesses, it defined as the conditional expectation of the return given that exceeds the VaR.

Using the concept of conditional probability, we can define the expected value of a loss, given an exceedance, as: (Miller M. B., 2019)

$$\mathbf{E[L|L \geq} \; \boldsymbol{VaR_y}] = \boldsymbol{S}$$

Where: **S** is the CVaR or Expected Shortfall

If the expected profit of a fund can be described by a probability density function given by f (x), and VaR is the VaR at the γ confidence level, we can find the CVaR as:

$$\mathbf{S} = -\frac{\mathbf{1}}{\mathbf{1}-\boldsymbol{y}} \int_{-\infty}^{\boldsymbol{VaR}} \boldsymbol{x f(x) dx}$$

IV. RESULTS AND DISCUSSION:

Table 1 provides the descriptive statistics of the Saudi stocks. We found that 05 stocks (27%) have average positive returns, the biggest loss that achieved by individual stock was 0.75% (SIFCO), and the biggest return was 0.70% (SIFCO).

Table 1: «Descriptive statistics of the 19 Saudi stocks returns»

Index	Min.	Mean	Max.
DAR ALARKAN	-1.086e-01	-5.094e-05	1.021e-01
ALINMA	-0.2356868	-0.0002148	0.0951171
SABIC	-0.1049348	0.0001703	0.0958324
SEERA	-0.1108732	-0.0007977	0.0958482
SAUDI KAYAN	-1.045e-01	-8.189e-05	9.726e-02
ALRAJHI	-0.0846092	0.0005371	0.0984401
BJAZ	-0.434795	-0.000027	0.431723
ZAIN	-0.4792768	-0.0000761	0.5135635
TASNEE	-0.1082136	-0.0006327	0.0996675
PETRO RABIGH	-0.1053605	-0.0003338	0.0946264
SFICO	-0.7485145	-0.0006442	0.6699088
JOUF CEMENT	-0.1053605	-0.0002914	0.1025591
SIPCHEM	-0.1046914	-0.0003233	0.0915151
CHEMANOL	-0.1139229	-0.0001717	0.1084093
SAUDI ELECTRICITY	-0.1057963	0.0002816	0.0941133
EMAAR EC	-0.1082136	-0.0003026	0.0971911
NAJRAN CEMENT	-0.1047686	-0.0002495	0.0953102
NCB	-0.1053605	0.0002249	0.1846222
RIBL	-0.1052200	0.0001644	0.0970458

Source: Author's computation

Risk-Return of MV portfolios and performance-risk metrics: The portfolio return and risk of the optimised portfolios (for both rebalanced and non-rebalanced) are calculated using historical data of the Saudi stock exchange, we used the Sharpe and Treynor ratios and CAPM's Alpha and Beta for the performance metrics and historical simulation VaR, CVaR for the risk metrics, we also used TASI index (Tadawul All Share Index) as the Benchmark.

Table 2 shows the return-risk, the performance and the risk metrics of the optimized MV portfolio under the single-period scenario (non-rebalanced). It is observed that the return of MV portfolio is bigger than the equal-weighted portfolio (0.00036015> -0.000148384), the standard deviation of MV portfolio is smaller (0.01258218 < 0.01431538) , we also found that the historical simulation VaR and CVaR for both 95% and 99% of MV portfolio are smaller than those in equal-weighted portfolio, it means that the maximum loss can MV portfolio faces at 99% level of confidence is -0.0341 and at 95% level of confidence is -0.0187 which is less than the maximum loss can the equal-weighted portfolio faces under both 99%, 95% level of confidence

(-0.04124, -0.0194 respectively), and we also found that the average loss beyond VaR of MV portfolio at both 99 and 95% level of confidence are less than those in equal-weighted portfolio (-0.052 and -0.03 < -0.062 and -0.0334 respectively), it is also observed that the MV portfolio performance is better than the equal-weighted one, the sharpe ratio of MV is bigger than equal-weighted it means that each unit of total risk in MV is better rewarded with return, Treynor ratio is greater in MV than in equal-weighted portfolio, it means that MV portfolio performs better than equal-weighted. Based on beta, we see that the MV portfolio is less volatile than the market, in contrary we see that equal-weighted portfolio is more volatile than the market, and Alpha showed that the MV portfolio is earning excess returns (positive value). In other words, the MV portfolio has beat the market with their stock-picking skills.

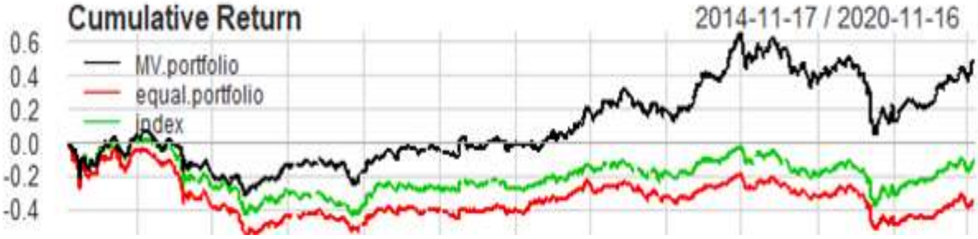
Table 2: «Performance and risk metrics of single-period MV versus equal-weighted portfolio»

	MV Port	Eq-We port
Mean	0.00036015	-0.000148384
STDV	0.01258218	0.01431538
VaR 99	-0.0341	-0.04124
VaR95	-0.0187	-0.0194
CVaR 99	-0.052	-0.062
CVaR 95	-0.03	-0.0334
Sharpe Ratio	0.454	-0.22
Alpha	0.00040079	-0.00012847
Beta	0.9449578	1.065286
Treynor	0.093021	-0.0425533

Source: Author's computation

Figure 03 shows that the MV portfolio performs better than the Saudi Stock index (TASI) and equal weighted portfolio based on cumulative return.

Figure (03): «MV portfolio performance»



Source: Source: By the authors

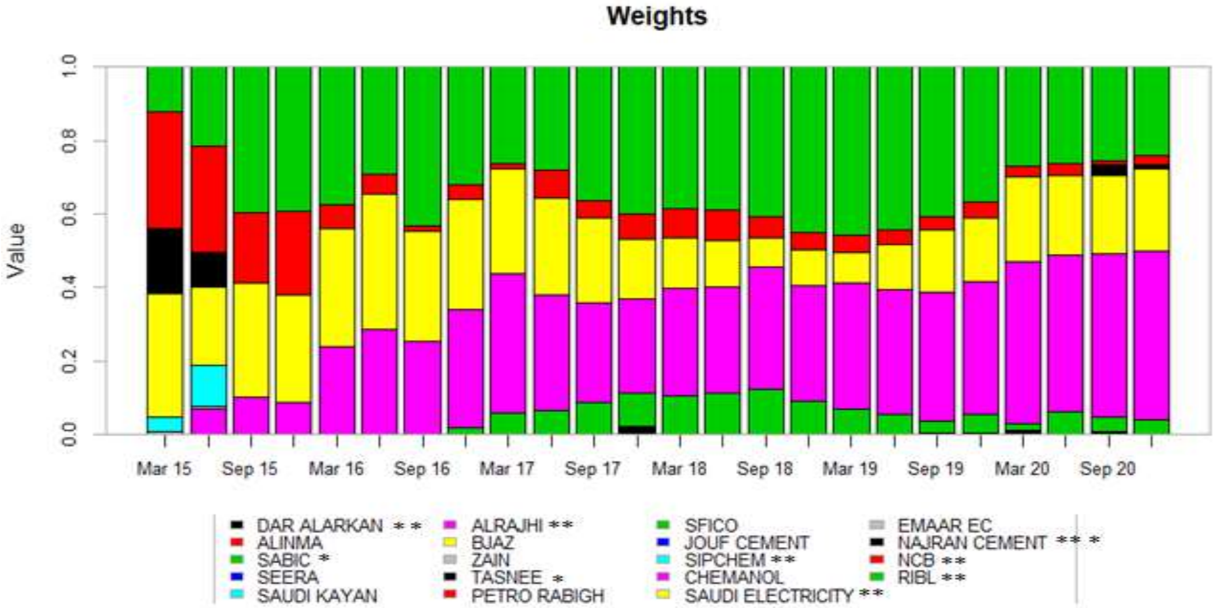
Table 3: «The rebalancing estimated portfolio return-risk for MV portfolio»

Period	Mean	StdDev
3/31/2015	0.000227	0.019294
6/30/2015	0.00018	0.015707
9/30/2015	-0.00043	0.016313
12/31/2015	-0.00059	0.015291
3/31/2016	-0.00063	0.014874
6/30/2016	-0.0002	0.014486
9/29/2016	-0.00053	0.01351
12/29/2016	-4.26E-05	0.013924
3/30/2017	4.05E-05	0.013635
6/29/2017	9.30E-05	0.013208
9/28/2017	3.05E-05	0.012509
12/31/2017	-4.54E-06	0.012105
3/29/2018	0.000145	0.011877
6/28/2018	0.000277	0.011729
9/30/2018	0.000283	0.011686
12/31/2018	0.000291	0.011712
3/31/2019	0.000447	0.011617
6/30/2019	0.000479	0.011704
9/30/2019	0.000409	0.011677
12/31/2019	0.000384	0.011685
3/31/2020	0.000191	0.012583
6/30/2020	0.000229	0.012745
9/30/2020	0.00029	0.012526
11/12/2020	0.00036	0.012582

Source: Author's computation

Table 03 Shows that the returns were positive in the two first periods, then they had negative values for a long period "from9/30/2015 to 12/31/2017". However, the portfolio has achieved positive returns again from the period "3/29/2018-11/12/2020".

Figure04: «Rebalance weights chart»



Source: prepared by the researchers

We notice from figure 04 that the program (Rstudio) was unable to identify 19 colors according to the number of companies, so it repeated the colors between companies. We explained this with the stars, as the higher number of stars for the company, this means that its relative weight is greater than the company with the same color.

Table 4 shows the annualised performance and risk measures of optimised MV versus equal-weighted portfolios using the rebalancing technique. It is observed that the annualised return of MV portfolio is bigger than the equal-weighted portfolio (0.04714542> -0.0384), the annualized standard deviation of MV portfolio is smaller (0.01258218 < 0.01431538) , we found that the historical simulation VaR and CVaR for both 95% and 99% of MV portfolio are smaller than those in equal-weighted portfolio when using rebalancing technique also, it means that the maximum loss and the average loss beyond it are bigger in equal-weighted portfolio than in MV portfolio, it is also observed that the MV portfolio performance is better than the equal-weighted portfolio based on (Sharpe and treynor ratios, CAPM Alpha and beta).

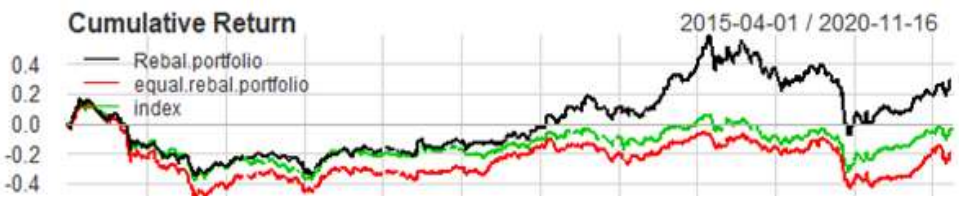
Table 4: «Performance and risk metrics of multi-period optimization versus the equal-weighted portfolio»

	Rebal port	Rebal-eq
Mean	0.04714542	-0.0384
STDV	0.1961555	0.2135
VaR 99	-0.0358	-0.0457097
VaR95	-0.0187	-0.0209101
CVaR 99	-0.0549	-0.0654168
CVaR 95	-0.0312	-0.0356795
Sharpe Ratio	0.24	-0.17
Alpha	0.0002466	-9.06E-05
Beta	0.9404973	1.080732
Treynor	0.0501282	-0.0355117

Source: Author's computation

Figure 05 shows that the rebalanced MV portfolio performs better than the Saudi Stock index (TASI) and rebalanced equal weighted portfolio based on cumulative return.

Figure 05: «Rebalance MV portfolio performance»



Source: Source: By the authors

Conclusion:

This study has estimated the Markowitz Mean-Variance optimization for portfolio weights using the linear programming technique. The two objectives of the optimization were to maximize the expected return with the minimization of risk under the constraints of "Full investment" and "Long only" using both single and multi-period techniques by rebalancing the weights. And after comparing the optimal portfolio with equal-weighted portfolio, it observed that the optimal one performed better in both single and multi-period after calculating Treynor, Sharpe ratios and CAPM Alpha and Beta, and it also observed that the maximum loss can the MV portfolio faces at 95 and 99% level of confidence is smaller than the maximum loss in equal-weighted portfolio, and also the average loss exceed VaR under 95 and 99% of the MV portfolio is smaller than the one in the equal-weighted portfolio.

The results tell us just how important is using optimization in reducing risks and also in improving the performance of the portfolio which makes the investors take this into account and avoid naïve diversification.

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