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Abstract *–A numerical study of Rayleigh-Bénard convection in a rectangular cavity has been presented. The onset of natural convection and the transition to oscillatory convection were considered in this study. The finite volume method was used to solve numerically the governing equations of the phenomenon. The pressure-velocity coupling was matched by SIMPLER Algorithm of Patankar. The study concerned has been made for the Rayleigh number varied from 10^3 to 10^6 in order to define the Ra_{c1} corresponding to the onset of convection for the different aspect ratio of the cavity. The transition threshold regime laminar-Oscillatory convection which is defined by Ra_{c-2} is determined and discussed. In addition a discussion of the different modes of bifurcation of convection were also determined and discussed.*

***Keywords**: Rayleigh-Bénard convection, natural convection, oscillatory flow, bifurcation.*

I. Introduction

Due to its practical importance in many general science and engineering applications, Rayleigh–Bénard convection has been the subject of many theoretical, experimental, and numerical studies. Since, Rayleigh–Bénard convection presents the evolution from the stationary state to the fully developed turbulent regime with many different flow patterns and sequences of bifurcations; it is widely investigated as the problems of different transition mechanisms in hydrodynamics [1-2]. Most of the published works covering natural convection in enclosures that exist today can be classified into two categories: differentially heated enclosures [3-5] and enclosures heated from below and cooled from above (Rayleigh Bénard problems) [6-8]. Benchmark solutions related to differentially heated enclosures (first group) can bee found in many numerical investigations [9-12]. However, numerical benchmark solutions related to the simplest case of 2D Rayleigh-Bénard convection are less encountered in the literature.

Some recent development in turbulent Rayleigh-Bénard convection was talked by Lappa, 2011 [13]. As usual, the case of two dimensional square enclosures is considered; he begins by the bifurcation and the symmetry breaking system. He found four cases of possible symmetries, one cell, two horizontal cells, two vertical cells, and four cells. At the beginning of bifurcation for Ra_{c1}, they can see the one cell case. For the time dependence, other forms can appear following the oscillatory mode, in second part he aboard the turbulence appearing for Ra= 10^8 and Pr = 15. Venturi et al., 2010 [14], studied the stochastic bifurcation and stability of the natural convection of Rayleigh-Bénard by different stochastic modelling approaches. They focused on fluid in the supercritical state and studied the value of Ra_{c1} in square cavity (2585). They focused also on the sensibility of the initial conditions, Bifurcation in steady state.

To controlling the amplitude of bifurcated solutions Chen et al., 1999 [15], showed that the amplitude of the bifurcated solutions is directly related to the so-called bifurcation stability coefficient. The bifurcation amplitude control is applied to the active control of Rayleigh-Bénard convection. Mas et al., 2004[16], studied the bifurcation and stability of the solutions of the Boussinesq equations, and the onset of the Rayleigh-Bénard convection. A nonlinear theory for this problem is established in this article using a new notion of bifurcation called attractor bifurcation.

Kao et al., 2007 [17], studied the Rayleigh-Bénard convection by the Lattice Boltzmann method. The critical Rayleigh number value Ra_{c1} is independent from the Prandtl number. In the fact, they have found a relationship between Prandtl, Rayleigh and Nusselt number using Pr between 0.71 and 70. They concluded that the method is simple and useful for study of Rayleigh-Bénard. Yachitaka et al., 2012 [18], giveed a method to verify the existence of bifurcating solutions of the two-dimensional problem and the bifurcation point itself.

In the work of Angeli et al., 2011 [19], the system considered is an air-filled, square-sectioned 2D enclosure containing a horizontal heated cylinder. The results are shown respecting the variation of the Rayleigh number, and changing the three values of the aspect ratio A. Chaotic flow features are detailed for the case of A = 2.5, for which the nature of the first bifurcations of the low Rayleigh number (Ra) fixed point solution, and the related critical Rayleigh number values were determined. Finally, the analysis of global heat transfer data showed that the Nu and Ra relationship is sensibly influenced by the transition mode associated with each a value. A correlating equation for the average Nusselt number on the cylinder, derived for the sub critical case, was found to be valid up to slightly supercritical Ra-values.

Recently, Raji et al., 2013 [20], presented a numerical results of the natural convection in a square cavity filled with air, the temperature of the lower horizontal surface is kept constant(hot), while that of the upper surface (cold temperature), the remaining upright walls are considered adiabatic. The Rayleigh number ($10^3 < Ra < 7\times10^6$, three different solutions are obtained (single-cell flow, two-cell vertical flow and horizontal flow bicellular).

The aim of this study is to propose two dimensional numerical solutions related to natural convection in a square enclosure heated from below and cooled from above. We are interested to the determination of the onset of natural convection and the transition from laminar to oscillatory convection in Rayleigh –Bénard configuration. The effect of aspect ratio on the two cases was also examined.

II. Geometry of the problem and Mathematical formulation

The cavity which is heated from below and cooled from above corresponds to the configuration of the Rayleigh-Bénard dealing with the stability and motion of a fluid confined between two horizontal plates that are maintained at uniform temperatures (Fig. 1).

We consider the flow is incompressible and satisfied the Boussinseq approximation. To give the conservation equation in dimensionless form, we have used the dimensionless variables respectively: $X=\frac{x}{H}$, $Y=\frac{y}{H}$, $U=\frac{u}{\nu/H}$, $V=\frac{v}{\nu/H}$ $P=\frac{p}{\rho(\alpha/H)^2}$, $\theta=\frac{T-T_f}{T_C-T_f}$, $\tau=\frac{t}{H^2/\alpha}$, for Cartesian coordinates, velocity components, the pressure, temperature and the time respectively.

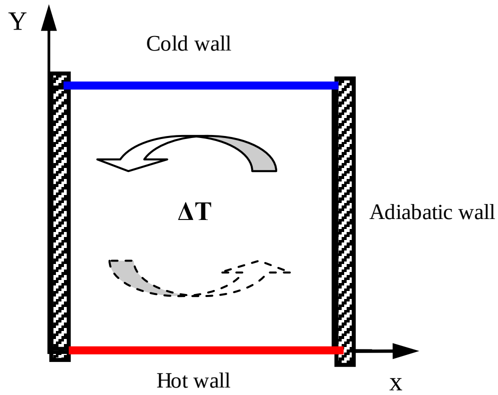


Figure 1. Schematic configuration of Rayleigh-Bénard probleme.

After, we obtained the dimensionless equation form as following:

- Mass conservation equation

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (1)$$

- X-momentum equation:

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \qquad (2)$$

- Y-momentum equation:

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{Ra}{\Pr}\theta+\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right) \qquad (3)$$

- Energy equation :

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\Pr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \qquad (4)$$

The obtained parameters of flow appear in the equation (1-4) are: Rayleigh number and Prandtl number defined respectively as: $Ra=\frac{g\beta H^3\Delta T}{\nu\alpha}$ and $\Pr=\frac{\nu}{\alpha}$.

The initial and boundary conditions are given as following equations:

At $\tau=0$: $U=V=0$ and $\theta=1$ (5a)

At $\tau>0$:

- $Y=0$: $0\le X\le 1$, $U=V=0$, $\theta=1$ (5b)
- $Y=1$: $0\le X\le 1$, $U=V=0$, $\theta=0$ (5c)
- $X=0,1$: $0\le Y\le 1$, $U=V=0$, $\partial\theta/\partial X=0$ (5d)

III. Numerical resolution

The systems equation (Eqs.1-4) associate with initial and boundary conditions (Eqs.5a-d) has been solved by finite-volume method [21]. The Quick scheme is used for discretization of convective and diffusive terms. The couple's velocity-pressure is solved by SIMPLER Algorithm. The obtained algebraic equations are solved by the line-by-line tri-diagonal matrix algorithm (TDMA). The convergence is declared when the maximum relative change between two consecutive iteration levels fell below than 10^{-4}. Calculations are carried out on a PC with CPU 3 GHz.

The grid independency of the numerical solution was established on a careful analysis of three grids (60×60), (80×80) and (100×100); we have used a refined grid in the lower and upper wall, due to the existence of a strong temperature and velocity gradients near this walls, refining value is equal to 1.05. We assume for this purpose the case of a steady natural convection flow with Ra = 10^4. The Table 1, show the grids effect on the deferent flow parameter (Ψ, U, V, and Nu). In table 2, we have compared our results with those of Ben Cheikh [3], and found good agreement with our results, even if our grid seems coarse, we find that it is consistent with the literature, like Venturi [14] and Gelfgat [22] which used the same kind of grid, and Shishkina[23] determine the minimum number of nodes to have an accurate result. This grid is considered to show the best compromise between computational time and precision.

Table 1. Flow Parameter: Ψ_{max}, Nu_{moy}, $|V_{max}|$, $|U_{max}|$,for different grid size (Ra = 10^4).

Grid size	Ψ_{max}	Nu_{moy}	$\|V_{max}\|$	$\|U_{max}\|$
60 × 60	321.79	2.1	26.54	25.7
80 × 80	316.61	2.02	27.81	26.07
100 × 100	316.43	1.98	27.29	26.18

Table 2. Comparison of our results, with those of [3] for Ra =10^4

Our results		Results of Ben Cheikh et al. [3]		Relative error %	
$\|V_{max}\|$	$\|U_{max}\|$	$\|V_{max}\|$	$\|U_{max}\|$	$\|V_{max}\|$	$\|U_{max}\|$
26.54	25.7	26.36	25.22	0.68	1.9

IV. Results and discussions

All the resultats presented herein are given in the dimensionless form. To allocate more confidence in our numerical results, we have established some comparisons with other studies available in the literature. One with the benchmark solution (Table 3), the work of Val Davis [10], and the second was made with Rayleigh-Bénard convection (Table 4), studied by Turan [24].

We begin with a comparison of Nusselt number progress in Rayleigh-Bénard convection with those of Turan [24] and the Benchmark results [24], as given in Table 4. We can see a little relative error between the results, less than the1.5% of our results.

The second comparaison is made with the works of Val Davis [10], presented on the Table 4. In this case the heat gradient is horizontal. The results compared shows a good agreement of the results with a very small difference error for all flow parameter.

Table 3. Comparison of our results with those of Turan [24] and Benchmark [10].

Ra	Our results	Results of Turan [24]	Benchmark Results [10]	Relative error with [24] (%)
10^3	1	1 .0004	1	0.039
10^4	2.2	2.1581	2.154	1.49
10^5	3.9	3.9103	3.907	0.26
10^6	6.4	6.3092	6.36	1.439

Table 4. Comparison of our results with those of Val Davis [10].

Ra	Ψ	Ψ [10]	Nu	Nu [10]
10^3	1.171	1.170	1.118	1.117
10^4	5.061	5.059	2.25	2.240
10^5	9.147	9.059	4.53	4.505
10^6	16.22	16.24	8.9	8.810

IV.1. Onset of natural convection

The heat convection mode is started by energy accumulation (conduction) then the flow motion. The beginning of the convection in the case of Rayleigh-Bénard convection, Then we will proceed to the consideration of the phenomenon begin by studying the threshold of the beginning of the convection in the case of Rayleigh-Bénard convection, and then calculate the critical value of Rayleigh and see the effect of the aspect ratio on this critical value, we will compare it with the theoretical result; and then make a close approach to the cavity of infinity, then we will consider the extension of the cavity.

A linear stability analysis of the Boussinesq equations about the linearly conducting profile between two infinite horizontal no-slip plates shows that the critical Rayleigh number $R_{c1} \approx 1707.76$, the values of R_{c1} is independent of the Prandtl number [22], This critical value is necessary to determinate the mode of heat transfer; it represents the threshold of the onset of convection and the first bifurcation flow structure.

We examine the effect of the aspect ratio on the first Rayleigh critical number (Rac_1). Four aspect ratios of the Rayleigh-Bénard cavity were examined in this work are A = 1, 2, 4 and 8. In Figure 2, we present the average Nusselt number along the hot wall in function of Rayleigh number. The value of Rac_1 on various aspect ratios is given in Table 5.

We note that although the aspect ratio has an influence on the value of Rac_1 therefore the fluid motion is depending with the geometry of the cavity. In the literature, they found that for infinite cavity Ra_{c1} = 1708 but for a square cavity (A = 1) the value of critical Rayleigh is much greater (Ra_{c1} = 2585.01). In Table 6, we compare our results of Ra_{c1} for a different aspect ratio with the results of Gelfgat [22]. A good agreement was obtained with results from the literature.

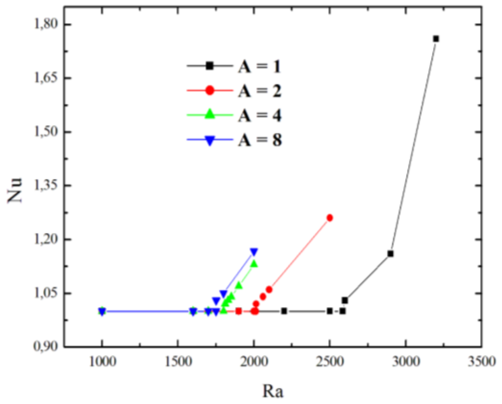


Figure 2. Average Nusselt Number in function of Rayleigh number for aspect ratio A = 1, 2, 4 and 8.

Table 5. Critical Raleigh number for different aspect ratio.

Aspect ratio	1	2	4	8
Rac_1	2585.01	2013.5	1810	1752

IV.2. Onset of oscillatory natural convection

At the beginning of the convection the flows are laminar and do not depend on time, but after a certain value Ryeigh number, there will appear an oscillating state and the beginning of the time dependence. We will confirm that the oscillations are physical in origin, and then make an evaluation for different probes into the cavity.

According to linear theory, a significant change in convection will take place as soon as the critical Rayleigh number is exceeded and Rayleigh-Bénard problem will become nonlinear. In our study we found that the time dependente flow begins at around 16 Ra_{c1}, it's the oscillatory mode. The critical value of this mode is Ra_{c2}= 4.58×10^4. To view the physical scillatroy flow, we have chosen many arbitrary probes in the cavity.

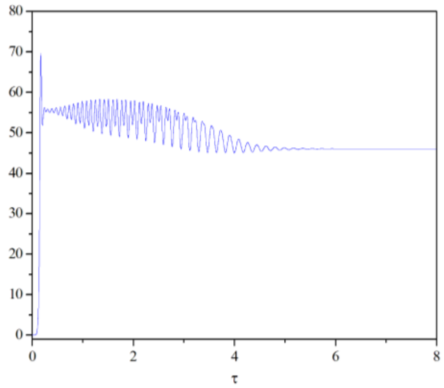
In order to determine the beginning of the unsteady state, we note that the steady state is obtained up to the value of Ra = 4×10^4 (Figure 3a). However, past the value of Ra_{c2} = 4.58×10^4, the flow becomes oscillatory (Figure 3b). To show that the oscillations are not numerical but physical, for that we will fixe a probe in our cavity with the same flow parameter but we reduce the time step to the half, the Figure 4 shows that there is no influence of the time step on the oscillation amplitude. So these instabilities are physical and not numerical. Another test of oscillatory mode is the phase portrait as showing in Figure 5. The phase portraits reflect the change of hydrodynamic and thermal parameters between them. So, for a periodic oscillatory regime at point in flow, these changes are closed circles reflect the periodicity of the flow regime. Moreover, for no periodically oscillations these changes appear become the endpoints or other unordered structure.

Oscillatory flow regime occurs for the critical value of Rayleigh number 4.58×10^4. We note that steady state is obtained up to the value of Rayleigh number Ra = 4×10^4. For illustrate the temporal evolution of the horizontal, vertical velocity component, and temperature, we chose the same probes (P_1, P_6, P_7 ,P_8) in the cavity to look into the field, and the results are shown in Figures 6a-c.

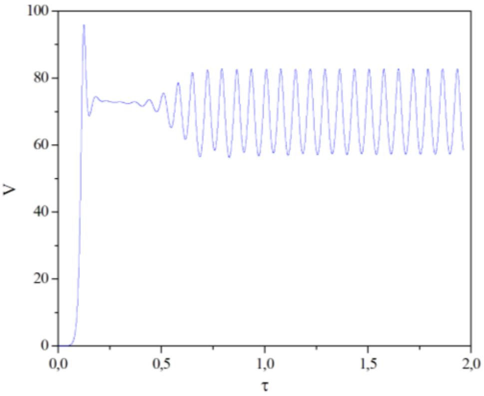
Table 6. Physical location of measurement probes.

/	P_1	P_2	P_3	P_4
X	0.5	0.75	0.75	0.25
Y	0.5	0.75	0.25	0.25
/	P_5	P_6	P_7	P_8
X	0.25	0.2	0.1	0.6
Y	0.75	0.1	0.4	0.9

It is clear that these profiles are oscillatory and periodic, so the flow regime is unstable. We observe that the amplitude of these oscillations changes from one point to another in the enclosure. The difference in the degree of oscillation depends on the location of probes in relation to walls adiabatic to the hot wall and the cold wall. For example, probe 1 (Table 6) is in the centre of the enclosure, the probes 6, 7 and 8. Probes 2, 3, 4 and 5 are used to explain the wave propagation flow.



a) Ra = 4× 10^4.



(b) $Ra_{c2} = 4.58\times10^4$

Figure 3.Time-dependente of dimensionless vertical velocity at the probe P_1 for two value of Rayleigh number.

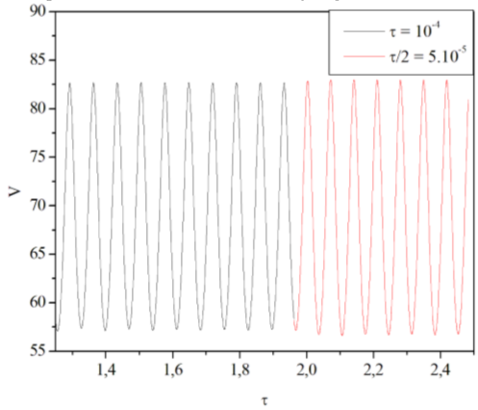


Figure 4. Time dependente of vertical velocity componente for two time setp in P_1 (chosed arbirary) and for Ra_{c2} = 4, 58 $\times10^4$

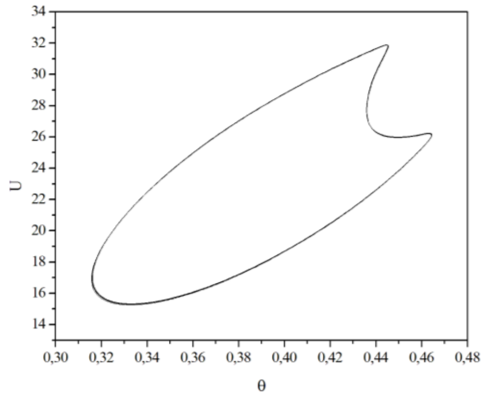
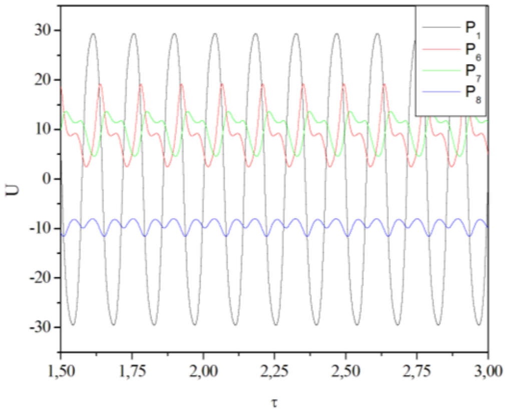
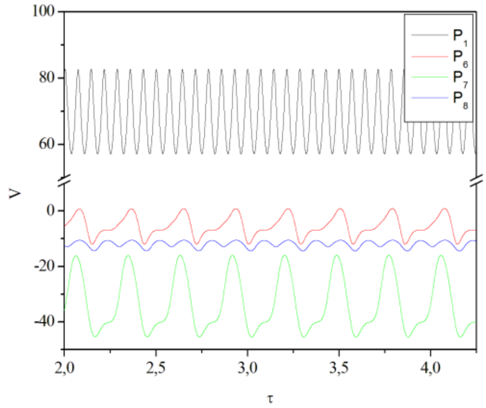


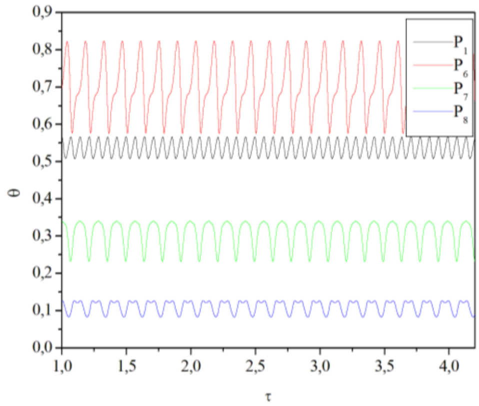
Figure 5. Phase portrait of horizontal velocity component in function of temperature at point P_2 (chosed arbirary), for Ra_{c2} = 4, 58 $\times10^4$.



(a) Dimensioless horizontal velocity component U.



(b) Dimensionless vertical velocity component V.



(c) Dimensionless temperature.

Figure 6. Time-dependente of dimensionless horizontal (a), vertical velocity component (b) and dimensionless temperature (c), in probes P_1, P_2, P_3, P_4, P_5, P_6, P_7 and P_8 (chosing arbirary), for Ra_{c2} = 4,58$\times10^4$.

The amplitude of the temperature is greater on the hot wall, as the particles back into the wall; it means average amplitude of the adiabatic wall. But there are small amplitude for the center point and the cold wall.

It is clearly noticed on the probes 2 and 5, the same amplitude, the same structure of the wave, except in phase shift is due to the wave propagation, similarly for the probes 3 and 4 with smaller amplitude.

To better understand the oscillatory flow we have presente the time-dependente flow in a period of time for Ra_{c2} = 4,58$\times10^4$ (Figure 7), and illustrate the structure of the flow by the stream line evolution over time in one period of time (at the time τ_a, τ_b, τ_c, τ_d, τ_e and τ_f). It was found that the flow remains unstable and unicellular streamlines which looks oscillatory (Figures 8a-g). we can see tow cells counter rotating, the left one is bigger than the Second, after they have the same shape in the symmetry case, after the right one becomes bigger.

For the isotherms, we can see that the head of the mushroom move slowly between the left side and the central axe of the cavity. In the next time period it will move from the central axe to the right side (Figure 9a-g).

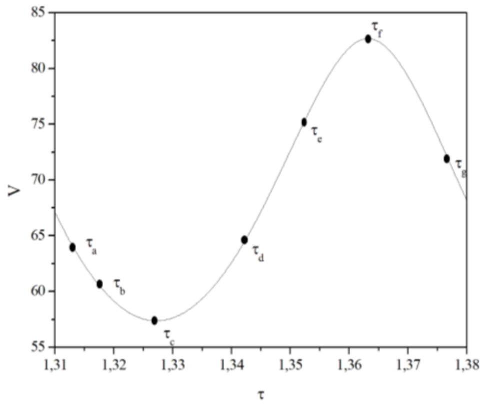
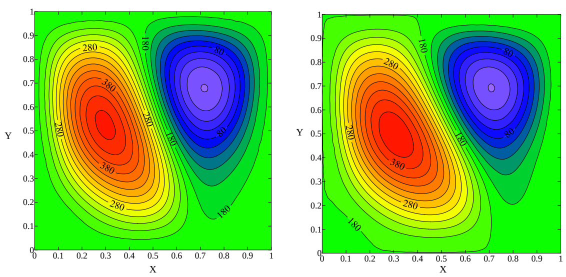
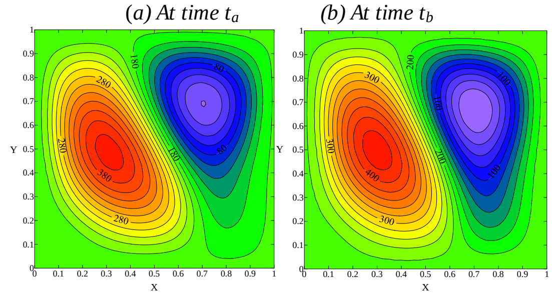


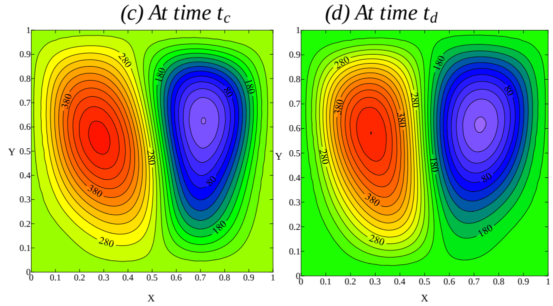
Figure 7. Time- evolution of the vertical velocity component in τ =1, for Ra_{c2} = 4, 58 × 10^4.



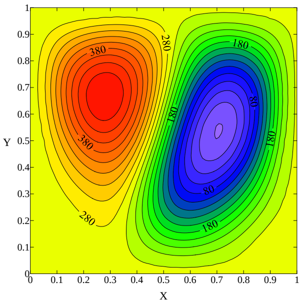
(a) At time t_a (b) At time t_b



(c) At time t_c (d) At time t_d

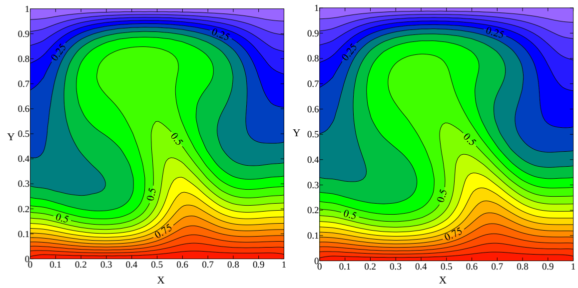


(e) At time t_e (f) At time t_f.

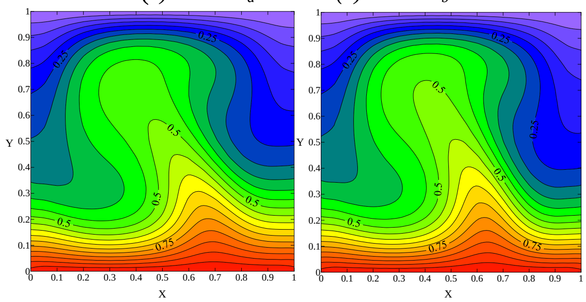


(g) At time t_g.

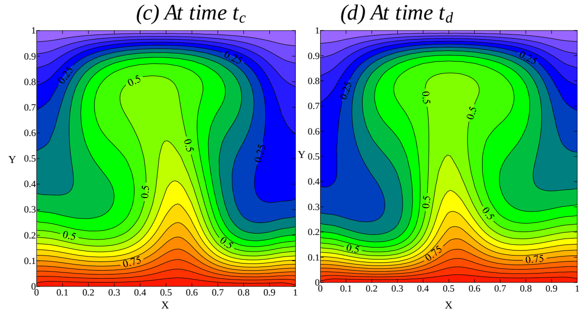
Figure 8. Stream line flow over time for one period of time for Ra_{c2} = 4.58× 10^4.



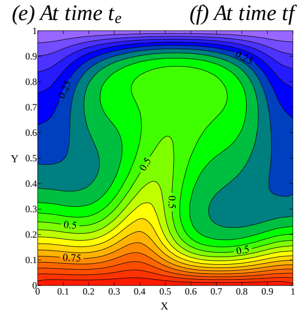
(a) At time t_a (b) At time t_b



(c) At time t_c (d) At time t_d



(e) At time t_e (f) At time tf



(g) At time t_g.

Figure 9. Isotherm line over time in one period of time for Ra_{c2} = 4.58×10^4.

IV.3. Flow Bifurcation

The bifurcation flow in the cavity and the different forms that may have: bifurcated cells, unicellular, two vertical cells, two horizontal cells. This study will be made up of Ra_{c1} to $Ra = 10^6$. Now, we are going to talk abut bifurcation as we have discussed in the first is a condition that causes a small change in the physical state a big change. We can observe different bifurcation flow structures which are shown in Table 7.

Table 7. Flow Structure for different Rayleigh numbers.

Ra	Cell number and disposition	Flow regime
$[2.5\times10^3, 1.8\times10^4]$	Unicellular (UC)	Steady state
$[1.9\times10^4, 4.57\times10^4]$	Two vertical cells (2VC)	Steady state
$[4.58\times10^4, 4.99\times10^4]$	Two vertical cells (2VC)	periodic oscillatory
$[5\times10^4, 4.1\times10^5]$	Unicellular (UC)	Steady state
$[4.2\times10^5, 10^6]$	Two horizontal cells (2HC)	periodic oscillatory

The Figure 10, represente the logarithmic bifurcation diagram of Rayleigh-Bénard convection. The figure also summarizes the stream lines and isothermal line according the Rayleigh number.

The first bifurcation is observed for Ra_{c1}, which represents the transition from conduction to convection (onset of convective). This bifurcation is represented by the one principal cell in the middle of the cavity and with a clockwise flow direction. Followed by another bifurcation appears for the value of 1.9×10^4. In this case the basic unit of our convection (UC) is divided in two, two vertical cell (2VC), one cell with a clockwise direction and the other has a counter clockwise, to move on impulse isothermal lines. In this case the particles do not lie near the adiabatic wall but in the center of the hot wall and the cold particles come down almost close of the two adiabatic walls, it will create the form that resembles a mushroom. Rayleigh-Bénard convection is very sensitive area which has a lot of bifurcations, as a result the two cell bifurcation; at 5×10^4, another single bifurcation cell appear so similar to the first. Finally, for the value 4.2×10^5, it has two cells but this case, they are horizontal (2HC), hot and cold particles, go up and down on one and the same adiabatic wall.

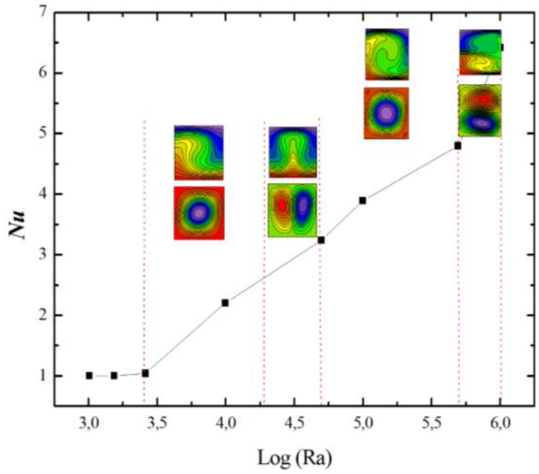


Figure 10. Logarithmic bifurcation diagram of Rayleigh-Bénard convection. *The figure also summarizes* Stream lines and isothermal line.

V. Conclusion

A numerical study of Rayleigh-Bénard convection in a rectangular cavity has been presented. The onset of natural convection and the transition to oscillatory convection were considered in this study.

The study concerned has been made for the Rayleigh number varied from 2.5×10^3 to 10^6 in order to define the Ra_{c1} corresponding to the onset of convection for the different aspect ratio of the cavity. The transition threshold regime laminar-Oscillatory convection which is defined by Ra_{c2} is determined. In the range studied there five modes bifircation of Rayleigh-Bénard convection in the range studied: unicellular (UC), two vertical cells (2VC) in steady state, two vertical cells (2VC) in oscillatory periodic state, and two horizontal cells (2HC) in oscillatory periodic state.

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Nomenclature

C : Specific heat at constant pressure [$J.kg^{-1}.K^{-1}$]
g : Acceleration of gravity [$m.s^{-2}$]
H : Dimensional height of the enclosure [m]
W: Dimensional width of the enclosure [m]
A : Aspect ratio [-]
P : Dimensionless pressure[-]
Q: Heat Flux [W]
u, v : Speeds components [$m.s^{-1}$]
U, V : Dimensionless speeds components [-]
T : Dimensional temperature [K]
x, y : Dimensional coordinate space [m]
X, Y : Dimensionless coordinate space [-]
t : Dimensional time [s]

Greek symbols

α : Thermal diffusivity [$m^2.s^{-1}$]
λ : Thermal conductivity of the fluid [$w.m^{-1}.K^{-1}$]
Γ : Diffusion coefficient [-]
υ : Kinematics viscosity [$m^2.s^{-1}$]
μ : Dynamic viscosity [$kg.s^{-1}.m^{-1}$]
ρ : Density [$kg.m^3$]
τ : dimensionless time [-]
β : Coefficient of thermal expansion at constant pressure [K^{-1}]
θ : Dimensionless temperature [-]
ΔT Temperature difference [K]

Dimensionless Numbers

Ra Rayleigh number
Ra_{c1} First critical Rayleigh number (onset of convection)
Ra_{c2} Second critical Rayleigh number (oscillatory convective flow)
Nu Nusselt number
$\Pr$ Prandtl number

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