Democratic and Popular Republic of Algeria
Ministry of Higher Education and Scientific Research
University of BECHAR

Printed from

Journal of Scientific Research

http://www2.univ-bechar.dz/jrs/

Dynamics contact between two Deformable Elastic Bodies

Slimane BENAYAD[1], Abdelmadjid BERGA[1], Nazihe TERFAYA[1]
[1] Laboratory FIMAS University of Bechar

Published on 10 December 2011

Journal of Scientific Research
ISSN 2170-1237

Journal of Scientific Research

The Editor, on behalf of the Editorial Board and Reviewers, has great pleasure in presenting this number of the Journal of Scientific Research. This journal (ISSN 2170-1237) is a periodic and multidisciplinary journal, published by the University of Bechar. This journal is located at the interface of research journals, and the vulgarization journals in the field of scientific research. It publishes quality articles in the domain of basic and applied sciences, technologies and humanities sciences, where the main objective is to coordinate and disseminate scientific and technical information relating to various disciplines.

The research articles and the development must be original and contribute innovative, helping in the development of new and advanced technologies, like the studies that have concrete ideas which are of primary interest in mastering a contemporary scientific concepts. These articles can be written in Arabic, French or English. They will not be published in another journal or under review elsewhere. The target readership is composed especially of engineers and technicians, teachers, researchers, scholars, consultants, companies, university lab, teaching techniques and literary ... The journal is obtainable in electronic form, which is available worldwide on the Internet and can be accessed at the journal URL:

http://www2.univ-bechar.dz/jrs/.

ISSN 2170-1237

Journal of Scientific Research

P.O.Box 417 route de Kenadsa
08000 Bechar - ALGERIA
Tel: +213 (0) 49 81 90 24
Fax: +213 (0) 49 81 52 44
Editorial mail: jrs.bechar@gmail.com
Submission mail: submission.bechar@gmail.com
Web: http://www2.univ-bechar.dz/jrs/

Journal of Science Research N 1, Vol. 1, p. 36-41

Dynamics contact between two Deformable Elastic Bodies

Slimane BENAYAD, Abdelmadjid BERGA, Nazihe TERFAYA

Laboratory FIMAS University of Bechar

Abstract – *The flexibility method has been applied successfully to the contact problems modeling with friction between two deformable elastic bodies for the static cases. This article presents the extension of this method to dynamic problems. The main objective of the suggested work is to develop an algorithm based on the non-smooth contact dynamic method to study the phenomenon and dynamic friction contact between two deformable elastic bodies and the use of the flexibility method. For friction, Coulomb's law is considered. The results obtained are encouraging and will undoubtedly pave the way for other aspects and areas of research in this area.*

Keywords: *Contact, friction, deformable bodies, finite element method, flexibility method, NSCD method*

1. Introduction

The problems of contact and friction present several inherent difficulties in their non-linear, irregular and multi-scale, making their analytical solution often impossible.

The impossibility to calculate analytically solutions to problems of contact, on the one hand, and development of methods and computational tools, on the other hand, nave to the approximate resolution of these problems. An abundant literature has been devoted to the numerical formulations, analyses, approximations and resolutions of the contact problems, since the beginning of the Seventies [1][2][3][4][5][6].

The main objective of our work is to develop an algorithm based on the non-smooth contact dynamics method to study the dynamics contact and friction phenomenon between two deformable bodies by using the flexibility method. For friction, Coulomb's law is considered. We limit the present study at the contact plane problems with friction generated by elastic strain.

2. Unilateral contact condition

Let us consider a contactor *A* of contour Γ_A which comes into contact with an obstacle *B* of contour Γ_B. To study the kinematics of the contact of *A*, we should only consider a point *P* on the contour Γ_A, at the vicinity of the obstacle *B*. The orthogonal projection of *P* on the surface Γ_B of the obstacle *B* defines a point P', which will be the origin of the local frame related to the obstacle. The normal coordinate x_n of the point *P* in the local frame is equal to $\overline{p'p}$. The unit vector $\{n\}$, oriented towards the outside of the obstacle, is normal to the surface at P'. The two unit vectors $\{t_1\}$ and $\{t_2\}$, orthogonal to $\{n\}$, define a plan tangent to the surface of the obstacle at the point P' (figure 1).

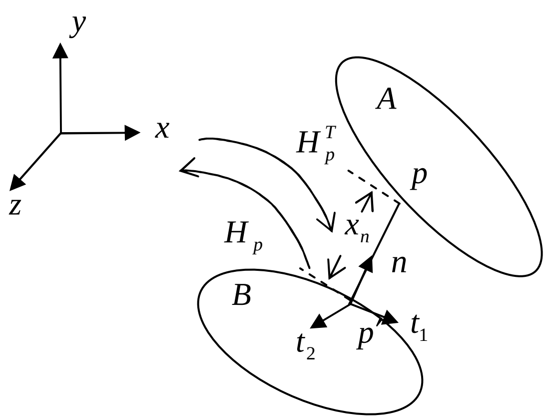

Figure 1: local contact Frame

The unilateral contact conditions are expressed in the local frame as follows:

a) *- Impenetrability:*

$$x_n \geq 0 \qquad (1)$$

When this condition is checked, we have two possible cases:

b) *- Static State of contact (non-adhesion):*

$$x_n = 0 \Rightarrow r_n \geq 0 \qquad (2)$$

r_n : Normal component of the contact reactions

***c)** - State of non-contact:*

$$x_n > 0 \Rightarrow r_n = 0 \qquad (3)$$

This set of relations can be gathered in the form of the Signorini conditions:

$$x_n \geq 0\,,\ r_n \geq 0\,,\ x_n r_n = 0 \qquad (4)$$

Or in the equivalent form [7] [8] [9]:

$$\forall \rho_n > 0\ \{r_n\} = proj_{R^+}(r_n - \rho_n x_n) \qquad (5)$$

Where ρ_n is a certain strictly positive real number.

3. Coulomb's friction law

The most used friction law today is that of Coulomb. This law connects, by the intermediary of a coefficient of friction presumably constant, the normal and tangential components of the forces of contact. In the case of isotropic contact and friction, it is written as follows:

$$\|r_t\| \leq \mu r_n \qquad (6)$$

μ : Friction coefficient;

r_t and r_n : Tangential and normal component of the contact reactions;

If the sliding velocity is different from zero, the force of friction is opposed to the sliding velocity:

$$\|v_t\| > 0 \Rightarrow r_t = -\mu r_n \frac{v_t}{\|v_t\|} \qquad (7)$$

v_t : Tangential component of relative (sliding) velocity;

In numerical calculation, one often uses the equivalent form [7][8][9]:

$$\forall \rho_t > 0\ \{r_t\} = proj_c(r_t - \rho_t u_t) \qquad (8)$$

Where *c* represents interval $\langle -\mu r_n, \mu r_n \rangle$ in the two-dimensional case; or the radius disc $D = \mu r_n$ in the three-dimensional case.

If $\|r_t\| < \mu r_n \Rightarrow$ (adherence)

If $\|r_t\| = \mu r_n \Rightarrow$ (sliding)

4. Modelisation

4.1. Dynamic resolution

The kinematics character of the contact phenomenon often implies the use of a dynamic model in the digital simulation; some problems of contact are even very difficult to solve into quasi-static. For example, in the case of a released deformable solid of the boundary geometrical conditions in contact with a rigid foundation, one obtains a singular system of equations. The solution consists of solving the problem in dynamics, because the inertia terms even though not very significant compared to the stiffness terms also make it possible to stabilize the badly conditioned equilibrium equations. In addition, the integration of the dynamics equations in the presence of contact and friction requires adapted additional algorithms [10].

In the case of linear elasticity and under the assumption of the small deformations, the equilibrium equations which control the dynamic response of the system are written:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{F\} \qquad (9)$$

$[M]$: The mass matrix.

$[C]$: The damping matrix.

$[K]$: The stiffness matrix.

$\{U\}, \{\dot{U}\}, \{\ddot{U}\}$: Are respectively, the vectors of displacement, the velocities and accelerations.

$\{F\}$: The external vector of load acting.

4.1.1. The non-smooth contact dynamic method

Method (NSCD) initiated and developed by J. J. Moreau [11] and M. Jean [12], is dedicated to the resolution of the problem relating to the dynamic systems in the presence of unilateral strains. The integration dynamics discretized is carried out in two steps. First, the classical differential connection (9) is rewritten in terms of differential measurements in a weak form. Then, one integrates by means of a θ-method the continuous terms (displacement, forces applied).

The dynamics equations are rewritten in the sense of measurement in the form [11]:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{F\} + \{R_c\} \qquad (10)$$

M. Jean proposed the following approximations basing himself on a traditional θ-method (for $\theta \in \left]0,1\right]$) and by direct integration of the equation (10) on the interval $[t, t + \Delta t]$ [13][14]:

$$\overline{K}\Delta U = \frac{1}{\Delta t\theta} M\dot{U}_t - KU_t + \overline{F} + R_c \qquad (11)$$

$$U_{t+\Delta t} = U_t + \Delta U \qquad (12)$$

with

$$\overline{K} = \frac{M}{\Delta t^2 \theta} + \frac{C}{\Delta t} + \theta K \qquad (13)$$

$$\overline{F} = \theta F_{t+\Delta t} + (1-\theta) F_t \qquad (14)$$

Velocity will be calculated at the end of the time step by the expression:

$$\dot{U}_{t+\Delta t} = \left(1 - \frac{1}{\theta}\right)\dot{U}_t + \frac{1}{\theta \Delta t}\Delta U \qquad (15)$$

This method is unconditionally stable for values of $\theta \geq 0.5$.

4.2. Resolution of the contact problem

The majority of the problems of contact require the introduction of the impenetrability condition, which is a unilateral condition on the borders of the solids in contact that leads to variational inequations [9].

The various methods of resolution of a contact and friction problem are characterized primarily by the treatment of the contact reactions in the equilibrium equations.

For the direct methods, the contact conditions are not introduced explicitly into the variational formulation of the problem. Compared to the rigidity methods, the contact reactions are calculated a priori, and then added in the discretized equations of balance, as known additional external forces.

Among these methods, the flexibility method was retained for the development of our contact system. This method was developed by Francavilla and Zienkiewicz [15] for an elastic contact problem without friction, and by Sachdeva and Ramakrishnan [16] for the contact with friction. It was later modified and improved by other authors namely M.Jean [8] Feng, Touzot and De Saxce [9][17], N.R. Terfaya, A. Berga [18], D. Douli [19], and S. Benayad [1].

The method principle is to calculate a matrix of flexibility, which relates only to the candidate nodes with the contact starting from the matrix stiffness. This matrix expresses displacements of the nodes of the grid, due to unit forces applied to the points of contact. In practice, it constitutes part of the reverse of the structure stiffness matrix, reduced to the contact nodes, transformed in the local frames. Then the reactions of contact are calculated repeatedly, by checking the contact conditions and the friction law.

The system (11) transferred in the local reference mark $(\vec{t}_1, \vec{n}, \vec{t}_2)$ gives the solution:

$$U = U_{lib} + WR_c \qquad (16)$$

Where the term U_{lib} represents displacements of the particles of the system with taking into account of the contact, transformed in the reference mark buildings:

$$U_{lib} = W\left(\frac{1}{\Delta t\theta} M\dot{U}^t - KU^t + \overline{F}_{ext}\right) \qquad (17)$$

With: $W = \overline{K}^{-1}$

By projection on the nodes of contact, equation (16) is written:

$$u_c = H^T WH.r_c \qquad (18)$$

u_c : Displacements of the nodes candidates to the contact

H : The passage matrix of the total frame to the local one.

W : The flexibility matrix.

In then seeks to calculate (u_c, r_c) while checking the conditions of unilateral contact and the law of friction.

5. Example of application

One considers two deformable bodies of the same elasticity module *E= 1GPa*, of the same Poisson's coefficient $\nu = 0{,}3$ and the same density $\rho = 0{,}01$ kg/cm^3. Body 1 is supposed to have an initial velocity v_y =15 cm/s. Simulation lasts four 0,04s for a step of time equal to 8.E-04. Dimensions and the boundary conditions are shown on (Figure 2). On the surface of contact we choose five nodes likely to come into contact with the deformable foundation A, B, C, D, E and an initial variation h= 1mm. The coefficient of friction μ = 1,0[20].

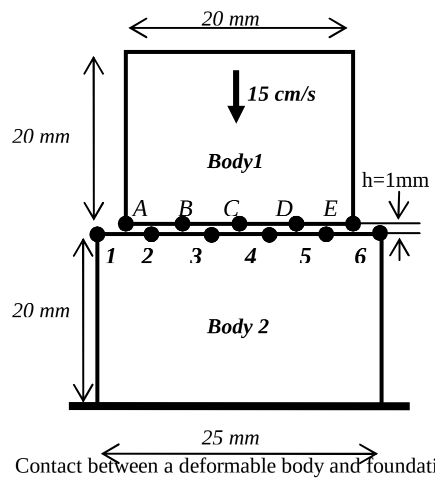

Figure2: Contact between a deformable body and foundation (punching).

Initially, and to test the developed algorithm, one regards the body 2 as being a rigid foundation. That is possible either by the increase in the elasticity module E or by fixing of the nodes of that upper surface.

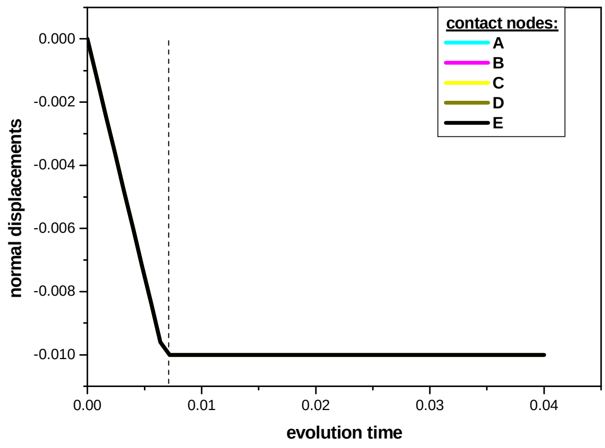

Figure 3: Normal displacements of the contact nodes.

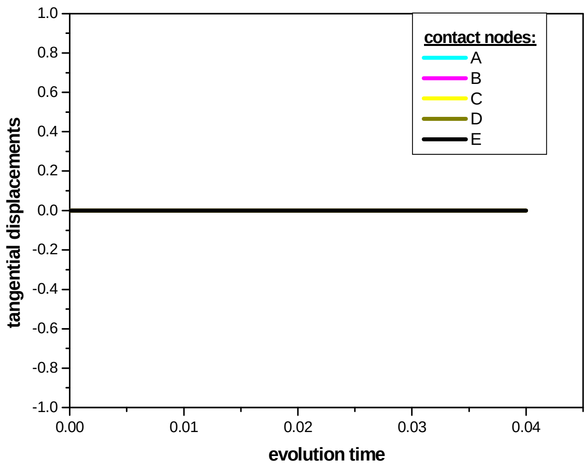

Figure 4: Tangential displacements of the contact nodes.

Figures 3 and 4 show that the nodes A, B, C, D, E come at the same time into contact at the moment t=7.103s and adheres with the obstacle that is translated by a tangential constant displacement equal to zero. No penetration is noticed. This proves the validity of the code for the rigid -deformable contact case.

In the second case, one considers two bodies regarded as deformable. On the figures below, one shows normal and tangential displacements for the contact nodes and the obstacle nodes.

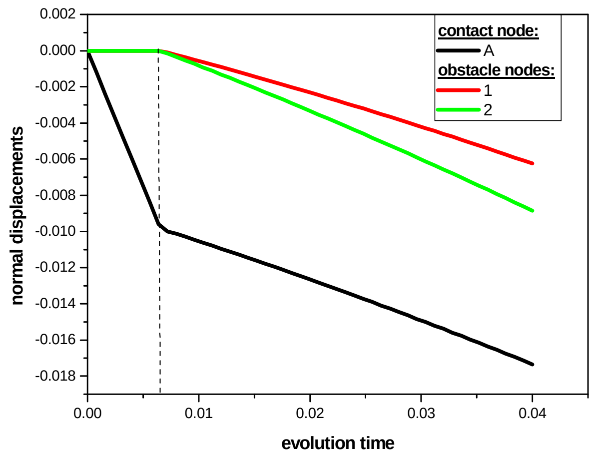

Figure 5: Normal displacements of the contact intermediate node A and obstacle nodes 1, 2.

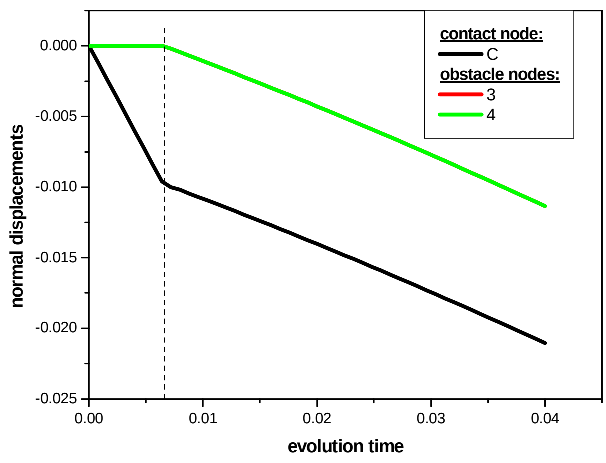

Figure 6: Normal displacements of contact node C and obstacle nodes 3, 4

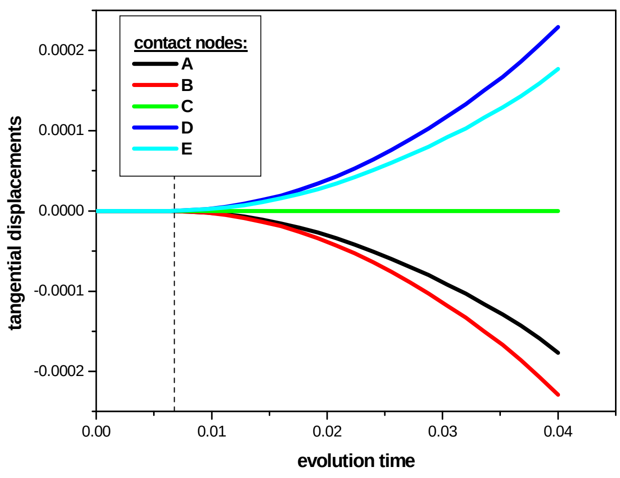

Figure 7: Tangential displacements of the contact nodes

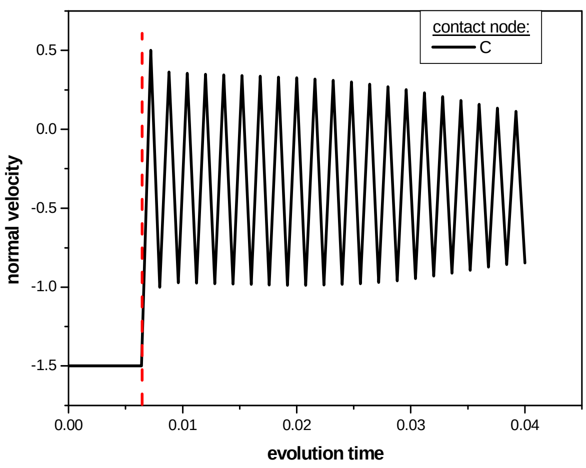

Figure 8: Normal velocity of the central node

The contact is established at the moment $t=7.10^{3}$ or one notices an abrupt change in the normal velocity (figure 8), and two states of contact can be noticed according to figure 7. Nodes A, B, D and E slide on the obstacle. While node C adhered with the obstacle where there is a constant tangential displacement equalize to zero. For the obstacle, and since the nodes A and B come into contact with the two segments characterized by the nodes (1, 2) and (2, 3), node 2 undergoes the greatest reactions and thus a larger normal displacement compared to that of node 1 (figure 5).

On the other hand and by symmetry, nodes 3 and 4 undergo the same the reactions, and therefore equal normal displacements (figure 6).

One must note here that the algorithm convergence was obtained by five iterations for local calculation only (calculation of the reactions).

6. Conclusion

We have presented, in this article, a treatment of the dynamic problems of contact and friction between two elastic deformable bodies.

One develops an algorithm based on the flexibility method. The friction law that we have considered, is the Coulomb law with a constant coefficient. But the open architecture of the developed code allows the taking into account of other laws of friction. The example has made it possible to show the limits of a master / slave traditional formulation.

The dynamic problem of unilateral contact with Coulomb friction of in linear elasticity is formulated so as to take into account possible velocity discontinuities at the time of the interaction.

We have tried to present the NSCD method for the dynamic case, and we have also developed a symmetrical approach in which any contact node will be considered as a candidate node to the contact with an obstacle. Such an approach simplifies the introduction of the data. It also makes it possible to satisfy with more precision the conditions of impenetrability between the two bodies in interaction.

In addition to the effectiveness on the level of the stability computing time, the algorithm shows how totally complex problems can be solved economically.

References

[1] Benayad S., Modélisation des problèmes de contact et frottement entre deux corps déformables, Thèse de magister, C U Bechar, 2008.

[2] Barber T. R., CIAVARELLA Ciavarella M., Contact mechanics, International Journal of solids and structures,Vol. 37, PP. 29 – 43, 2000.

[3] Chan S. H., Tuba I. S., A finite element method for contact problems of solid bodies, Int. J. Mech. Sci; 13, 615-639, 1971.

[4] Bathe K. J., Chaudhary A., A solution method for planar and axisymmetric contact problems,Int. Jou. Num. Meth. Eng, Vol 21, 65-88.1985.

[5] Wriggers P., Simo J. C., A note on tangent stiffness for fully nonlinear contact problems, Communication in applied numerical methods. Vol 1, 199-203 1985.
[6] Dumon G., Algorithme de contraintes actives et contact unilatéral Sans frottement, Revue européenne des éléments finis. Vol. 4-no 1/ 1995.
[7] Wronski M., Couplage du contact et du frottement avec la mécanique non linéaire des solides en grandes déformations, Thèse de doctorat, UTC, France, 1994.
[8] Jean M., Simulation numérique des problèmes de contact avec frottement, Matériaux et techniques, journée de la société tribologique française, 9-10 Avril, 1992.
[9] Feng Z. Q., Contribution à la modélisation des problèmes non linéaires : contact, plasticité et endommagement, Thèse de doctorat, UTC, France, 1991.
[10] Terfaya N., Berga A., Une approche dynamique pour les problèmes de contact, CIMA'04, Boumerdés (Algérie) 30-XI/2-XII- 2004.
[11] Moreau J. J., Some numerical methods in multibody dynamics : application to granular materials, Eur. J. Mech. A/Solids 13 n°4-suppl. 93-114, 1994.
[12] Jean M., The non-smooth contact dynamics method, Comput. Methods App. Mech. Eng. 177(1999)235-257. 1999.
[13] Jean M., Implementation of unilateral contact and dry friction in computer code dealing with large deformation problems, J. Theo. Appl. Mech. Vol. 7, pp 145-160. 1988.
[14] Stromberg N., A method for structural dynamic contact problems with friction and wea, Int. J. Numer. Meth. Eng. 2003; 58:2371-2385.
[15] Francavilla A., Zienkwicz O. C., A note on numerical computation of elastic contact problems, Int. J. Num . Meth . Eng. no 9, pp. 913-924, 1975.
[16] Sachdeva T.D., Ramakrishnan C. V., A finite element solution for the two – dimensional elastic contact problems. Int. J. Num. Meth. Eng. 17, 1257-1271, 1981.
[17] Feng Z. Q., Touzot G., Desaxce G., A finite element solution for the elastic contact problems with friction, Division Modèles Numériques en Mécanique, Université de Technologie de Compiègne,France.
[18] Terfaya N., Berga A., Contribution à la résolution des problèmes de contact et de frottement entre un solide élastique et une fondation rigide par la méthode de flexibilité, Implantation en C++, Séminaire National de Génie Civil, Sid Bel Abes, 15-16 mai 2001 Algerie.
[19] Douli D., Analyse dynamique du problème de contact avec frottement entre un corps élastique et une fondation rigide, Thèse de magister, C U Bechar, Algerie 2006.
[20] Terfaya N., Contribution à la modélisation des problèmes de contact et de frottement bi–dimensionnels, Thèse de magister, C U Bechar, Algerie 2000.

Journal of Scientific Research

P.O.Box 417 route de Kenadsa
08000 Bechar - ALGERIA
Tel: +213 (0) 49 81 90 24
Fax: +213 (0) 49 81 52 44
Editorial mail: jrs.bechar@gmail.com
Submission mail: submission.bechar@gmail.com
Web: http://www2.univ-bechar.dz/jrs/