Iterative method with Successive over relaxation & Statistical method With Anova one to detect the lesion

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ملخص:

في هذه الورقة قمنا باستعمال طريقتين رياضيتين : الطريقة الستاتيكية لانوفا (التحاليل التباين) وهي الطريقة القديمة والمأخوذ فكرتها من برنامج spm والطريقة الجديدة المقترحة من طرفنا وهي SOR وأسلوب التكرار التي تقوم على فحص سطح الصورة واكتشاف الاورام بدقة وسرعة كبيرة مقارنة مع انوفا، البرنامج المطبق هنا هو الماتلاب.

Abstract

In this paper we apply two different mathematical methods: statistical method with ANOVA (analyzis of variance) which is older method & used in Spm logicial & iterative method which is our idea with SOR technique (successive over relaxation) which detected all surface of image in accuracy time & faster in front of Anova. The sofware used here is matlab.

key words: Iterative , statistical, S.O.R, Anova.

Introduction

The successive over relaxation method (SOR) was devised simultaneously by David M. Young, Jr. and by H. Frankel in 1950 for the purpose of automatically solving linear systems on digital computers. Over-relaxation methods had been used before the work of Young and Frankel for instance, the method of Lewis Fry Richardson, and the methods developed by R. V. South well. However, these methods were designed for computation by human calculators, and they required some expertise to ensure convergence to the solution which made them inapplicable for programming on digital computers. These aspects are discussed in the thesis of David M. Young, Jr. [1].

From iterative methods come our idea which used in processing of image to detect the place of lesion then compared them with statistical method of Anova one way.

The idea comes from SPM logiciel which used to detect the lesion with statistical methods [2]. Since the time of execution is long we thought for another method which was iterative methods of linear resolution with Successive Over-Relaxation (S.O.R) which has a good solution in few time in front of Anova technique.

The base of these methods defined in the background title.

The description of itterative method used for given a square system of n linear equations :(Hadjidimos, A) (Makinson, G.J. and Shah, A).

$$A * X = b$$

Where

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ . \\ x_n \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_2 \\ . \\ b_n \end{bmatrix}$$

Then *A* can be decomposed into a diagonal component *D*, and the remainder *R*:

A=D+R

Where

$$D = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ \vdots & a_{22} & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix} \qquad R = \begin{bmatrix} 0 & a_{12} & \cdots & a_{1n} \\ a_{21} & 0 & \ddots & a_{2n} \\ a_{n1} & a_{n2} & \cdots & 0 \end{bmatrix}$$

The solution is then obtained iteratively via:

Iterative resolution with Successive Over-Relaxation method (S.O.R)

The SOR is a variant of the Gauss–Seidel method for solving a linear system of equations, resulting in faster convergence. A similar method can be used for any slowly converging iterative process.[3][4][5][6]

$$X^{(k+1)} = wD^{-1}\left(b - RX^{(k)}\right) + (1 - w)X^{(k)}$$

Where : w is the relaxation factor.

The choice of relaxation factor ω is not necessarily easy, and depends upon the properties of the coefficient matrix. In 1947, Ostrowski[11] proved that if $A$is symmetric and positive-definite then $\rho(L_\omega) < 1$for $0 < \omega < 2$. Thus convergence of the iteration process follows, but we are generally interested in faster convergence rather than just convergence.

Values of $\omega > 1$ are used to speedup convergence of a slow-converging process, while values of $\omega < 1$ are often used to help establish convergence of a diverging iterative process or speed up the convergence of an overshooting process. [11][12]

Note that the computation of $x_i^{(k+1)}$ requires each element in $\mathbf{x}^{(k)}$ except itself. Unlike the Gauss–Seidel method, we can't overwrite $x_i^{(k)}$ with $x_i^{(k+1)}$, as that value will be needed by the rest of the computation. The minimum amount of storage is two vectors of size n.

A.1. Convergence

The standard convergence condition (for any iterative method) is when the spectral radius of the iteration matrix is less than 1:

$$\rho(D^{-1}R) < 1$$

A.2. Error

Let $e^{(k)}$ be the error vector

$$e^{(k+1)} = x^{(k+1)} - x^{(k)}$$

Analysis of Variance (ANOVA)

In statistics, analysis of variance (ANOVA) is a collection of statistical models, and their associated procedures, in which the observed variance in a particular variable is partitioned into components attributable to different sources of variation.

Its expression can be calculated as follows: [7][8][9][10][11]

$$F_0 = \frac{MS_R}{MS_E}$$

Where, MSR are the mean squares and MSE are the error mean squares.

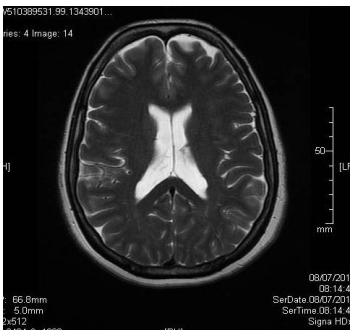
I. EXPRIMENTAL RESULTS

A. *Algorithm*

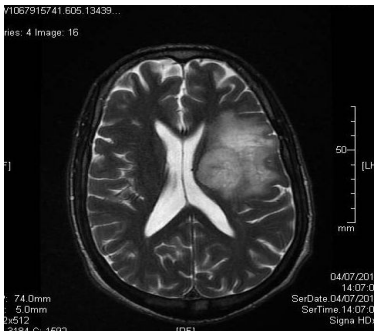
- Read of images.
- Selection of sample of image in order to identify the images.
- Applied the iterative method with successive over relaxation (S.O.R).
- Applied the statistical method with Anova techniques.
- Comparison between the both methods.

The following examples & for SOR: we chose w=1.1, the precision e=0.001, maximum iteration=200

Example1:Surface 480X480



a. Normal image



b. Pathological image

Fig.1. Global face of the both images

Figure (1) presented the global face of normal & pathological images & we need applied iterative with SOR technique then statistical method with Anova one way. After execution, figure (2) displayed only the SOR technique. no result for anova technique, here from ref (2,12,13) we can understand that anova can't detect big surface (>200), plus the long-time of execution for surface between(100& 200).

Fig (3) display The elapsed time for the S.OR. equal: **65.140155 seconds**. The elapsed time with Anova is: no result**.**

So, we can see clearly that SOR able to detect the lesion in few time & with excellent result in front of Anova.

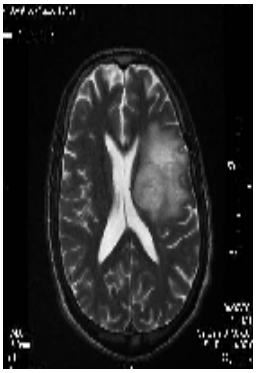


Fig.1.Pathological image

Fig.2.Result of SOR

Fig.2. the location of disease after applied SOR & Anova techniques

```
epsilon =

  1.0000e-003


w =

    1.1000

Elapsed time is 65.140155 seconds.
end  SOR
```

Fig.3. elapsed time for SOR technique

Now we minimised the surface of image in the second example.

Example 2: Surface of 200X200

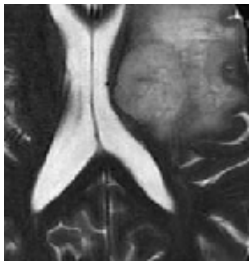


Fig.1.Pathological image

Fig.2.Result of SOR

Fig.4.Result ANOVA

Fig4.Location of disease for SOR & Anova for surface of 200X200

In this surface, the SOR & Anova gave the same results but SOR detected the lesion in 2.633953s (fig.5) where anova detected in 439.372992s (fig (6), & here you can see the big difference between the both methods. (See fig5)

```
epsilon =

   1.0000e-003


w =

    1.1000

Elapsed time is 2.633953 seconds.
end   SOR
```

Fig5.Elapsed time for the SOR

```
fcal < 1
h0 accept
Elapsed time is 439.372992 seconds.
end anova
>> n

n =

    200

>>
```

Fig 6. Elapsed time for Anova

The next example is for surface 124X124 where the image minimised

Example3: 124X124

The same result for the surface 124X124, where, Fig (7) present the place of lesion for the both techniques but always the SOR detected faster then anova .see (fig(8-9)).

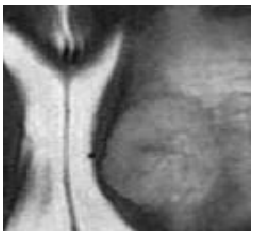


Fig.1.Pathological image

Fig.2.Result of SOR

Fig.4.Result ANOVA

Fig.7. location of lesion for surface 124X124 & for the both techniques

```
epsilon =

  1.0000e-003


w =

    1.1000

Elapsed time is 0.620910 seconds.
end  SOR
```

Fig.8.elapsed time for SOR (surface 124X124)

```
fcal < 1
h0 accept
fcal < 1
h0 accept
fcal < 1
h0 accept
fcal < 1
h0 accept
fcal < 1
h0 accept
Elapsed time is 36.305703 seconds.
end anova
>>
```

Fig.9.elapsed time for Anova (surface 124X124)

III.5.3.2. Comparisons between the results of the both techniques:

As we notice in the results that the S.O.R results more precise then the ANOVA.

The table below display the comparison between the elapsed time and application domain for each method.

Methods		Elapsed time	Kind of image	
Example1				
S.O.R	480X480	65.140155s	excellent	
ANOVA	480X480	No result	bad	
Example2				
SOR	200X200	2.633953s	good	
Anova	200X200	439.372992s	good	
Example3				
SOR	124X124	0.620910s	good	
Anova	124X124	36.305703s	good	

Table (1): Comparaison between SOR & Anova from the table.

From thus results we can say that the S.O.R is faster than ANOVA and with high level of precision as we notice before.See tab(1)

Also S.O.R it's the better method for the linear model application in front of Anova

III.6. Conclusion

The SOR technique detecte the lesion for the both surfaces (small and big ones) with high level of speed and precision . Anova could not work in big surface (>200X200) & it can't display any answer. For small surface (<=200X200) anova spent a long time before display the result. Anova & SOR detects well the lesion for small surfaces.

As perspective we proposed for the future researcher to compare the SOR with anova two ways.

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