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Meta-heuristic algorithms for the multi-item transshipment problem

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Abstract

Differential Evolution (DE) and the Particle Swarm Optimization (PSO) are two evolutionary algorithms that confirmed their efficiency in resolving complex problems. In this paper, we intend to adopt these algorithms to resolve a complex inventory management problem, known in the literature by the transshipment problem. This problem concerns networks of collaborative retailers selling items and they collaborate by exchanging items between them. The transshipment problem consists in deriving the optimal replenishment quantity, for each retailer, while a transshipment policy is adopted. A huge body of literature works has addressed this problem where several configurations are investigated. A few of them has addressed the multi-item and the multi-location configuration because of its complexity. We focus in this paper on this complex configuration and we resolve it by the PSO and DE algorithms. Secondly, we compare between the performances of these algorithms according to a set of criteria. Thirdly, we analyze the impact of the studied transshipment parameters on the inventory system performance measures.

Keywords: multi-item, multi-location; transshipment problem; Particle swarm optimization; Differential Evolution;

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1. Introduction

Inventory management aims to satisfy demands while optimizing inventory performances which are generally expressed in terms of profit or cost functions. When companies evolve in an uncertain and competitive environment, this objective becomes hard to be achieved. Consequently, many inventory flexibility techniques are practiced in order to cope with these constraints and they help companies to achieve their primary objective. The substitution and the transshipment are two well-known inventory flexibility techniques largely practiced in industry. The first consists to replace the unavailable items by alternatives ones having the same functionalities. However, the second consists to transfer the items from locations in excess to ones in need. These two flexibility techniques help companies to improve their fill rate and to reduce, simultaneously, their inventory cost. The transshipment is hugely practiced in many domains as spare parts and fashion items sold in several locations. Generally, these kinds of items are replenished from suppliers, where lead times are expressed in terms of weeks or months, and transshipment is practiced to serve customers requiring, from a location, items which are out of stock. For example, when a customer looks for a specific item from 'Zen la Soukra' and this item is out of stock in this location, then the required item could be transshipped from 'Zen Manar 2' where the item is in excess. So, the customer demand is satisfied, the profit in 'Zen la Soukra' is improved and the inventory cost of 'Zen Manar 2' is reduced. The transshipment problem has been studied since 1965 where several configurations, parameters and approaches are investigated. The multi-item transshipment variant is considered as a complex problem and there are a few of works that treated it. Generally, these works focused mainly on the two-location or the two-item configurations and they looked for a transshipment policy for all items simultaneously. In this paper, we focused on the multi-location and the multi-item configuration.

Our contributions here are threshold: first we studied the multi-location and multi-item transshipment problem considering periodic review and we formulate the studied problem, second we resolve the problem with PSO and DE and we compare between their performances and we study the impact of transshipment and uncertainty on the inventory system performance.

The rest of the paper is organized as follows: the second section presents a literature review of the transshipment. In the third section, we present the studied problem and its formal model. The fourth section is dedicated to the meta-heuristics algorithms PSO and DE and their application to resolve the studied problem and finally the fifth section is related to experimentation.

2. Literature review

The transshipment problem consists to derive the optimal replenishment quantities where transshipment policy is adopted, has been studied. So, many configurations, parameters and approaches are investigated, and many transshipment policies are identified. Paterson et al., 2011 overviewed works related to transshipment problem and they proposed a classification based on a set of criteria linked to: replenishment parameters, transshipment policies and environmental parameters. They identified two classes of works according to the transshipment policies: the reactive transshipment and the proactive one in Seidscher et al., 2013. The reactive is triggered once the demands are observed and the locations in need and other in excess are identified. In contrast, the proactive transshipment is started before the realization of demand and it aims to redistribute stock in order to avoid a possible shortage. Two streams of works are identified according to number of locations involved: the two-location and the multi-location transshipment problem. The first stream adopts exact methods in order to derive the optimal inventory decisions. Krishnan and Rao, 1965 are the first that studied the two-location transshipment problem. They developed a single period model aiming to minimize an inventory cost function expressed in terms of holding and shortage costs. Since that, many researches focused

on this stream and investigated many configurations. Among recent ones, we quote Olsson F. in 2015, who considered the transshipment lead times for two-location inventory system adopting continuous reviewing. Yao et al., 2016 studied the two-location transshipment problem with a single replenishment, at the beginning of a season, while reactive transshipment is practiced during the season. Feng et al., 2019 studied the two-location transshipment problem in a competitive context and with dynamic demand information's. The second stream of works focused on the multi-location configuration and it adopts simulation-based methods to derive approximate solutions proposed by KÖCHEL, P., 1998 and Kochel, P. et al., 2005. The meta-heuristic is a simulation-based optimization approach which is widely adopted to resolve complex problem as the multi-location problem considering uncertain demands. Miao Z., 2008 resolve the transshipment problem with fixed schedules with a genetic algorithm. Hochmuth and Kochel, 2012 resolve the multi-location transshipment problem with many realistic parameters with particle swarm optimization (PSO) Algorithm. Danloup et al., 2018, compared the performances of two meta-heuristics applied for the transshipment problem: Local Neighborhood Search and genetic algorithm. All the mentioned works above, focused mainly on the single item configuration. Few of works have interested in the multi-item configuration. They investigated in the two-location network with periodic reviewing or the multi-location configuration with continuous reviewing.

In this paper, we focus on the transshipment problem for multi-location and multi-item configuration considering uncertain demands. We aim to resolve the studied problem with meta-heuristic approach. We compare between the performances of two algorithms, Particle swarm optimization (PSO) and Differential Evolution (DE), according to a set of criteria.

3. Problem description

We study an inventory system composed of N locations selling many products. At the beginning of the period, these locations are replenished from a common supplier and over the period demands are observed and satisfied. At the end of the period, a location L_i could be in excess related to product P_k and in need for P_m . However, location L_j could be in need for P_k and in excess for P_m . Transshipment, from Locations L_i to L_j of P_k units, corrects P_k shortage at L_j and it reduces the P_k holding cost at L_i . Here, we consider the fixed transshipment cost. The goal is to determine the transshipped quantities between locations and the replenishment quantities of products at each location optimizing a profit function. In order to introduce our studied problem, we present an illustrative example, shown by Figure 1, of three locations L_1 , L_2 and L_3 selling two products P_1 and P_2 . P_1 is in need (-3) at L_1 , in excess (+4) at L_2 and in need (-2) at L_3 . However, P_2 is in excess (+4) at L_1 , in excess (+3) at L_2 and in need (-6) at L_3 . Shortages at L_1 and L_3 could be corrected by transshipping items from L_1 (P_1) and L_2 (P_1, P_2).

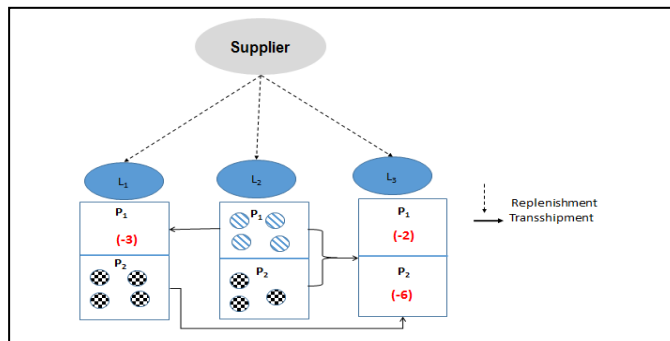


Fig. 1. Illustrative example of three locations.

3.1. Notations

Throughout this paper, we adopt the following notations

1) Indexes

- i index of location $i \in \{1, 2, \dots, N\}$.
 k index of product, $k \in \{1, 2, 3, \dots, K\}$.

2) Parameters

- P_i^k The unit selling price of the product k at location i .
 r_i^k The shortage unit cost of product k at location i .
 s_i^k The salvage unit cost of product j at location i .
 c_i^k The replenishment unit cost of product k at location i .
 tc_{ij}^k The transshipment unit cost between locations i and j for product k
 D_i^k The demand of the **product k at location i** .
 $f(D_i^k)$ The probability density function of the demand of the **product k at location i** .

3) Performance measure functions

- $\pi(X, D)$ The total profit function.
 $TC(X, D)$ The total Inventory cost.
 $TR(X, D)$ The total revenue generated before transshipment execution.
 $TP(X, D)$ The total transshipment profit.

3.2. Problem formulation

$$\pi(X, D) = TR(X, D) - TC(X, D) + TP(X, D) \quad (1)$$

$$TR(X, D) = \sum_{i=1}^N \sum_{k=1}^K p_i^k \times \min\{D_i^k, Y_i^k\} \quad (2)$$

$$TC(X, D) = \sum_{i=1}^N \sum_{k=1}^K c_i^k \times Y_i^k + \sum_{i=1}^N \sum_{k=1}^K s_i^k \times (Y_i^k - D_i^k)^+ + \sum_{i=1}^N \sum_{k=1}^K r_i^k \times (D_i^k - Y_i^k)^+ \quad (3)$$

$$TP(X, D) = \sum_{i=1}^N \sum_{\substack{j=1 \\ i \neq j}}^N \left(\sum_{k=1}^K (s_i^k + P_j^k - tc_{ij}^k) \times T_{ij}^k \right) \quad (4)$$

$$\sum_{i=1}^N T_{ij}^k \leq (D_j^k - Y_j^k)^+ \quad \forall j \in \{1, 2, \dots, N\}, \forall k \in \{1, 2, \dots, K\} \quad (5)$$

$$\sum_{j=1}^N T_{ij}^k \leq (Y_i^k - D_i^k)^+ \quad \forall i \in \{1, 2, \dots, N\}, \forall k \in \{1, 2, \dots, K\} \quad (6)$$

4. DE and PSO for the Transshipment problem

Here, we resolve the studied problem described and formulated above by two Evolutionary Algorithms, Differential Evolution (DE) and Particle Swarm Optimization (PSO), that confirmed their efficiency in resolving complex problem.

4.1. Differential Evolution

Differential Evolution (DE) was introduced for the first time by Storn and Price, 1997, as a stochastic and population-based optimization algorithm. DE was proved to be the fastest evolutionary algorithm (EA) and it was used for solving nonlinear optimization problem over continuous spaces. DE has been shown having a good convergence and very simple but very powerful for optimizing continuous functions. As many others evolutionary algorithms, DE uses three operations: mutation, crossover and selection that guide the individuals of (population) to move toward a global optimum. DE was used to solve diverse optimization problems and it has proved his performance. The DE algorithm results depend on the mutation strategy and the control parameters: the population size (NP), the crossover operator (CR) and the differential weight factor (F).

The general structure of DE algorithm is composed by the steps shown in figure 2. The steps are executed sequentially till stop continuation is met. These steps are:

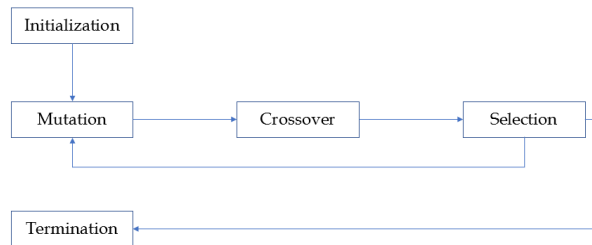


Fig. 2. DE general structure.

- Initialization: DE starts with initialization of the population by producing NP individuals in the problem space domain. Each individual X_i is presented by a vector of D values (each value associated to one variable of the problem dimension):

$$X_i = \{x_i^1, x_i^2, \dots, x_i^N\}, i \in \{1, 2, \dots, NP\}, \text{ where } N \text{ is the number of locations.}$$

$$x_i^j = (x_i^{j1}, x_i^{j2}, \dots, x_i^{jK}), \text{ where } k \text{ is the number of items.}$$

- Mutation: After the initialization and in each generation g , DE uses the mutation operation to create a mutant vector $V_{i,g}$ associated to each target vector $X_{i,g}$ (X_i at generation g), $V_{i,g} = \{V_{i,g}^1, V_{i,g}^2, \dots, V_{i,g}^D\}$ with $i \in \{1, 2, \dots, NP\}$ and $g \in \{1, 2, \dots, G\}$, (G is the maximum number of generation). There are different DE variants identified according to mutations operation formula. Here, we are limited to the DE best/1 having the following mutation formula:

$$V_{i,G} = X_{best,G} + F * (X_{r1,G} - X_{r2,G}) \quad (7)$$

- Crossover operation: The crossover step, as presented by the formula below, is used to introduce some diversity in the population during each generation in order to look for the optimum. In this phase DE

produce, at each generation g , a trial vector $U_{i,g} = \{u_{i,g}^1, u_{i,g}^2, \dots, u_{i,g}^D\}$ associated to each individual $X_{i,g}$. The binomial Version of the crossover operation is presented below:

$$u_{i,g}^j = \begin{cases} v_{i,g}^j & \text{if } rand < CR \text{ or } j == Rand \\ x_{i,g}^j & \text{otherwise} \end{cases} \quad (8)$$

CR represents the crossover parameter ($CR \in [0.1]$), Rand is a random integer value with $Rand \in [0.D]$.

- Selection: During this step, at the generation g DE have to decide which vector to keep between the pair X_g and U_g depending on their fitness values, for the generation $g+1$.

$$X_{g+1} = \begin{cases} U_g & \text{if } fitness(U_g) > fitness(X_g) \\ X_g & \text{otherwise} \end{cases} \quad (9)$$

This selection formula is in case of maximization problem.

The algorithm restarts the cycle from the mutation phase until the stop condition is reached.

The main DE parameters are:

- The Cross-over probability (CR).
- The differential weight factor (F)
- The population size (NP).

Table 1. DE algorithm

DE algorithm (BEST/1)
P (NP) \leftarrow Initialize Population
Evaluate each individual of P by algorithm 2
While (Stop-condition not met) Do
For $i=1$ to NP
Randomly choose {individual ₁ and individual ₂ } from P
Individual _{best} \leftarrow Look for the best individual from P
R \leftarrow RandomInteger [0, N*k]
For $j=1$ to (N*K)
CrossoverProbability \leftarrow random()
If (crossoverProbability < CR or R== j)
Candidate _j \leftarrow Individual _{best,j} + W * (individual _{1,j} - individual _{2,j})
Else
Candidate _j \leftarrow individual _{1,j}
End for
Evaluate candidate fitness by algorithm 2
If (Candidate Fitness > Individual ₁ Fitness) Then
Individual ₁ \leftarrow Candidate
End for
End while

4.2. PSO algorithm

The Particle swarm optimization (PSO) is one of the most known population-based algorithms proposed by Eberhart and Kennedy, 1995. Since its appearance, several improvements were introduced to the basic version and was used for solving global optimization problems. The basic operation of Particle Swarm Optimization consists in searching for the optimal solution in a search space D (number of dimension) where each particle 'i' is characterized by its position (X_i) and its velocity (V_i) which are represented as follows:

$X_i = \{x_i^1, x_i^2, \dots, x_i^N\}$, $i \in \{1, 2, \dots, NP\}$. Where N is the number of locations.

$X_i^j = (x_i^{j1}, x_i^{j2}, \dots, x_i^{jk})$, where k is the number of items

Each particle must also keep track of its best previous position $P_{besti} = (p_{i1}, p_{i2}, p_{id})$ as the best among all the particles of the population $P_{gbest} = (p_{g1}, p_{g2}, p_{gd})$. At each iteration, each particle in the population adjust its velocity and calculate its new positions vector using the best fitness in the population according to the following two formulas:

$$v_{id} = wv_{id} + c_1r_1(p_{id} - x_{id}) + c_2r_2(p_{gd} - x_{id}) \quad (10)$$

$$x_{id} = x_{id} + v_{id} \quad (11)$$

Where c_1 and c_2 are the acceleration constants and w is the inertia weight parameter. r_1 and r_2 are two random generated numbers in the interval $[0, 1]$ according to the uniform law.

Table 2. PSO algorithm

PSO algorithm
$P(NP) \leftarrow$ Initialize Population
While (Stop-condition not met) Do
Update pBest of each particle
Update gBest of the population
For $i = 1$ to NP
Candidate \leftarrow Particle _{i} (copy Particle _{i} in Candidate)
Calculate new_Velocity for Candidate according to (10)
Calculate new_Location for Candidate according to (11)
Evaluate Candidate fitness by algorithm 2
if (Calculate Fitness > Particle _{i} Fitness) Then
Particle _{i} \leftarrow Candidate
End for
End while

4.3. Expected profit algorithm

Using the below expected profit algorithm, we estimate the profit generated by the system for a specific combination of a predetermined inventory vector and a set of generated demand vectors.

Table 3. Expected profit algorithm

Expected profit algorithm
Expected_profit \leftarrow initialize_to_zero()
X inventory vector
For i= 1 to Number of Simulation (NS)
D \leftarrow generate_Demand ()
Expected _profit \leftarrow Expected _profit + TP(X,D)
End for
Expected _profit \leftarrow Expected _profit /NS
Return Expected _profit

5. Experimental Results

In this experimental section, we focus on three different problem configurations of transshipment problem identified according to the number of items. We are interested in four locations selling 2, 4 or 8 items. Characteristics of the studied configurations are presented in Appendix A.

We note here, that in addition to the uniform distribution of demand mentioned in table 1, we used the normal distribution with the same parameters.

5.1. Parameters settings

The parameters of the used algorithms are tested in Noomen et al., 2020 and summarized in the Table below:

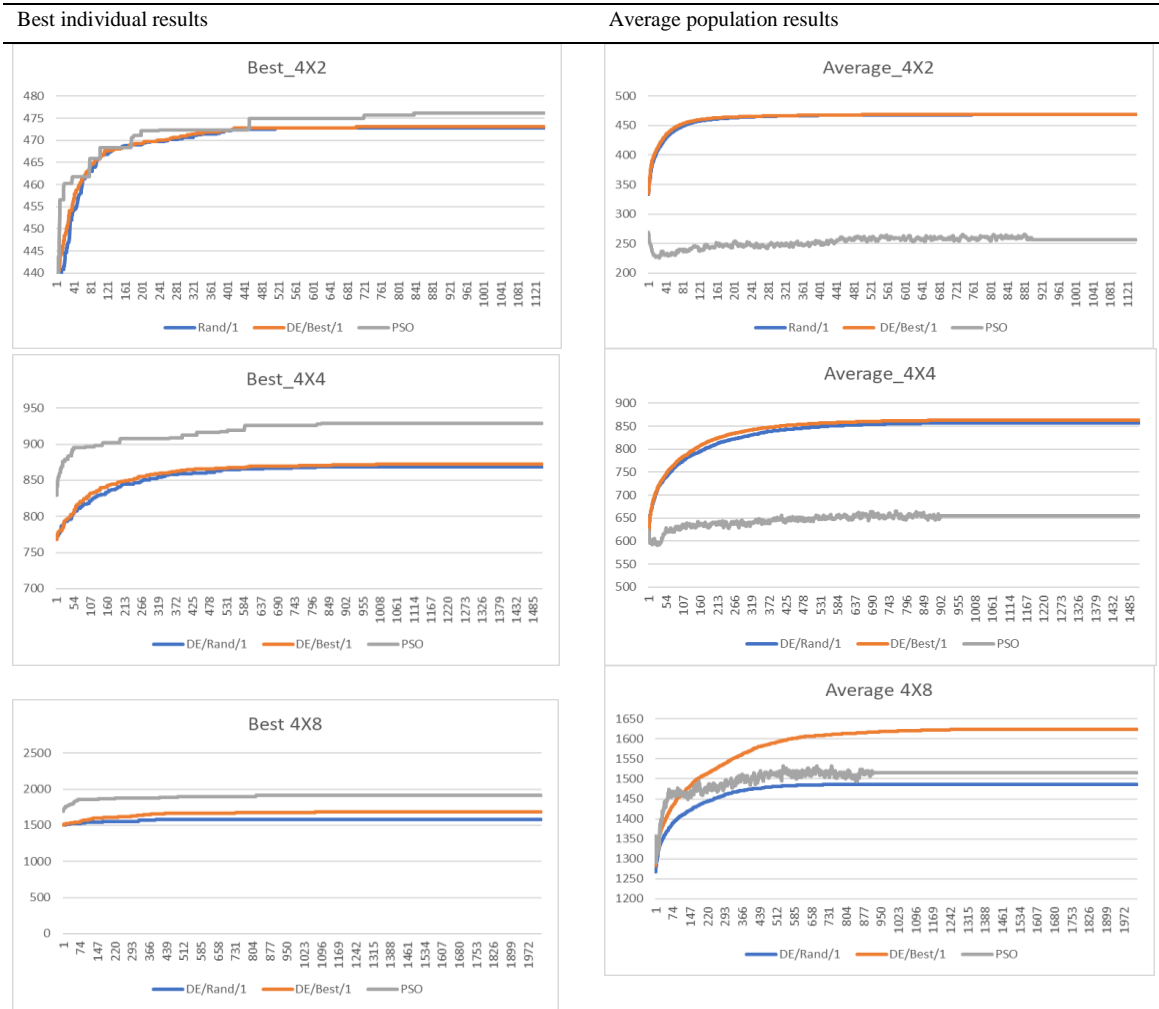
Table 4. List of used parameters for PSO and DE

DE Parameters		PSO Parameters	
Parameter	Best value	Parameter	Best value
CR	0.7	W	0.6
F	0.6	C1=C2	1.5
NP	30	NP	30*
N. generation	Stop stability condition*	N. generation	900

5.2. DEs and PSO comparison

We treated here 3 different problem configurations (3 models) and we present below the results of DE/Rand/1, DE/Best/1 and PSO showed in the table below. This table contains respectively the results of the best and the average system fitness value realized by each algorithm corresponding to the three different configurations (4X2, 4X4 and 4X8).

Table 5. DE/Rand/1, DE/Best/1 and PSO results



The curves show that PSO always takes the top compared to DE in all models in the case of the best individual results, which proves the advance of PSO compared to DE variants. On the other hand, the average value achieved by individuals of PSO population is almost lower than the others of DE algorithms.

System performance studies: We study in this section the impact of two different parameters (fixed cost transshipment and the demand uncertainty) on the system performance. We chose also to use two demand types distribution (uniform demand distribution and normal demand distribution). We present below the results of these experimentations

5.3. Demand Impact on system performance

We present below the results of the study of the impact of the variation of normal and uniform demand distribution on the system performance using DE/Rand/1, DE/Best/1 and PSO:

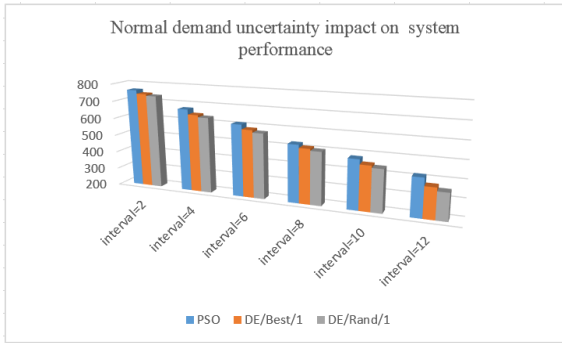


Fig. 3. Impact of normal demand on system performance

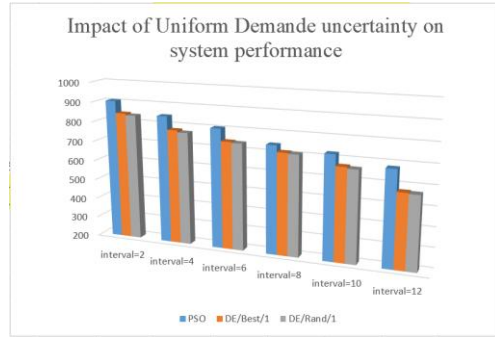


Fig. 4. Impact of uniform demand on system performance

Figure 3 and 4 show that the performance of the PSO is always better than that of the DE even with the variation of the uncertainty of the demand in both normal and uniform cases. Results show also that DE/Best/1 proves to be better than DE/Rand/1 in all experimentations. Our experimentations prove also that the impact of normal and uniform demand uncertainty on system performance is very remarkable; we notice also a rapid decrease of the system profit in the case of normal demand faster than in the case of uniform demand, this proves that our system depends directly on the nature of demand and its domain: The more the uncertainty of the demand increases the more it causes a decrease of the system performance and affects the result of the fitness function regardless of the algorithm.

5.4. Transshipment Impact on system performance using DE/Best/1

We present below the results of the study of the impact of the transshipment on system performance in case of normal and uniform demand distribution uncertainty variation using DE/Best/1:

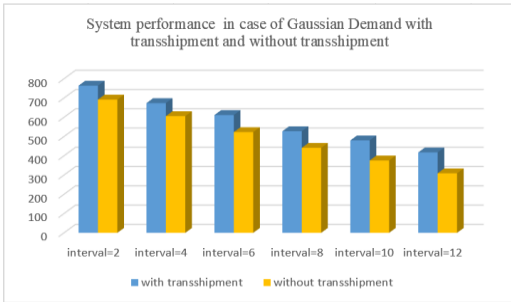


Fig. 5. Impact of transshipment on system performance in case of normal demand

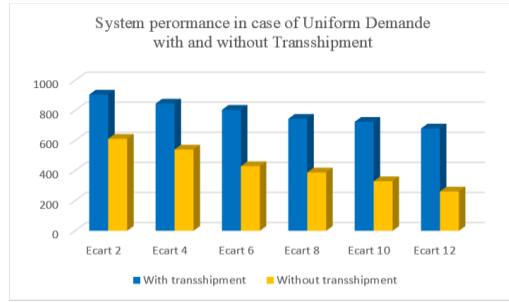


Fig. 7. Impact of transshipment on system performance in case of uniform demand

Figure 6 and 7 show that the impact of transshipment, in both cases normal and uniform demand, on system performance is very remarkable; we notice also a rapid decrease of the system profit when the demand uncertainty increases. Our experimentations show also a very clear effect of the transshipment on system results which proves its added value.

We present below the gain realized by transshipment using normal and uniform demand distribution using DE/Best/1:

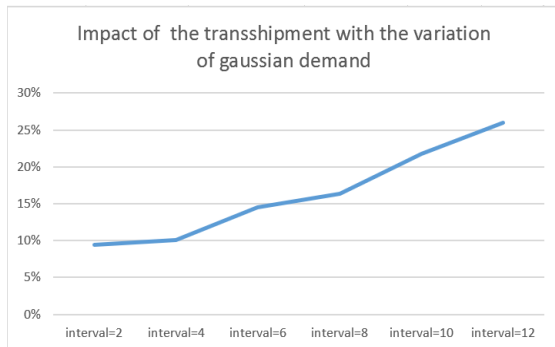


Fig. 8. Gain made by transshipment in case of normal demand

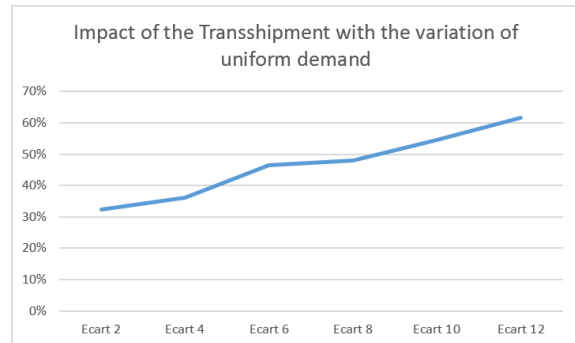


Fig. 9. Gain made by transshipment in case of uniform demand

Figure 8 and 9 show the gain realized when using transshipment, in both cases normal and uniform demand, on system performance is very important. We notice also that the gain realized in case of uniform demand is more important compared to the normal demand case.

6. Conclusion

In this paper, we studied the multi-location and multi-item inventory management considering lateral transshipment. We proposed two evolutionary algorithms (DE/Rand/1, DE/best/1 and PSO), to resolve the studied problem. The model considered many items which could be transshipped between locations. We proposed a simulation algorithm to derive the expected profit value of a replenishment vector. Our experimental study shows that the PSO algorithm performs better results than those of DEs. We studied the uncertainty demand effect on the system performance. We noted that the transshipment impact on the system performance is more significant for high level of demand uncertainty. Our actual research could be extended by: (1) considering the fixed cost of transshipment and (2) integrating a substitution flexibility technique allowing to customer to replace their first choice by an alternative one when it is out of stock.

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Appendix A.

Configurations data from Noomen et al., 2020.

Location		L1	L2	L3	L4
P1	P_i^-	15	15	14	9
	S_i^+	4	4	6	4
	r_i^+	8	5	4	5
	C_i^+	10	10	9	9
	D_i^+	u[12, 26]	u[10, 20]	u[5, 36]	u[15, 35]
P2	P_i^-	17	17	18	11
	S_i^-	5	5	5	4
	r_i^-	6	6	6	6
	C_i^-	12	12	13	11
	D_i^-	u[13, 26]	u[12, 20]	u[4, 36]	u[16, 35]
P3	P_i^o	15	15	14	9
	S_i^o	4	4	6	4
	r_i^o	8	5	4	5
	C_i^o	10	10	9	9
	D_i^o	u[12, 26]	u[10, 20]	u[5, 36]	u[15, 35]
P4	P_i^+	17	17	18	11
	S_i^+	5	5	5	4
	r_i^+	6	6	6	6
	C_i^+	12	12	13	11
	D_i^+	u[13, 26]	u[12, 20]	u[4, 36]	u[16, 35]
P5	P_i^o	15	15	14	9
	S_i^o	4	4	6	4
	r_i^o	8	5	4	5

	C_i°	10	10	9	9
	D_i°	u[12, 26]	u[10, 20]	u[5, 36]	u[15, 35]
	P_i°	17	17	18	11
	S_i°	5	5	5	4
P6	Γ_i°	6	6	6	6
	C_i°	12	12	13	11
	D_i°	u[13, 26]	u[12, 20]	u[4, 36]	u[16, 35]
	P_i°	15	15	14	9
	S_i°	4	4	6	4
P7	Γ_i°	8	5	4	5
	C_i°	10	10	9	9
	D_i°	u[12, 26]	u[10, 20]	u[5, 36]	u[15, 35]
	P_i°	17	17	18	11
	S_i°	5	5	5	4
P8	Γ_i°	6	6	6	6
	C_i°	12	12	13	11
	D_i°	u[13, 26]	u[12, 20]	u[4, 36]	u[16, 35]