



EXACT SOLUTION FOR NORMAL DEPTH COMPUTATION IN SOME OPTIMAL CHANNEL CROSS-SECTIONS

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ABSTRACT

The normal depth is an important hydraulic element in the design, operation and maintenance of canals. The calculation of this normal depth is based on Manning's formula, which has long been considered applicable only in the rough turbulent domain, and Manning's coefficient is considered in several works as a constant of the problem. The objective of this study is to express the normal depth in different semipolygonal canals as the optimal cross-section (semicircular, rectangular, triangular and trapezoidal), taking into account the variation in Manning's coefficient as a function of all the parameters governing the uniform flow, including the viscosity of the liquid. The study led, through the application of the rough model method (RMM), to a dimensionless expression of Manning's coefficient, written as a function of the characteristics of the rough model, allowing the construction, for each chosen cross-section, of a diagram similar to Moody's. The diagrams give Manning's coefficient in the whole turbulent domain (smooth, transition and rough), which is considered a generalization of Manning's formula in the whole turbulent domain instead of only the rough domain. The exact solution of normal depth by using the equation of Manning's coefficient has therefore been proposed.

Keywords: Normal depth, Manning's resistance coefficient, Optimal cross section, Rough model method (RMM), Semipolygonal channels.

INTRODUCTION

In uniform open channel flow, the normal depth is an important hydraulic element in the design of the channels, analysis of gradually varied flow profiles (backwater), operation and maintenance (Swamee, 1994). The normal depth is determined by using many numerical methods with no explicit solution (Zhang and Wu, 2014), and one of the most resistance equations applied to calculate the normal depth is Manning's equation (Chow, 1959). Manning's formula is generally used to calculate the average flow velocity V (m/s) in open channels as a function of hydraulic radius R_h (m), the bed slope of the channel S_0 (m/m) and a resistance coefficient of Manning n ($s/m^{1/3}$), which represents the friction applied to the flow by the inner wall of a channel or a pipe (Chow, 1959) and is also used to calculate the discharge Q (m^3/s) with $Q = VA$, where A (m^2) is the wetted area of flow (García-díaz, 2005).

In practice, and in many studies, Manning's formula is more generally applied to calculate the normal depth of uniform flow in open channels and pipes. Chow (1959) used the Manning formula for the computation and analysis of uniform flow and provided that when the discharge Q , longitudinal slope S_0 and resistance coefficient called roughness n are known, the Manning formula gives the section factor $AR_h^{2/3}$ and hence the normal depth y_n .

Barr and Das (1986) provided a numerical procedure for the direct solution of normal depth for rectangular channels and provided both numerical and graphical procedures for trapezoidal channels and circular conduct using Manning's formula. Babaeyan-Koopael (2001) determined a graphical solution for calculating the normal depth of different channel cross-sections (triangular round-bottom sections, parabolic sections and rectangular round-bottom sections) by determining a dimensionless parameter using the Manning equation. Abdulrahman (2007) used Manning's formula to determine the minimum and optimum characteristics of a composite section formed by a trapezoidal section at the bottom and a rectangular section at the top (half octagon) and showed that it is the most efficient section. Froehlich (2008) determined the dimensions of standard two-section trapezoidal shapes with rounded bottom vertices by applying two different methods, the analytical method and the graphical method, using the cross-sectional factor for uniform flow from the Manning equation.

Using Manning's equation, Raikar et al. (2010) proposed a regression analysis method to calculate the normal and critical depths for an ovoid duct model recommended by the Indian Standard Code of Practice IS-4880 (1976). Li and Gao (2014) used an iterative formula to calculate the nondimensional normal depth ϵ [0.04, 400] with a maximum relative error of 0.34% for a parabolic open channel section. Liu et al. (2010), by applying

Manning's formula, presented an iterative formula to calculate the normal depth for all types of tunnels with a standard horseshoe cross-section with a maximum relative error of 0.5%, and Han and Easa (2016) and Elhakeem (2017) proposed explicit solutions for calculating the normal depth of channels with cubic and trapezoidal cross-sections, respectively. Dai et al. (2020) determined the normal depths of three parabolic-shaped channels using three methods. The first is the Simpson integral numerical method giving wetted perimeters for the calculation of the three normal depths. The second method is used to calculate three alternative depths with a maximum error of 0.13%, while the third is used to determine three conjugate depths with a maximum error of 0.34%. In the studies cited above, the value of the Manning roughness coefficient n is considered constant and given in the literature by a table depending on the kind of channels. This can only constitute an approximation of the problem because the constancy of this coefficient is no longer physically justified; it must vary according to all the parameters governing the uniform flow, particularly the normal depth, which is generally the unknown of the problem (Achour and Amara, 2020).

Some researchers have investigated the value of Manning's roughness coefficient n by considering it nonconstant and have developed solutions using different methods: Yen (1992) determined an iterative solution and a graphical solution for calculating the dimensionally homogeneous n_g of Manning's roughness coefficient n as a function of hydraulic radius (or normal depth). Akgiray (2005) discussed explicit approximate solutions of Manning's equation for two assumptions: the first assumes that n is constant, and the second assumes that n varies with depth flow, as indicated by Camp (1944, 1946). Azamathulla et al. (2013) worked with genetic expression programming algorithms (GEPs) to estimate the Manning roughness coefficient n in an open channel as a function of the Reynolds number, Froude number, flow depth and channel slope for partially filled circular pipes. The authors showed that the GEP approach is in good agreement ($\pm 10\%$) with the experimental results compared to the classical models based on regression analysis (traditional formula). Achour and Amara (2020) presented a new relation of the Manning roughness coefficient n for partially filled circular pipes derived from the combination of the rational Darcy-Weisbach and Colebrook-White equations. The variable nature of Manning's coefficient as a function of the different parameters governing it, notably the relative roughness and kinematic viscosity, shows that it is also affected by the flow regime.

In all cited works, Manning's formula is used; this relation is only valid for the rough turbulent regime with average relative roughness (Hager, 1989). In this domain, the Reynolds number has no influence because the kinematic viscosity is not considered in the expression of Manning's formula. Some researchers have investigated the application of Manning's formula in the whole domain of turbulent flow and have developed an explicit equation for Manning's roughness coefficient n , including Achour and Bedjaoui (2006), who developed a relationship for the volume flow Q in turbulent flow to obtain a general expression for Manning's coefficient n as a function of the relative roughness ε/R_h , the hydraulic radius R_h and the Reynolds number \overline{Re} of the rough model. Loukam

et al. (2019) determined Manning's resistance coefficient n in uniform flow for egg-shaped conduits as a function of the filling ratio η , the relative roughness $\varepsilon/\overline{D}_h$ and the full-state Reynolds number characterizing the referential rough model.

In this study, and for optimal channel cross sections, an exact solution for determining the normal flow depth y_n using Manning's formula is sought, which is done by generalizing Manning's coefficient n to the whole turbulent flow domain (smooth, transition and rough). Where the semicircular shape of the open channels is the section with the best hydraulic conductivity, the larger these sections are, the more difficult their construction. To overcome the construction difficulties of large semicircular channels and to approach the best conductivity. Vatankhah (2014) presented a semiregular polygon section for one half of a regular polygon with an adequate number of sides, which includes the semisquare, the semihexagon, the semioctagon and the semidecagon, where each semiregular polygon has two types of cross-section: the type I semiregular polygon with a flat bottom and the type II semiregular polygon with an angular bottom. Both types of cross sections (type I and type II) have the same wetted perimeter and flow area. Among the semiregular polygons, the semicircular, the type I semisquare, which is a rectangular cross-section, the type II semisquare, which is a triangular cross-section, and the type I semihexagon, which is a trapezoidal cross-section, are the cross-sections that have been employed for this study in the rough turbulent domain. Semicircular sections are generally used for irrigation channels with small flows, rectangular sections are designed for moderate flow in most cases, triangular sections are generally used to carry small flows and to carry large flows, and trapezoidal sections are preferred for side slope stability (Swamee and Chahar, 2015).

The method used to attain our objective is the rough model method, called the RMM method. Much work has been done in this regard, such as Achour (2007), Lakehal and Achour (2014; 2017), and Zegait and Achour (2016), which is applicable to all domains of the turbulent regime. To express the Manning roughness equation and calculate the exact solution of the normal depth of different proposed channel cross-sections, the following steps are necessary:

First, the Manning roughness equation is established. Second, the dimensionless Manning's coefficient (N) equation is established. Third, a diagram (N diagram) is constructed, giving Manning's coefficient as a function of the Reynolds number and relative roughness. Fourth, the normal depth equation is established for the rough reference model. Finally, the normal depth equation is established.

GEOMETRIC AND HYDRAULIC PROPERTIES OF CHANNEL SECTIONS

Fig. 1 shows the cross-sections of an open channel, which include semicircular, rectangular, triangular and trapezoidal cross-sections. These cross-sections are characterized by the normal flow depth y_n and the top width of the surface water T .

Similarly, the rectangular and trapezoidal sections are characterized by the width of the channel bottom b , and the trapezoidal and triangular sections are characterized by the side slope z , which is given to each section as $z_{Trapezoidal} = 1/\sqrt{3}$ and $z_{Triangular} = 1$.

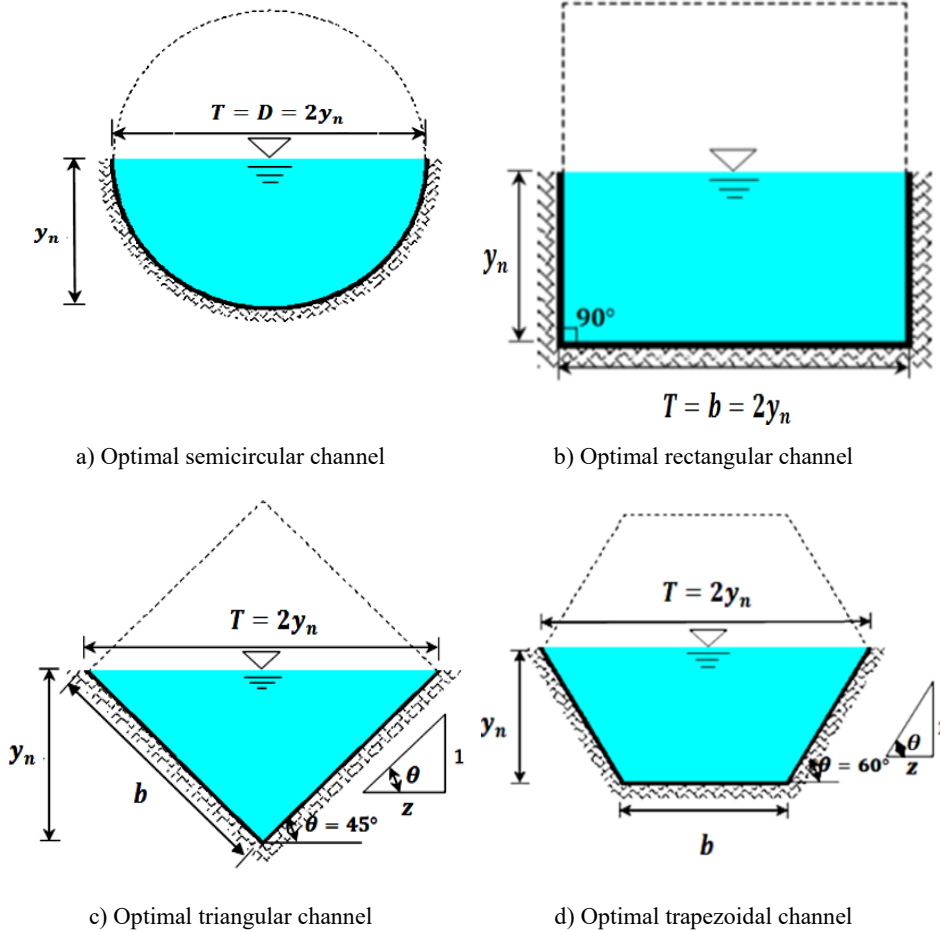


Figure 1: Channel profile of the selected sections

The geometric elements are the wetted area A (m^2) and wetted perimeter P (m) of the channel section, as shown in Figure 1, which are given by:

$$A = c_A y_n^2 \tag{1}$$

$$P = c_P y_n \tag{2}$$

The hydraulic elements are the hydraulic radius $R_h = A/P$ and the hydraulic diameter $D_h = 4R_h$ of the channel section, as shown in Figure 1, which are given by:

$$R_h = c_{R_h} y_n \tag{3}$$

$$D_h = c_{D_h} y_n \tag{4}$$

where c_A, c_P, c_{R_h} and c_{D_h} are coefficients related to the shape studied in Fig. 1, as given in Table 1.

Table 1: Geometric coefficients and hydraulic elements of chosen cross sections c_A, c_P, c_{R_h} and c_{D_h}

Cross section	c_A	c_P	c_{R_h}	c_{D_h}
Semicircular	$\pi/2$	π	1/2	2
Rectangular	2	4	1/2	2
Triangular	1	$2\sqrt{2}$	$1/(2\sqrt{2})$	$\sqrt{2}$
Trapezoidal	$\sqrt{3}$	$2\sqrt{3}$	1/2	2

GENERAL RELATION OF THE NORMAL FLOW DEPTH

The uniform flow in an open channel is expressed by the Manning formula as follows:

$$Q = \frac{\lambda}{n} R_h^{2/3} S_0^{1/2} A \tag{5}$$

where Q (m^3/s) is the flow discharge, S_0 is the longitudinal bed slope (m/m), n is Manning’s roughness coefficient ($s/m^{1/3}$) and λ is the unit conversion constant equal to 1 for the SI unit and 1.49 for the CU unit.

Substituting A and R_h from Eqs. (1) and (3), respectively, into Eq. (5) and simplifying, Manning’s equation is written as follows:

$$Q = \frac{\lambda}{n} c_{R_h}^{2/3} y_n^{8/3} S_0^{1/2} c_A \tag{6}$$

Solving Eq. (6), the normal depth y_n can be expressed by the following relation:

$$y_n = \Omega \left(\frac{n Q}{\sqrt{S_0}} \right)^{3/8} \quad (7)$$

where $\Omega = (c_A c_{R_h}^{2/3})^{-3/8}$ are shown in Table 2.

Eq. (7) allows the calculation of normal depth for semicircular, rectangular, triangular and trapezoidal channels using known values of flow discharge Q and channel bed slope S_0 .

However, as the value of the Manning roughness coefficient n depends on all the parameters governing the uniform flow, it cannot be used as a problem datum. More precisely, the normal depth y_n is the unknown of the problem.

To solve this problem, it is necessary to apply the RMM method to express Manning's coefficient n independently of the normal depth y_n . This aspect will be addressed in the next paragraph.

Table 2: The constant Ω of the chosen cross sections

cross section	Ω
Semicircular	$\pi/2$
Rectangular	$2^{-1/8}$
Triangular	$2^{3/8}$
Trapezoidal	$2^{5/8}/\pi^{3/8}$

MANNING'S ROUGHNESS COEFFICIENT EQUATION USING THE RMM METHOD

Referential rough model

All the geometric and hydraulic characteristics of the rough model are distinguished by the symbol " $\bar{\quad}$ ", where the reference rough channel is characterized by strong relative roughness, arbitrarily chosen equal to $\bar{\varepsilon}/\bar{D}_h = 3.7 \times 10^{-2}$, for a rough turbulent regime (Achour, 2007).

For a rough turbulent regime, the friction factor is given by the following Nikuradse equation, where $\varepsilon/D_h = \bar{\varepsilon}/\bar{D}_h$ and $f = \bar{f}$:

$$\bar{f} = \left[-2 \log \left(\frac{\bar{\varepsilon}/\bar{D}_h}{3.7} \right) \right]^{-2} \quad (8)$$

Substituting into Eq. (8) for the chosen value $\bar{\varepsilon}/\bar{D}_h = 3.7 \times 10^{-2}$, the friction factor \bar{f} takes the following constant value:

$$\bar{f} = \left[-2 \log \left(\frac{3.7 \times 10^{-2}}{3.7} \right) \right]^{-2} = (4)^{-2} = \frac{1}{16} \quad (9)$$

Any linear dimension L of the channel is related to the homolog linear dimension \bar{L} of the rough model by the following fundamental relationship, applicable to the turbulent domain (Achour, 2007):

$$L = \psi \bar{L} \quad (10)$$

where ψ is a dimensionless correction factor of the linear dimension varying in the following range [0; 1]; ψ is governed by the following explicit relationship (Achour and Bedjaoui, 2006):

$$\psi \cong 1.35 \left[-\log \left(\frac{\bar{\varepsilon}/\bar{D}_h}{4.75} + \frac{8.5}{\bar{R}_e} \right) \right]^{-2/5} \quad (11)$$

where \bar{D}_h is the hydraulic diameter in the rough model, $\bar{\varepsilon}$ is the absolute roughness and \bar{R}_e is the Reynolds number characterizing the flow in the rough model, which can be expressed as follows:

$$\bar{R}_e = 32\sqrt{2} \frac{\sqrt{g S_0 \bar{R}_h^3}}{\nu} \quad (12)$$

where ν is the kinematic viscosity (m^2/s).

Taking into account Eq. (10) R_h and \bar{R}_h are related as follows:

$$R_h = \psi \bar{R}_h \quad (13)$$

Eq. (10) can be rewritten as:

$$L^2 = \psi^2 \bar{L}^2 \quad (10a)$$

where L^2 and \bar{L}^2 are proportional to the water areas A and \bar{A} , respectively. Thus, one may obtain the following:

$$A = \psi^2 \bar{A} \quad (14)$$

Manning's roughness equation

Substituting the parameters R_h and A from Eqs. (13) and (14) into Eq. (5) and simplifying, Manning's equation becomes as follows:

$$Q = \frac{\lambda}{n} \psi^{8/3} \bar{R}_h^{2/3} S_0^{1/2} \bar{A} \quad (15)$$

For the rough model, Manning's equation expressed by Eq. (5) can be written as follows:

$$\bar{Q} = \frac{\lambda}{\bar{n}} \bar{R}_h^{2/3} \bar{S}_0^{1/2} \bar{A} \quad (16)$$

According to the RMM fundamental principle (Achour and Bedjaoui, 2006), the flow rates Q and the bed slope S_0 of the channel are the same in the rough model, i.e., $\bar{Q} = Q$ and $\bar{S}_0 = S_0$.

The combination of Eqs. (15) and (16) show that the Manning roughness coefficient n of the studied channel and \bar{n} of the rough model can be linked by the following relation (Zegait and Achour, 2016):

$$n = \psi^{8/3} \bar{n} \quad (17)$$

Manning's roughness coefficient \bar{n} in the rough model is given by Chezy's resistance coefficient as (Zegait and Achour, 2016):

$$\bar{n} = \frac{\bar{R}_h^{-1/6}}{\bar{C}} \quad (18)$$

Chezy's resistance coefficient \bar{C} was derived from the study of Achour and Bedjaoui (2006) as follows:

$$\bar{C} = \sqrt{\frac{8g}{\bar{f}}} \quad (19)$$

Where $\bar{f} = 1/16$ according to Eq. (9). Thus, Eq. (19) reduces to $\bar{C} = 8\sqrt{2g}$. The hydraulic radius \bar{R}_h of the rough model can be written by analogy with Eq. (3) as follows:

$$\bar{R}_h = c_{R_h} \bar{y}_n \quad (20)$$

Considering Eq. (20) and $\bar{C} = 8\sqrt{2g}$, Eq. (18) becomes:

$$\bar{n} = \frac{c_{R_h}^{1/6} y_n^{-1/6}}{8 \sqrt{2g}} \tag{21}$$

By introducing \bar{n} and ψ from Eqs. (21) and (11) into Eq. (17), Manning’s roughness coefficient n equation becomes:

$$\frac{1}{n} = \sigma \sqrt{g} y_n^{-1/6} \left[-\log \left(\frac{\varepsilon/\overline{D_h}}{4.75} + \frac{8.5}{R_e} \right) \right]^{16/15} \tag{22}$$

where:

$$\sigma = 1.35^{-8/3} 8 \sqrt{2} c_{R_h}^{-1/6} \tag{22a}$$

Table 3, along with Table 1, gives the values of σ for the considered channel cross-sections.

Table 3: Values of σ for the four selected cross sections

Cross section	σ
Semicircular	5.705
Rectangular	5.705
Triangular	6.044
Trapezoidal	5.705

Discussion of Manning's roughness equation

Eq. (22) can be written in dimensionless terms as follows:

$$\frac{y_n^{-1/6}}{n \sqrt{g}} = \sigma \left[-\log \left(\frac{\varepsilon/\overline{D_h}}{4.75} + \frac{8.5}{R_e} \right) \right]^{16/15} \tag{23}$$

Let us be the dimensionless parameter N as follows:

$$N = \frac{y_n^{-1/6}}{n \sqrt{g}} \tag{24}$$

Therefore, Eq. (23) can be written as follows:

$$N = \sigma \left[-\log \left(\frac{\varepsilon/\overline{D_h}}{4.75} + \frac{8.5}{R_e} \right) \right]^{16/15} \tag{25}$$

As a result, the dimensionless coefficient N is expressed as a function of the relative roughness $\varepsilon/\overline{D_h}$ and Reynolds number $\overline{R_e}$. It is worth noting that equation (25) applies to

the four selected channel shapes. Equation (25) is graphically presented using a semilogarithmic axis of the coordinate system. The resulting diagram (Fig. 2) shows the variation in N as a function of Reynolds number $\overline{R_e}$ for various values of relative roughness $\varepsilon/\overline{D_h}$ and different selected open channels.

Eq. (25) is evidently applicable in all domains of the turbulent flow corresponding to $\overline{R_e} \geq 2300$ and $0 \leq \varepsilon/\overline{D_h} \leq 0.05$.

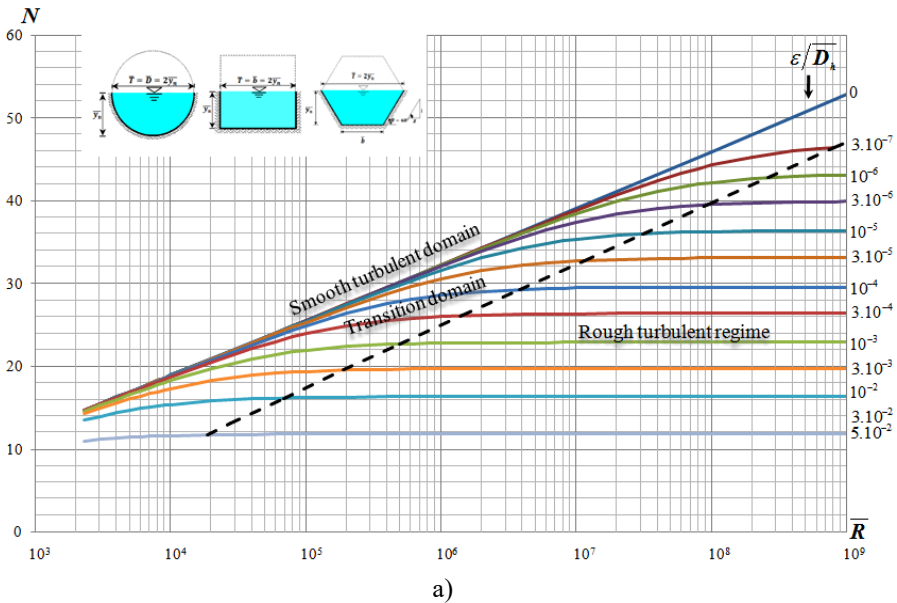
Therefore, for the smooth domain corresponding to $\varepsilon \rightarrow 0$, Eq. (25) is written as follows:

$$N = \sigma \left[-\log \left(\frac{8.5}{\overline{R_e}} \right) \right]^{16/15} \quad (26)$$

Additionally, for the rough turbulent domain, corresponding to $\nu \rightarrow 0$ or $\overline{R_e} \rightarrow \infty$, Eq. (25) is written as follows:

$$N = \sigma \left[-\log \left(\frac{\varepsilon/\overline{D_h}}{4.75} \right) \right]^{16/15} \quad (27)$$

The discontinuous curve in Fig. 2 represents the limit between the transition and the rough turbulent domains.



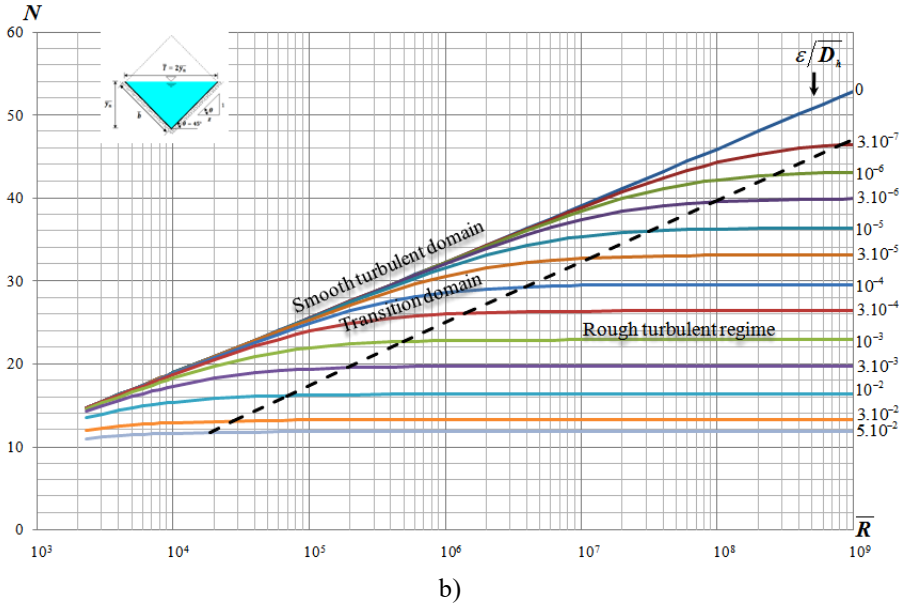


Figure 2: Variation in N as a function of the Reynolds number \overline{R}_e for fixed values of relative roughness $\varepsilon/\overline{D}_h$ according to Eq. (25) for the four chosen open channels. a) Semicircular, rectangular and trapezoidal channels, b) triangular channel. (—) Practical limit curve between transition and rough turbulent domains

The normal depth of the rough reference model

The rough reference model of different considered open channels chosen are characterized by their top width \overline{T} of the surface water and the normal flow depth \overline{y}_n .

Taking into account Eq. (1), the wetted area of the rough reference model \overline{A} (m^2) is as follows:

$$\overline{A} = c_A \overline{y}_n^2 \tag{28}$$

Additionally, taking into account Eq. (2), the wetted perimeter \overline{P} (m) is expressed as follows:

$$\overline{P} = c_P \overline{y}_n \tag{29}$$

Taking into account Eq. (4), one may write the following:

$$\overline{D}_h = c_{D_h} \overline{y}_n \tag{30}$$

For flow in open channels, the Darcy-Weisbach relationship is written as follows:

$$S_0 = \frac{f}{2g} \frac{Q^2}{A^2 D_h} \quad (31)$$

Applying Eq. (31) to the reference rough model, with $\bar{Q} = Q$ and $\bar{S}_0 = S_0$, one may obtain the following:

$$S_0 = \frac{\bar{f}}{2g} \frac{Q^2}{\bar{A}^2 \bar{D}_h} \quad (32)$$

With $\bar{f} = 1/16$ and taking into account Eqs. (28) and (30), Eq. (32) becomes as follows:

$$S_0 = \frac{1}{\chi g} \times \frac{Q^2}{y_n^5} \quad (33)$$

where χ is a constant expressed as follows:

$$\chi = 32 c_A^2 c_{D_h} \quad (34)$$

Table 4 gives the values of the constant χ for the considered cross-sections.

Table 4: Values of the constant χ for the considered cross-sections

Cross section	χ
Semicircular	16π
Rectangular	256
Triangular	$32\sqrt{2}$
Trapezoidal	192

The normal flow depth \bar{y}_n of the reference rough model is obtained from Eq. (33) as follows:

$$\bar{y}_n = \chi^{-1/5} \times \left(\frac{Q}{\sqrt{g S_0}} \right)^{2/5} \quad (35)$$

Eq. (35), along with Eqs. (12), (22), and (30), allows computing Manning's roughness coefficient n . Thus, the normal depth sought y_n is worked out using Eq. (7) for the known value of the parameter Ω selected from Table 2.

CONCLUSION

This study aims to propose the exact solution of the normal depth in four semipolygonal cross sections, namely, semicircular, rectangular, triangular and trapezoidal cross sections. Indeed, the general equation of the normal depth as a function of discharge, bed slope, and Manning coefficient applicable to the four chosen shapes can be explicitly expressed. Manning's roughness coefficient was expressed based on the RMM method according to the characteristics of the reference rough model, namely, normal depth $\overline{y_n}$, relative roughness $\varepsilon/\overline{D_h}$ and Reynolds number $\overline{R_e}$. Manning's coefficient was also written in dimensionless form as a function of the last two parameters.

Two diagrams similar to Moody's were developed as a graphic depiction of the result. The semicircular, rectangular, and trapezoidal shapes are all covered by the first diagram, while the triangle shape is covered by the second. These diagrams show that the Manning coefficient is not constant and may be observed in the rough, smooth, and transitional turbulent domains. This result permitted Manning's formula to be generalized throughout the whole turbulent domain.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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