Larhyss Journal, ISSN 1112-3680, n°53, Mars 2023, pp. 95-115

DISCHARGE COEFFICIENT RELATIONSHIP FOR THE SMBF FLUME

ACHOUR B.[1], AMARA L.[2]

[1] Professor, Research Laboratory in Subterranean and Surface Hydraulics (LARHYSS), University of Biskra, Algeria
[2] Associate Professor, Department of Civil Engineering and Hydraulics, Faculty of Science and Technology, University of Jijel, Algeria

(*) bachir.achour@larhyss.net

Research Article – Available at http://larhyss.net/ojs/index.php/larhyss/index
Received September 16, 2022, Received in revised form February 25, 2023, Accepted March 2, 2023

ABSTRACT

The discharge coefficient C_d, defined as the ratio of the experimental flow rate to the ideal flow rate, is an important parameter. It is the ultimate correction factor for the theoretical stage-discharge relationship when it is determined on the basis of simplifying assumptions. To understand the behavior of the flow rate Q, the plot of Q as a function of the stage h is insufficient because the influence parameters are not detected. Plotting the experimental discharge coefficient against the relative upstream flow depth, related to the width of the approach channel, is the best way to know which are the parameters that influence the flow rate and hence the stage-discharge relationship. This can sometimes reveal the existence of unsuspected influential phenomena.

Regarding the SMBF flume, the literature does not report any relationship likely to govern the discharge coefficient of the device. Studies have focused on the stage-discharge relationship without alluding to the discharge coefficient. The stage-discharge relationships available in the literature are of two types. There are formulas of complex form, totally locked, which do not allow any possibility of development or expansion to extract the discharge coefficient relationship hidden inside. There are formulas that are rather simple in form but require transformations to highlight the unapparent discharge coefficient relationship.

It is the last type of formula that will be addressed in this study. For each stage-discharge relationship proposed in the literature, the main objective is to associate it with the relationship that governs the discharge coefficient and to highlight the influential parameters.

The different models describing the discharge coefficient of the device will be compared with the observations available in the literature, and interesting conclusions will be drawn.

Keywords: SMBF flume, Flow measurement, Stage-discharge, Discharge coefficient.

INTRODUCTION

Whether it is a liquid or gas, the measurement of flow rate is usually of the utmost importance in many processes. Some crucial applications require the ability to conduct accurate flow measurements to ensure the safety of hydraulic structures and installations (Tadda et al., 2020; Burcharth et al., 2007). Even in the least demanding operations from the safety point of view, such as the design of an open channel, knowledge of the maximum evacuated flow rate is decisive. Determining the flow rate is also fundamental for the proper irrigation of the plots, as it allows ascertaining that the right water quantity is going to the right place.

With the many varieties of flowmeter technology available in the literature (Henderson, 1966; Bos, 1976; Bos, 1989; Hager, 1986; Achour et al., 2003; Kulkarni and Hinge, 2021; Achour and Amara, 2022a), it can be difficult to decide which device is best for the application. As a rule, devices with high precision are expensive and are therefore excluded from common applications that do not require as much precision. What is desired by the operators is to have available measuring devices that are inexpensive, fairly accurate over a wide range of flow rates and easy to implement. To meet these requirements, operators preferably select flume devices (Bos, 1976; Hager, 1986), including the SMBF flume (Samani and Magallanez, 2000; Ferro, 2002), which is the major concern of the current study. In addition to having the aforementioned advantages, the SMBF flume requires a reduced space compared with many flumes usually used, such as the Parshall flume, Venturi flume or Montana flume (Cone, 1917; Parshall, 1926; Westesen, 1992).

Although it is known today and widely disseminated in the specialized literature (Ferro, 2002; Baiamonte and Ferro, 2007; Di Stefano et al., 2008; Vatankhah, 2017; Vatankhah and Mohammadi, 2020), it is nevertheless worth recalling the description of the device even briefly. Samani and Magallanez (2000) are the initiators of this device, which was called the SM flume by relatedness since the device bears the initials of the author's names. The name of the device then evolved to be called SMBF, where the initials BF are those of the authors Baiamonte and Ferro (2007) who carried out further investigations.

The principle of operation of the device is based on the deliberate creation of a critical flow in the constricted section of the device, called the “control section”. This is the *sine qua non* condition for the correct functioning of the device, allowing production of the stage-discharge relationship; the flow rate is thus determined using only one upstream depth reading, provided the geometry of the device is well defined and its installation is properly executed. In other words, the flow rate Q is a single-valued function of the upstream measured depth h well known as the stage (Fig. 1), meaning that each upstream depth belonging to the depth domain maps to a single well-defined discharge of its range. The resulting single-valued function can be expressed as $Q = \psi(h)$, where ψ is an algebraic function of stage.

The installation of the device in an approach channel raises the upstream water level, and the flow is then subcritical (Fig. 1). The narrowed section of the device accelerates the flow, which becomes supercritical downstream of the contraction.

The SMBF flume was originally designed to be used for flow measurement in horizontal rectangular channels (Samani and Magallanez, 2000); however, the SMBF flume is also applicable to sloping channels (Carollo and Pampalone, 2021). The contraction of the device is obtained by cutting a pipe in half and placing half on each side of the approach channel opposite each other with the depth reading gauge set on the upstream side of the flume. This creates a critical flow between the two half-pipes of diameter d (Fig. 1), forming the narrowed section allowing the flushing of sediment and debris through the flume. Thus, the device is endowed with converging rectangular sections that vary according to the equation of a circle. The ratio of the width b of the narrowed section to the width B of the approach channel defines the contraction rate β, i.e., $\beta = b/B$.

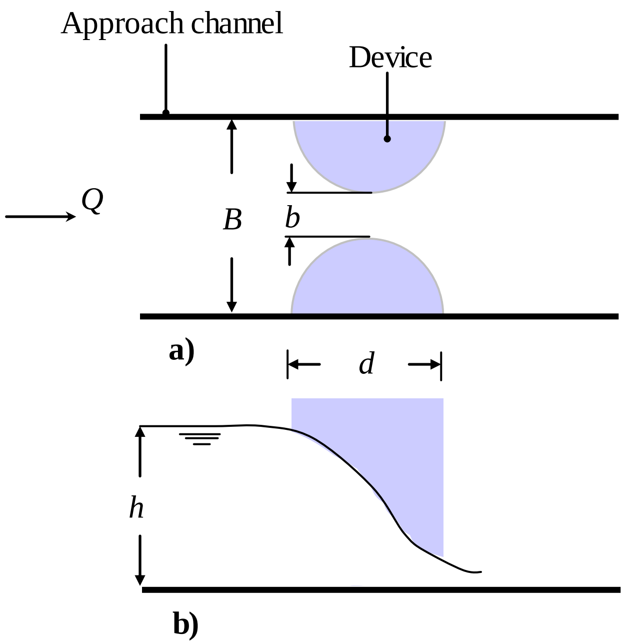

Figure 1: a) Plan view of the approach channel and the device; b) Longitudinal profile of the resulting flow

Defined as the ratio of the actual discharge to the ideal discharge and denoted C_d, the discharge coefficient can at times be poorly, incorrectly or mistakenly calculated. In practice, a constant value of the discharge coefficient is often preferred for simplicity and simplification. However, this approximation can lead to deceptive discharge estimations adversely affecting the design of hydraulic installations. Depending on the kind of measurement flow device and the upstream flow conditions (Henderson, 1966; Bos, 1976; Bos, 1989; Hager, 1986), the discharge coefficient value is different for each installation. The approach usually adopted consists of experimentally measuring the actual discharge coefficient by calibration tests and consequently correcting the theoretical discharge. The discharge coefficient is then used as an ultimate correcting factor of the theoretical stage-discharge relationship to counteract errors due to some assumptions made while developing the theoretical relationship.

For some flow measurement devices, the discharge coefficient can be a constant depending on the geometry of the device, independent of the upstream flow depth. However, the discharge coefficient of certain devices depends not only on their geometry but also on the upstream flow depth, which is most often related to the width of the approach channel to make it dimensionless.

One may then understand why it is useful to observe the variation in the discharge coefficient to define the main influential parameters and to better understand the flow behavior. This allows building the relevant governing model, especially when the theory is unable to produce a complete stage-discharge relationship, as is precisely the case with the SMBF flume.

Despite its great importance in the field of flow measurement, studies carried out on SMBF flumes have either concealed or disregarded the discharge coefficient of this device (Samani and Magallanez, 2000; Ferro, 2002; Baiamonte and Ferro, 2007; Di Stefano et al., 2008; Vatankhah, 2017; Vatankhah and Mohammadi, 2020), both experimentally and theoretically. Despite the availability of experimental tests, no analysis of the discharge coefficient of the SMBF flume has been envisaged, which could have nevertheless enabled the derivation of appropriate conclusions.

The experimental data available in the literature were exploited to derive the most accurate stage-discharge relationship that governs the device. Thus, some models have been adopted, and the resulting stage-discharge relationships are of great complexity in their form, which makes them cumbersome and unwieldy (Samani and Magallanez, 2000; Ferro, 2002; Baiamonte and Ferro, 2007; Di Stefano et al., 2008; Vatankhah, 2017; Vatankhah and Mohammadi, 2020). In addition, derived stage-discharge relationships are hardly consistent with flow-metering devices as advocated by fundamentals, where the stage must be to the 3/2 power when the device is formed of a rectangular section, as is the case with the SMBF flume. Furthermore, due to the complexity of their form, the available stage-discharge relationships cannot be developed or expanded to extract the relationship that governs the discharge coefficient C_d of the device, through which one could have observed the main influential parameters. Thus, these complex formulas will not be discussed in this study.

Therefore, the present study plans to reconsider the experimental measurements available in the literature along with the main objective of deriving the discharge coefficient relationship of the device related to the main affecting parameters. The resulting stage-discharge relationship is expected to be simple and easy to handle, which would further conform to flow measurement devices in open channels as prescribed by the fundamentals. The study will focus more particularly on Ferro's simple model (2002) describing the stage-discharge relationship, along with Baiamonte and Ferro's observations (2007). Judicious manipulations will be made to deduce the governing discharge coefficient relationship. In addition, an accurate discharge coefficient relationship derived from the analysis of the more complete Vatankhah and Mohammadi observations (2020) will be suggested.

Particular attention will be given to the degree of accuracy of the resulting discharge coefficient relationships in comparison with that obtained by previous studies.

To avoid any confusion in the interpretation of our calculations, the percent error (PE), considered in this study, does not relate to measurement, in the sense of evaluating the percent error on the experimental or measured parameter such as a flow rate Q or a flow depth h, for instance. This will be about evaluating the percent error on a value given by a predictive theoretical model, corrected for the effect of an adjustment factor derived from the experiment, such as $x_{Exp} = \alpha\, x_P$, where x_{Exp} and x_P are the experimental and the predictive value, respectively, while α is the correction factor. This allows us to write that $|1 - \alpha|$ is the relative error. The more coefficient α approaches 1, the more accurate a model prediction. The predictive value x_P is sometimes called the “accepted value” denoted x_a, the “true value”, or even the “theoretical value” denoted x_{Th}. Thus, the study refers to the following PE formula, which can be written under two versions meaning the same thing, using different wording, namely, PE = $100|x_a - x_{Exp}|/x_a$ or PE = $100|x_{Th} - x_{Exp}|/x_{Th}$.

ANALYSIS AND DEVELOPMENT OF THE AVAILABLE STAGE-DISCHARGE RELATIONSHIPS

Using dimensional and experimental analyses, the relationship governing the discharge Q through the SMBF flume was determined to be under the following form (Ferro, 2002):

$$Q = 0.701 b^{5/2} \sqrt{g} \left(\frac{h}{b}\right)^{1.59} \tag{1}$$

where b is expressed in feet or meters and can be written as follows: $b = (B - d)$ (Fig. 1), and g is the acceleration due to gravity. It is worth noting that width b is denoted B_c in the literature, where the subscript “c” denotes the critical condition, while the contraction rate β is designated by r.

In Eq. (1), the constant 0.701 and the exponent 1.59 are subject to uncertainties since they were determined using the digitized experimental data provided by Samani and Magallanez (2000), involving three tested devices characterized by the following contraction rates $\beta = b/B$: 0.40, 0.457, and 0.597.

However, at first glance, the flow rate given by Eq. (1) is strangely independent of the contraction rate β, and such an insufficiency was rightly pointed out by Vatankhah and Mohammadi (2020) and Carollo et al. (2016).

Apart from the fact that it is advantageously simple compared to available models, Eq. (1) does not comply with what is recommended by the fundamental principles of flow measurement. Actually, when the section is rectangular, as is the case with the current device, the flow rate should be directly proportional to the 3/2 power of the stage, meaning that $Q \propto h^{3/2}$. When the power exceeds the value 3/2 in the case of a rectangular section, then it is almost established by practice that the discharge coefficient C_d is also dependent on the upstream flow depth h, generally related to the width B of the approach channel to form the dimensionless parameter h/B.

Let us write the stage-discharge relationship of the device in the following form, which is consistent with flow measurement devices based on a rectangular control section (Achour and Amara, 2022b):

$$Q = C_d \sqrt{2g} B h^{\frac{3}{2}} \tag{2}$$

After some manipulations, Eq. (1) can be rewritten as follows:

$$Q = 0.4957 \sqrt{2\,g}\; \beta^{\,0.91} \left(\frac{h}{B} \right)^{0.09} B\, h^{3/2} \tag{3}$$

It is thus easy to recognize the discharge coefficient C_d of the device such that:

$$C_d = 0.4957\, \beta^{\,0.91} \left(\frac{h}{B} \right)^{0.09} \tag{4}$$

The discharge coefficient C_d of the device thus depends on both the contraction rate β of the device and the relative upstream depth h/B. In previous studies, it has already been shown that the contraction rate β has a significant influence on the stage-discharge relationship governing the device (Vatankhah, 2017; Carollo et al., 2016).

Fig. 2 shows the variation in $C_{d,Exp}$ derived from Baiamonte and Ferro's observations (2007) as a function of the experimental relative upstream depth h_{Exp}/B, where the subscript "*Exp*" denotes "Experimental". The observations are also compared to the predictive model expressed by Eq. (4), represented by the solid line curves in Fig. 2.

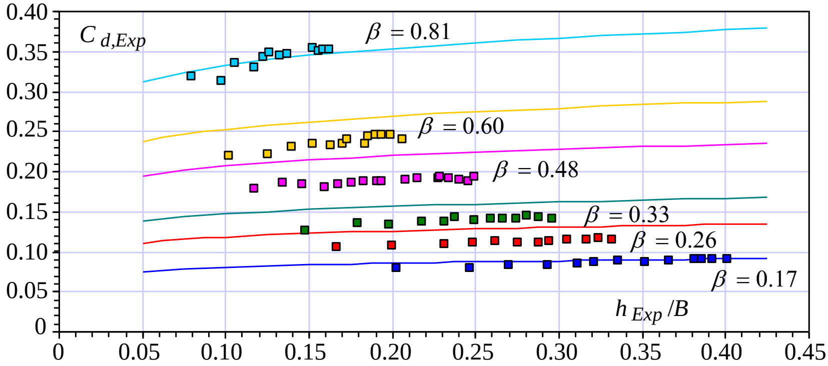

Figure 2: Variation in $C_{d,Exp}$ with h_{Exp}/B according to Baiamonte and Ferro observations (2007). Solid line curves: C_d values predicted according to Eq. (4).

Fig. 2 first allows us to give an opinion on the quality of the measurements collected during the tests (Baiamonte and Ferro, 2007). With the exception of a few measurement points, especially for $\beta = 0.81$, the observations seem to have been carried out correctly given the rather uniform distribution of the experimental points. However, the rather mediocre experimental data relating to beta = 0.81 are likely to generate significant inaccuracy for any model expressing either the flow rate or the discharge coefficient.

Fig. 2 reveals that the discharge coefficient C_d is dependent not only on the contraction rate β but also on the upstream relative depth h/B. However, it seems that for low values of the contraction rate β, such as $\beta = 0.17$ and $\beta = 0.26$, the effect of the ratio h/B on C_d is not significant, as the corresponding predictive curves are close to horizontal.

In addition, it may be observed in Fig. 2 that the predictive model described by Eq. (4) overestimates the discharge coefficient C_d since the observations, on the whole, are below the corresponding predictive curves. It is then logical to assume that the constants featured in the predictive model expressed by Eq. (4) are not the most appropriate ones. This is confirmed by the analysis of Baiamonte and Ferro's observations (2007), which show poor agreement with the predicted C_d values given by Eq. (4); the resulting deviations are grouped in Table 1 and are represented in Fig. 3 in detail.

Table 1: Deviations between Baiamonte and Ferro's C_d observations (2007) and C_d predicted values according to Eq. (4)

β	Deviation in C_d (%)		
	Minimum	**Maximum**	**Average**
0.17	0.364	10.40	3.956
0.26	12.39	15.64	13.95
0.33	10.58	17.65	13.18
0.48	12.65	16.78	14.71
0.60	8.88	14.38	11.23
0.81	0.016	5.76	1.77

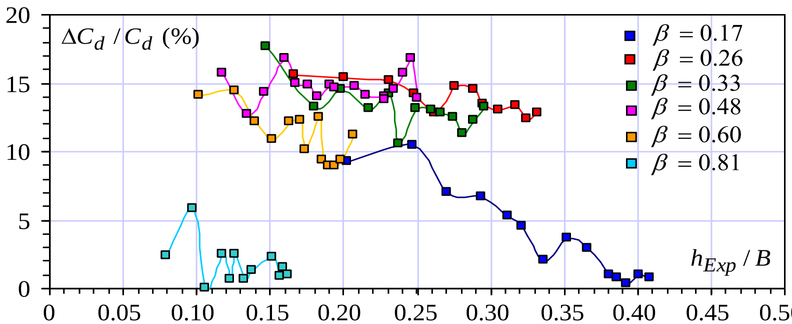

Figure 3: Deviations between predicted C_d values according to Eq. (4) and Biamonte and Ferro's observations (2007)

Thus, except for β = 0.81 and some values corresponding to β = 0.17, for which the deviations are below 5%, Fig. 3 confirms a poor agreement between the predicted and experimental C_d values according to Baiamonte and Ferro observations (2007).

Using the model expressed by Eq. (4), whose form is assumed to be the most appropriate, the observations of Baiamonte and Ferro (2007) were subjected to an optimization process to minimize the maximum deviation for each value of the contraction rate β. New model constants were then obtained and are presented as follows:

$$C_d = 0.506\, \beta^{1.0435} \left(\frac{h}{B}\right)^{0.108} \tag{5}$$

Eq. (5) was compared with the observations of Baiamonte and Ferro (2007), and the final result is shown in Fig. 4. One may perceive a clear improvement in the agreement between the values predicted by the new model described by Eq. (5) and the observed values compared to Fig. 2 involving the model expressed by Eq. (4).

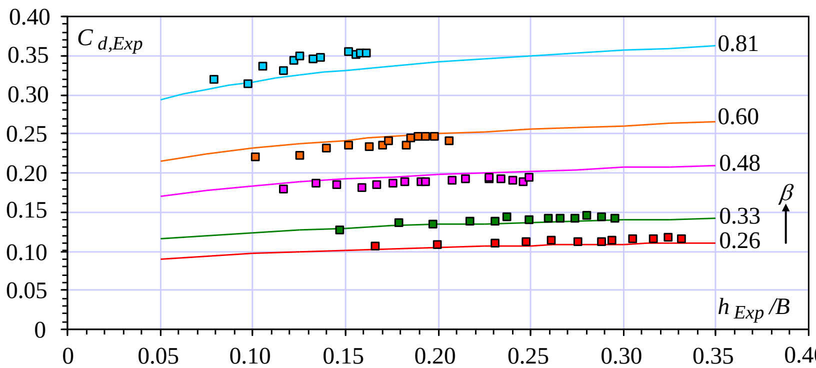

Figure 4: Variation in $C_{d,Exp}$ with h_{Exp}/B according to Baiamonte and Ferro (2007). Solid line curves: C_d values predicted according to Eq. (5)

Table 2 summarizes the deviations between the predicted values given by Eq. (5) and the observations, as shown in Fig. 5. Notably, the observations related to β = 0.17 were excluded from our analysis because abnormally high deviations were observed, varying between 13.5% and 26.66% depending on the h/B value.

Table 2: Deviation between C_d computed using Baiamonte and Ferro's observations (2007) and predicted C_d values according to Eq. (5)

β	Deviation in C_d (%)		
	Minimum	**Maximum**	**Average**
0.26	2.076	4.836	3.343
0.33	0.007	4.235	1.692
0.48	2.162	7.784	5.064
0.60	1.545	6.791	3.838
0.81	0.987	7.142	4.745

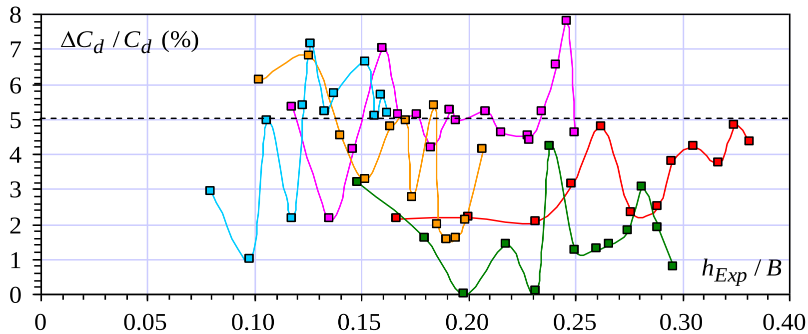

Figure 5: Deviations between predicted C_d values according to Eq. (5) and Baiamonte and Ferro's observations (2007). Same notations as in Fig. 4

A sample of sixty-eight measurement points was involved in the drawing of Fig. 5. More than 69% of deviations are less than 5%, while more than 97% are less than or equal to 7%. This quality result confirms the good agreement between the observations and the predictive model expressed by Eq. (5). This is valid in the following experimental relative upstream depth range of $0.08 \leq h_{Exp}/B \leq 0.332$ and for contraction rates β such as $0.26 \leq \beta \leq 0.81$. Outside the range of h_{Exp}/B previously indicated, additional tests are needed to confirm the advocated predictive model or to correct it if necessary.

Reconsidering the observations of Biamonte and Ferro (2007), Vatankhah and Mohammadi (2020) proposed new constants for the model developed by Ferro and expressed by Eq. (1). The derived model is as follows:

$$Q = 0.612 b^{5/2} \sqrt{g} \left(\frac{h}{b}\right)^{1.585} \tag{6}$$

Let us transform Eq. (6) to put it in the form of Eq. (2). Inserting the contraction rate $\beta = b/B$ and operating some manipulations results in the following:

$$Q = 0.43275 \beta^{0.915} \left(\frac{h}{B}\right)^{0.085} \sqrt{2g}\, B h^{3/2} \tag{7}$$

Accordingly, the discharge coefficient C_d can be identified as follows:

$$C_d = 0.43275\, \beta^{0.915} \left(\frac{h}{B}\right)^{0.085} \tag{8}$$

The model thus obtained also shows that the discharge coefficient C_d depends on both the contraction rate β and the relative upstream flow depth h/B. Although they were determined by the same process, the constants of the model expressed by Eq. (8) are different from those defined by the model expressed by Eq. (5).

The observations of Baiamonte and Ferro (2007) are compared in Fig. 6 to the predictive model expressed by Eq. (8) by solid line curves. The observations related to $\beta = 0.17$ were excluded from data processing because abnormal deviations, varying between 2.82% and 15.84%, were computed between the observations and the values predicted by Eq. (8).

It appears that the model expressed by Eq. (8) underestimates the discharge coefficient for at least the contraction rate $\beta = 0.81$. To determine to what value of β the model is acceptable, other tests are needed, involving contraction rates varying between 0.60 and 0.81. It is then recommended to use Eq. (8) in the range $0.26 \leq \beta \leq 0.60$, excluding the contraction rates $\beta = 0.17$ and $\beta = 0.81$. Therefore, the range of applicability for Eq. (8) is more restricted than that of the model expressed by Eq. (5). The manipulator should also ensure that the range of relative upstream flow depths is such that $0.102 \leq h_{Exp}/B \leq 0.332$.

Deviations between Eq. (8) and Baiamonte and Ferro's observations (2007) are reported in Table 3, which confirms that the use of the model expressed by Eq. (8) is not recommended for the contraction rate $\beta = 0.81$. More details on the resulting deviations can be seen in Fig. 7.

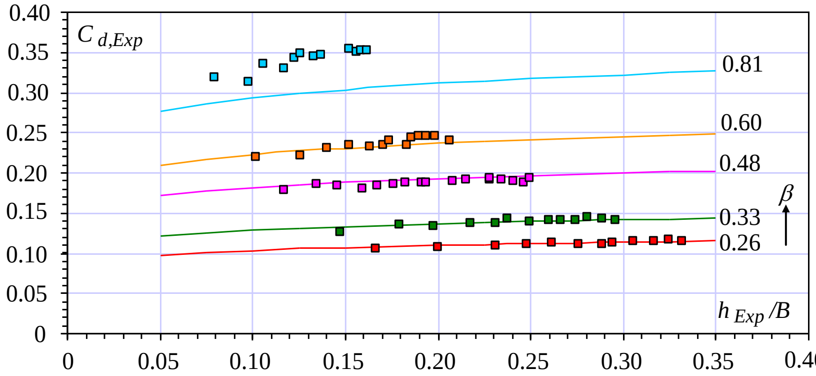

Figure 6: Variation in $C_{d,Exp}$ with h_{Exp}/B according to Baiamonte and Ferro (2007). Solid line curves: C_d values predicted according to Eq. (8)

Table 3: Deviation between Baiamonte and Ferro's C_d observations (2007) and predicted C_d values according to Eq. (8)

β	Deviation in C_d (%)		
	Minimum	Maximum	Average
0.26	0.055	3.579	1.504
0.33	0.211	6.055	1.395
0.48	0.583	5.142	2.757
0.60	0.0167	3.781	1.895
0.81	6.818	16.26	6.818

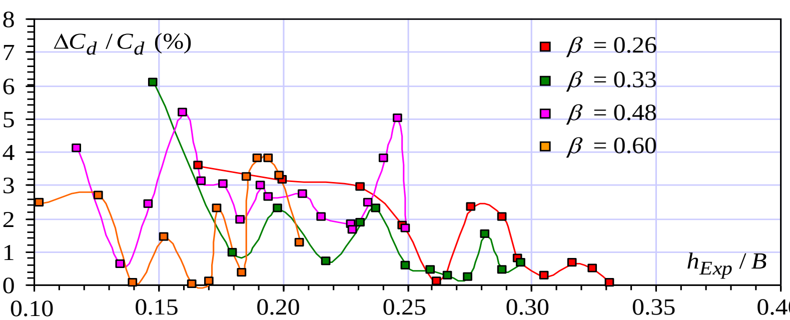

Figure 7: Deviations between predicted C_d values according to Eq. (8) and Baiamonte and Ferro's observations (2007)

As shown in Fig. 7, practically 100% of the deviations are less than or equal to 5%, which allows us to affirm that the model expressed by Eq. (8) is acceptable within the following ranges: $0.26 \leq \beta \leq 0.60$ and $0.102 \leq h_{Exp}/B \leq 0.332$.

Another simple model expressing the stage-discharge relationship that governs the device is the following (Vatankhah and Mohammadi, 2020):

$$Q = 0.65\,\beta^{\,0.05}\,b\,\sqrt{g}\left(\frac{h}{b}\right)^{0.11} h^{\,3/2} \tag{9}$$

The form of Eq. (9) complies with flowmeters in open channels, based on a rectangular section, since the flow rate is directly proportional to the 3/2 power of the stage. To transform Eq. (9) into the form of Eq. (2), it is necessary to carry out some manipulations. Replacing b by βB and rearranging yields the following:

$$Q = 0.4596\,\beta^{\,0.94}\sqrt{2g}\left(\frac{h}{B}\right)^{0.11} B\,h^{\,3/2} \tag{10}$$

Compared to Eq. (2), Eq. (10) allows us to deduce that the discharge coefficient C_d is as follows:

$$C_d = 0.4596\,\beta^{\,0.94}\left(\frac{h}{B}\right)^{0.11} \tag{11}$$

Eq. (11) again shows that the discharge coefficient C_d depends on the contraction rate β and the relative upstream flow depth h/B. The model expressed by Eq. (11) is presented in the same simple form as the models developed previously. It is not expected that this model can be more accurate than that described by Eq. (5). Fig. 8 shows the comparison between the predicted values of C_d given by Eq. (11), represented in solid line curves, with Baiamonte and Ferro's observations (2007). The experimental values collected on the device characterized by $\beta = 0.17$ have been excluded from the data processing because the deviations observed are as abnormally high as in the previous cases. According to the h_{Exp}/B ratio, the deviations vary between 6.58% and 16.97%.

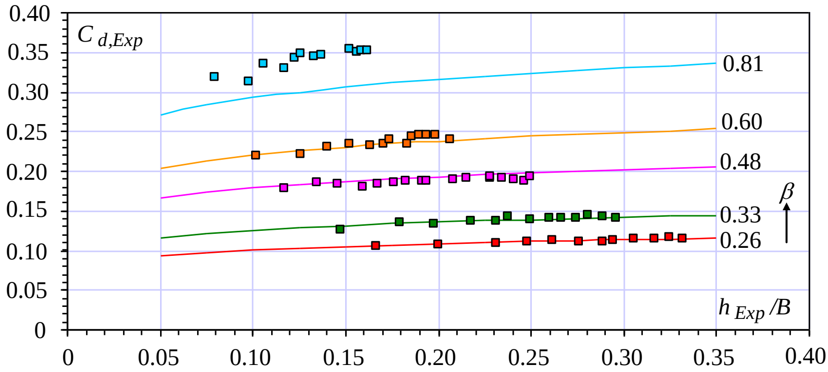

Figure 8: Variation in $C_{d,Exp}$ with h_{Exp}/B according to Baiamonte and Ferro (2007). Solid line curves: C_d values predicted according to Eq. (11)

It is thus observed in Fig. 8 that, as in the case of the model expressed by Eq. (8), the model described by Eq. (11) underestimates the values of the discharge coefficient C_d for the contraction rate $\beta = 0.81$. Thus, the conditions of applicability of Eq. (11) are the same as those for Eq. (8), especially with regard to the β and h_{Exp}/B ranges.

Vatankhah and Mohammadi (2020) also proposed a nonlinear form model describing the stage-discharge relationship. It reads as follows:

$$Q = \left[0.407\,\beta^{-0.16} \left(\frac{h}{b} \right)^{0.263} + 0.407\,\beta \right] b\sqrt{g}\, h^{3/2} \tag{12}$$

The model constants were derived from the processing of Baiamonte and Ferro observations (2007). The particularities of this new model are that it is simple, subject to development and expansion, and conforms to flow measurement devices in open channels based on a rectangular section. It is also of a more elaborate form than the original Ferro model expressed by Eq. (1).

After performing some transformations and manipulations, Eq. (12) has been put into the standard form expressed by Eq. (2), as follows:

$$Q = 0.2878\,\beta^{0.577} \left[\left(\frac{h}{B} \right)^{0.263} + \beta^{1.423} \right] B\sqrt{2g}\, h^{3/2} \tag{13}$$

Therefore, the discharge coefficient C_d can be expressed as follows:

$$C_d = 0.2878\,\beta^{0.577} \left[\left(\frac{h}{B} \right)^{0.263} + \beta^{1.423} \right] \tag{14}$$

Fig. 9 shows the comparison between the predicted values of C_d given by Eq. (14), represented in solid line curves, with Baiamonte and Ferro's observations (2007).

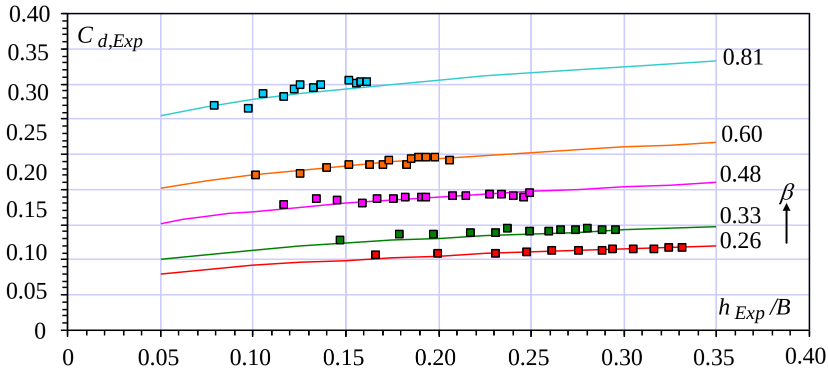

Figure 9: Variation in $C_{d,Exp}$ with h_{Exp}/B according to Baiamonte and Ferro (2007). Solid line curves: C_d values predicted according to Eq. (14)

As shown in Fig. 9, good agreement between the predicted values and the observations is noted. To better appreciate the accuracy of the model expressed by Eq. (14), the calculated deviations in the discharge coefficient C_d are reported in Table 4, while details can be seen in Fig. 10 involving the tested h_{Exp}/B and β experimental ranges.

Table 4: Deviation between Baiamonte and Ferro's C_d observations (2007) and predicted C_d values according to Eq. (14)

β	Deviation in C_d (%)		
	Minimum	**Maximum**	**Average**
0.17	0.012	2.378	1.000
0.26	0.020	3.188	1.983
0.33	0.669	4.917	1.978
0.48	0.041	5.183	1.799
0.60	0.054	2.908	1.289
0.81	0.566	4.364	1.890

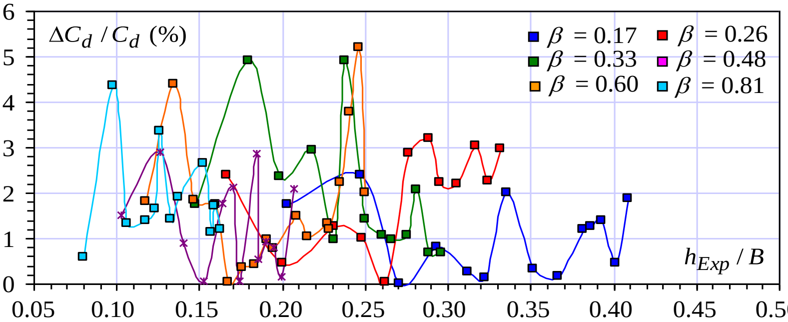

Figure 10: Deviations between predicted C_d values according to Eq. (14) and Baiamonte and Ferro's observations (2007)

As shown in Table 4, the maximum deviations in the calculation of the discharge coefficient C_d are quite acceptable, and it can thus be concluded that the model expressed by Eq. (14) is reliable. This is confirmed by Fig. 10, from which one may conclude that almost 100% of the deviations are below the acceptable value of 5%. It should be noted, however, that the model expressed by Eq. (14) is valid in the following ranges: $0.17 \le \beta \le 0.81$, $0.08 \le h_{Exp}/B \le 0.409$.

Other constants can be recommended to make the model expressed by Eq. (14) slightly more accurate since the maximum deviation is reduced to 4.357%. The proposed model expresses the discharge coefficient C_d as follows:

$$C_d = 0.267\,\beta^{0.5718}\left[\left(\frac{h}{B}\right)^{0.1937} + \beta^{1.435}\right] \tag{15}$$

The model expressed by Eq. (15) must be applied within the validity range of h_{Exp}/B tested, i.e., $0.08 \le h_{Exp}/B \le 0.409$. The deviations between Eq. (15) and the observations of Baiamonte and Ferro (2007) are summarized in Table 5, while their distribution is shown in detail in Fig. 11. As seen, 100% of the deviations are below 4.4%.

Table 5: Deviation between C_d computed using Baiamonte and Ferro's observations (2007) and predicted C_d values according to Eq. (15)

β	Deviation in C_d (%)		
	Minimum	Maximum	Average
0.17	0.114	4.258	1.627
0.26	0.656	3.332	2.102
0.33	0.103	4.349	1.504
0.48	0.090	4.366	1.382
0.60	0.059	3.480	1.547
0.81	0.584	4.357	2.712

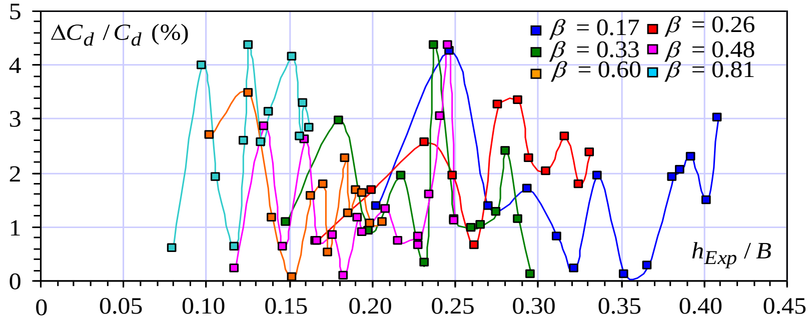

Figure 11: Deviations between predicted C_d values according to Eq. (15) and Baiamonte and Ferro's observations (2007)

In the recent past, Vatankhah and Mohammadi (2020) proposed new experimental data on SMBF flumes to extend the range of h_{Exp}/B provided by Baiamonte and Ferro (2007). The proposed range for h_{Exp}/B is such that $0.20 \le h_{Exp}/B \le 1.137$, collected on only four devices characterized by β = 0.30, 0.40, 0.60, and 0.884.

The experimental data provided allowed us to calculate the experimental discharge coefficient $C_{d,Exp}$, which is shown in Fig. 12, depending on h_{Exp}/B. Four corresponding curves are obtained, each of which corresponds to the tested value of the contraction rate β.

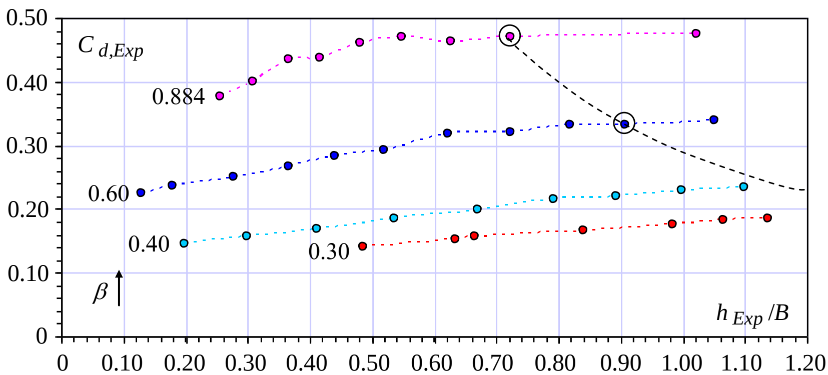

Figure 12: Variation in $C_{d,Exp}$ as a function of h_{Exp}/B according to Vatankhah and Mohammadi observations (2020). (- - -) Upper limit of the zone of influence of h/B

It can be observed that some points slightly deviate from their counterparts by distorting, somewhat, the trend of the curves, especially for the values 0.60 and 0.884 of β. These points are probably marred by some handling or measurement errors. However, the overall trend of the curves shows that the measurements are correct.

The meaningful observation that must be pointed out is that Fig. 12 seems to indicate that, for a given value of β, there is a limit $(h/B)_{lim}$ value beyond which the discharge coefficient remains constant regardless of h/B; this means that beyond this limit, the influence of h/B disappears, and only β is the influential parameter. In other words, inside the zone of no influence of h/B, the discharge coefficient is constant for a given device. The limit of influence of h/B on the discharge coefficient C_d is represented intuitively in Fig. 12 by the curve in the broken line, as well as the limit points surrounded by a circle corresponding to the values 0.60 and 0.884 of β.

It appears in Fig. 12 that the greater the value of the contraction rate β increases, the greater the limit value of influence of h/B decreases, meaning that the zone of no influence of h/B tends to extend toward the left of the graphics area of Fig. 12 when β increases.

Unfortunately, the available data, including the observations under consideration, are not sufficient to define the relationship that would help evaluate the influence limit value of h/B for a given contraction rate β. Further testing is obviously needed involving more β and h/B values.

The Ferro-type models, previously developed, cannot reveal the phenomenon observed in Fig. 12 given their reduced validity range of h/B; undoubtedly, the involved values of h/B are well below the influence limit, even for the large β value of 0.81 used during tests.

More generally, all available models describing the stage-discharge relationship, including those previously listed and developed to extract the corresponding discharge coefficient relationship, do not take into account the influence zone highlighted in Fig. 12.

The unsuspected behavior of the discharge coefficient as described could largely explain why the highest deviations caused by the available stage-discharge relationship models are observed for large β values, along with relative upstream depths h/B that have probably reached or even exceeded the influence limit value.

One may propose fairly accurate stage-discharge relationship models without taking into account the actual behavior of the discharge coefficient, but it is preferable to adapt them to reality for more accuracy and for a better mathematical representation and interpretation of the role of each influential parameter.

Somewhat far from the zone of no influence of h/B on the discharge coefficient C_d, the following model, derived from the observations of Vantankhah and Mohammadi (2020), could be recommended, paying attention to its applicability being restricted to the range $0.30 \leq \beta \leq 0.60$.

$$C_d = \left(\frac{0.446}{\frac{h}{B} - 0.034} \right)^{0.672} \left(\frac{h}{B} \right) \beta^{1.04} \tag{16}$$

The range of validity of Eq. (16) could be interesting insofar as the corresponding values of β are neither small nor too large. These β values would, in practice, generate quite high upstream flow depths, which allows a better accuracy of the gauge reading, especially since the measurement section involved is rectangular. In addition, such β values would cause a "tranquil" flow upstream of the device at the right of the measurement section, which would allow depth measurement in the best conditions. On the other hand, higher values of beta would inevitably lead to shallower depths, which could increase the inaccuracy of the gauge reading. The choice of lower contraction rates could lead to undesirable side effects, particularly the influence of surface tension.

The C_d model represented by Eq. (16) causes a maximum deviation of 4.872% in the resulting C_d predictive values compared to the recent observations of Vatankhah and Mohammadi (2020). The distribution of the resulting deviations, shown in Fig. 13 according to h_{Exp}/B and β experimental values, indicates that 100% of the deviations are below 5%, which is a relative error value generally accepted for such a device.

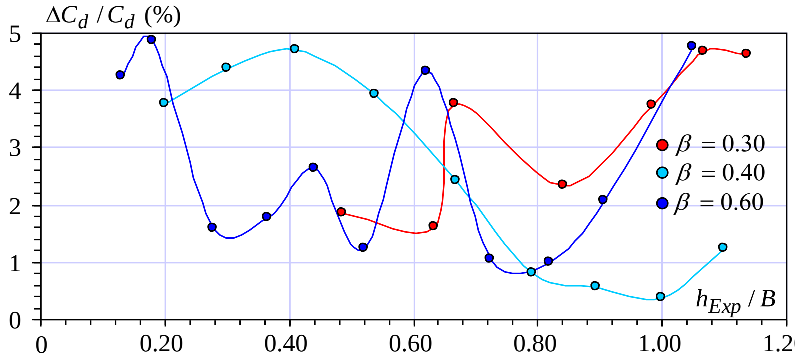

Figure 13: Deviations between predicted C_d values according to Eq. (16) and Vatankhah and Mohammadi observations (2020) for $0.30 \le \beta \le 0.60$

CONCLUSION

The discharge coefficient of a measuring device is a crucial parameter that not only allows us to identify the influential parameters but also to better understand and model their degree of influence. The discharge coefficient is a decisive correction factor of the theoretical discharge, called the ideal discharge, taking into account the simplifying assumptions that were the basis of the theoretical stage-discharge relationship. In regard to modeling a stage-discharge relationship based on the analysis of experimental data, it is worth considering the behavior of the discharge coefficient, as it could provide a guide for building a complete and reliable model. Unfortunately, this approach was not adopted during previous studies performed on the SMBF flume, so no relationship that governs the discharge coefficient of the device is available, even empirical, despite the availability of some significant experimental data.

As noted, previous studies have focused exclusively on modeling the stage-discharge relationship of the device with the help of available experimental data, without addressing the issue of its discharge coefficient. This approach has resulted in many empirical stage-discharge relationships, some of which are complex in form and not consistent with flow measurement devices in open channels. In addition, these formulas were conceived in such a way that no development or expansion is possible to allow the discharge coefficient relationship to be extracted. For all reasons, these formulas were not addressed in our study.

Fortunately, stage-discharge relationships that are much more handleable due to their simplicity, allowing transformation and development, are available in the literature, some of which are quite recent. The judicious exploitation of these formulas, whose degree of accuracy is varied, allows us to extract the corresponding governing discharge coefficient relationship. These relationships revealed two main influential parameters, namely, the contraction rate β of the device and the relative upstream flow depth h/B. In addition, the graphical representation of the discharge coefficient highlighted the influence of each of the abovementioned parameters.

Moreover, the graphical analysis of recent observations available in the literature has highlighted the limit beyond which the influence of h/B on the discharge coefficient C_d disappears. Inside the no influence zone, the discharge coefficient solely depends on the value of the contraction rate β, meaning that it is a constant for a given device.

The discharge coefficient relationships developed herein were discussed, and their range of validity was presented. To provide more accurate relationships, considering the effects of the zone of no influence of h/B on the discharge coefficient C_d, the authors recommend carrying out additional tests involving more contraction rates β and a wider range of h/B.

Although the discharge coefficient C_d can be analytically expressed by considering the theoretical approach already applied by the authors on certain devices, currently, there is no theory capable of predicting a mathematical model that can faithfully translate the influence of the relative upstream flow depth h/B on the discharge coefficient C_d. It is in this direction that future research should be directed without disregarding the zone of no influence of h/B.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

REFERENCES

ACHOUR B., AMARA L. (2022). Accurate discharge coefficient relationship for the Crump weir, Larhyss Journal, No 52, pp. 93-115.

ACHOUR B., AMARA L. (2022b). Theoretical and experimental investigation of a lateral broad-crested contraction as a flow measurement device, Flow Measurement and Instrumentation, Vol. 86, Paper 102175.

ACHOUR B., BOUZIANE M.T., NEBBAR K. (2003). Broad-crested triangular flowmeter in rectangular channel, Larhyss Journal, No 2, pp. 7-43. (In French)

BAIAMONTE G., FERRO V. (2007). Simple flume for flow measurement in sloping open channel, Journal of Irrigation and Drainage Engineering, Vol. 131, Issue 1.

BOS M.G. (1976). Discharge Measurement Structures, Laboratorium Voor Hydraulica AanAfvoerhydrologie, Landbouwhogeschool, Wageningen, the Netherlands, Report 4, May.

BOS M.G. (1989). Discharge Measurement Structures, third ed., Publication 20, Int. Institute for Land Reclamation and Improvement, Wageningen, Netherlands.

BURCHARTH H.F., HAWKINS S.J., ZANUTTIGH B., LAMBERTI A. (2007). Environmental Design Guidelines for Low Crested Coastal Structures, Chapter 13-Design tools related to engineering, Elsevier, pp. 203-333.

CAROLLO F.G., DI STEFANO C., FERRO V., PAMPALONE V. (2016). New stage–discharge relationship for the SMBF flume, Journal of Irrigation and Drainage Engineering, Vol. 142, Issue 5, Paper 04016005.

CAROLLO F.G., PAMPALONE V. (2021). Testing the Stage-Discharge Relationship in Sloping SMBF Flumes, Journal of Irrigation and Drainage Engineering, Vol. 147, Issue 5.

CONE V.M. (1917). The Venturi Flume, Journal of Agricultural Research, Washington D.C., Vol. IX, Issue 4, pp. 115-130.

DI STEFANO C., DI PIAZZA G.V., FERRO V. (2008). Field testing of a simple flume (SMBF) for flow measurement in open channels, Journal of Irrigation and Drainage Engineering, Vol. 134, Issue 2, pp. 235–240.

FERRO V. (2002). Discussion of 'Simple flume for flow measurement in open channel' by Zohrab Samani and Henry Magallanez, Journal of Irrigation and Drainage Engineering; Vol. 128, Issue 2, pp. 129–131.

HAGER W.H. (1986). Discharge Measurement Structures, Communication 1, Department of Civil Engineering, Federal Polytechnic School of Lausanne, Switzerland.

HENDERSON F.M. (1966). Open Channel Flow, the McMillan Company, New York, N.Y, USA.

KULKARNI K.H., HINGE G.A. (2021). Performance enhancement in discharge measurement by compound broad crested weir with additive manufacturing, Larhyss Journal, No 48, pp. 169-188.

PARSHALL R.L. (1926). The improved Venturi flume, Transactions ASCE, Vol. 89, pp. 841–880

SAMANI Z., MAGALLANEZ H. (2000). Simple flume for flow measurement in open channel, Journal of Irrigation and Drainage Engineering, Vol. 126, Issue 2, pp. 127-129.

TADDA M.A., AMIMUL I.A., MONZUR A., SHITU A., DANHASSAN U.A., ALIYU I.M. (2020). Operation and maintenance of hydraulic structures, Part of the edited volume Hydraulic Structures, Eds Amimul Ahsan, Swinburne University of Technology, 80 p.

VATANKHAH A.R. (2017). Discussion of "new stage–discharge equation for the SMBF flume, by Francesco Giuseppe Carollo, Costanza Di Stefano, Vito Ferro, and Vincenzo Pampalone, Journal of Irrigation and Drainage Engineering, Vol. 143, Issue 8, Paper 07017011.

VATANKHAH A.R., MOHAMMADI M. (2020). Stage–discharge equation for simple flumes with semi-cylinder contractions, SN Applied Sciences, No 2, Article number 510.

WESTESEN G.L. (1992). Montana (Short Parshall) Flume (Part 1), Water meas. MT 9121 (AG).