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DISCHARGE COEFFICIENT OF A PARABOLIC WEIR THEORY AND EXPERIMENTAL ANALYSIS

COEFFICIENT DE DEBIT D'UN DEVERSOIR PARABOLIQUE THEORIE ET ANALYSE EXPERIMENTALE

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ABSTRACT

The study is devoted to the parabolic-shaped thin-plate weir. It is the flow coefficient which is of particular interest to the study. This is planned to be determined from a theoretical point of view using the energy equation. This is applied between two well-chosen sections, the first of which is located in the approach channel upstream of the weir and the second is considered at the location of the weir. The state of the flow over the weir is subject to the realistic assumption that it is critical. Other simplifying assumptions are adopted such as the head loss which is neglected between the chosen sections, or that the pressure is considered to be hydrostatic. The effect of the curvature of the flow streamlines above the weir is also neglected. The strength of the theory lies in the fact that the approach flow velocity is taken into consideration.

Taking into account all the simplifying assumptions made during the study, it is obvious that the theoretical discharge coefficient must be different from the experimental discharge coefficient. In order to correct the deviation observed between these two discharge coefficients, experimental data available in the literature are used. The corrected theoretical discharge coefficient relationship was established on the basis of 157 significant measuring points, giving rise to an average error of less than 1.3% on the discharge coefficient calculation, while the maximum deviation is of the order of 6%. It is worth noting that 92.85% of the calculated values of the deviation are below 3%, meaning that 7.15% only are greater than 3%, while 82.5% are less than 2%.

Keywords: Parabolic weir, discharge coefficient, approach channel, approach velocity.

RESUME

L'étude est consacrée au déversoir à paroi mince de forme parabolique. C'est le coefficient de débit qui présente un intérêt particulier pour l'étude. Il est prévu que celui-ci soit déterminé d'un point de vue théorique en utilisant l'équation de l'énergie. Celle-ci est appliquée entre deux sections bien choisies, dont la première est située dans le canal d'approche en amont du déversoir et la seconde est considérée à l'emplacement du déversoir. L'état de l'écoulement au-dessus du déversoir est soumis à l'hypothèse réaliste qu'il est critique. D'autres hypothèses simplificatrices sont émises comme la perte de charge qui est négligée entre les sections choisies, ou que la pression est considérée comme étant hydrostatique. L'effet de la courbure des lignes de courant au-dessus du déversoir est également négligé. La force de la théorie réside dans le fait que la vitesse d'approche de l'écoulement est prise en considération.

Compte tenu de toutes les hypothèses simplificatrices faites lors de l'étude, il est évident que le coefficient de débit théorique doit être différent du coefficient de débit expérimental. Afin de corriger l'écart observé entre ces deux coefficients de débit, des tests expérimentaux disponibles dans la littérature sont utilisés. La relation corrigée du coefficient de débit théorique a été établie sur la base de 157 points de mesure significatifs, donnant lieu à une erreur moyenne inférieure à 1,3% sur le calcul du coefficient de débit, alors que l'écart maximal est de l'ordre de 6%. Il est à noter que 92,85% des valeurs calculées de l'écart sont inférieures à 3%, ce qui signifie que 7,15% seulement sont supérieurs à 3%, tandis que 82,5% sont inférieurs à 2%.

Mots clés : Déversoir parabolique, coefficient de débit, canal d'approche, vitesse d'approche.

INTRODUCTION

A weir is a device used to measure the volumetric rate of water flow that passes through an approach channel with a given cross section shape. The flow rate can be determined by measuring the height of the upstream water level. Weirs can be classified according to the shape of their notch such as rectangular, triangular, circular, trapezoidal, and parabolic (Ackers et al., 1978; Bos, 1976). It is this last form that interests the present study. As with the whole range of weirs, the parabolic weir is designed as a device for measuring flow in open channels.

The parabolic weir considered herein is of degree two as the exponent of the flow depth. Because of the exponent two, the manual calculation of the discharge is thus very easy to perform. Parabolic weirs encompass the advantages of rectangular weirs as well as those of triangular weirs. The discharge capacity of parabolic weirs is 33.3% greater than that of triangular weirs, for similar flow velocity and weir dimensions (Badhe et al., 2015). Since the parabolic shape is closer to a triangular shape than a rectangular shape, parabolic weirs should be characterized by the precision and sensitivity of standard triangular weirs. Due to the fact that the parabolic shape lies between the triangular and rectangular shapes, the parabolic weir can be expected to exhibit the advantages of the latter two combined shapes. Villemonte (1947) has shown that the parabolic is a more precise flow measuring device than many other types of weirs such as proportional and rectangular weirs.

The objective of this study is to examine, from a theoretical and experimental point of view, the discharge coefficient of a parabolic weir. The theoretical development is based on the energy equation applied between two well chosen sections of the flow occurring in a rectangular supply channel. The first section is located upstream of the weir, while the second section is taken at the location of the weir assumed to be crossed by a critical flow. Indeed, installing a weir in an open channel causes critical depth to form over the weir. However, some assumptions are made with regard to head loss that is neglected and pressure distribution of the flow passing over the weir assumed to be hydrostatic. A final hypothesis consists in not considering the effect of the flow streamlines curvature over the weir. For this reason, the experimental flow rate is not equal to the theoretical one and a discharge coefficient, denoted C_d , representing the ratio of the two flow rates must be determined. The discharge coefficient C_d needs to be determined experimentally for each weir to account for errors in estimating the flow rate that is due to these assumptions.

In order to validate or correct the relationships resulting from the theoretical development, the study is confronted with the experimental tests available in the literature, in particular those carried out recently by Vatankhah and khamisabadi (2019).

DESCRIPTION OF THE DEVICE

Fig. 1 describes the studied device representing a contracted parabolic weir. The central opening is of a maximum top width T_m and of a height y_m, while the approach channel is of width B. The ratio $\beta = T_m / B$ is the lateral contraction rate and the ratio $\tau = h_1 / y_m$ is the filling rate.

The weir is also characterized by a crest height P above which the upstream depth h_1 is measured. The discharge flowing through the channel is noted Q. Fig. 2 is a plan view showing both the rectangular approach channel and the weir. Section 1-1 is located upstream of the weir and in which the depth h_1 is measured.

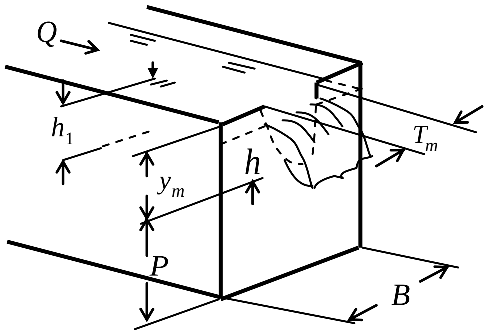

Figure 1: Definition sketch of the studied contracted weir

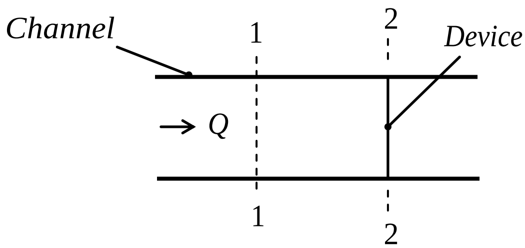

Figure 2: Plan view of the channel and the device

THEORETICAL DISCHARGE COEFFICIENT RELATIONSHIP

The critical depth in the rectangular cross-section 1-1 (Fig. 2) is written as:

$$h_{1c} = \left(\frac{Q^2}{g B^2} \right)^{1/3} \tag{1}$$

where the subscript « c » denotes the critical conditions.

On the other hand, the critical depth in the parabolic cross-section 2-2 (Fig. 2) is as (Achour and khattaoui, 2008):

$$h_{2c} = \left(\frac{27 Q^2}{8 g \sigma} \right)^{1/4} \tag{2}$$

where σ has a dimension of length since it is defined as $\sigma = T_m^2 / y_m$ (Fig. 1).

Eliminating the discharge Q betweens Eqs. (1) and (2) yields:

$$h_{2c} = \left(\frac{27B^2}{8\sigma}\right)^{1/4} h_{1c}^{3/4} \tag{3}$$

Assume that there is no head loss between sections 1-1 and 2-2. Equal total heads between sections 1-1 and 2-2 translates into:

$$H_1 = H_2 = \frac{4}{3} h_{2c} \tag{4}$$

Combining Eqs. (3) and (4) results in:

$$H_1 = \frac{4}{3} h_{1c}^{3/4} \left(\frac{27B^2}{8\sigma}\right)^{1/4} \tag{5}$$

Hence:

$$\frac{H_1}{h_{1c}} = \frac{4}{3h_{1c}^{1/4}} \left(\frac{27B^2}{8\sigma}\right)^{1/4} \tag{6}$$

Taking into account the effect of the approach velocity, the total head H_1 in section 1-1 can be written as:

$$H_1 = h_1 + \frac{Q^2}{2gB^2\left(h_1 + P\right)^2} \tag{7}$$

Implying that:

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{Q^2}{2gB^2\left(h_1 + P\right)^2 h_{1c}} \tag{8}$$

Eq. (8) can be written as:

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{Q^2}{2gB^2 h_1^2\left(1 + P / h_1\right)^2 h_{1c}} \tag{8a}$$

Eq. (1) allows writing that:

$$\frac{Q^2}{gB^2} = h_{1c}^3 \tag{9}$$

Combining Eqs. (8a) and (9) yields :

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{1}{2(h_1/h_{1c})^2(1+P/h_1)^2} \tag{10}$$

Eqs. (6) and (10) give what follows:

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{1}{2(h_1/h_{1c})^2(1+P/h_1)^2} = \frac{4}{3h_{1c}^{1/4}}\left(\frac{27B^2}{8\sigma}\right)^{1/4} \tag{11}$$

Let us adopt the following non-dimensional parameter:

$$h_1/h_{1c} = h_1^* \tag{12}$$

Inserting Eq. (12) into Eq. (11) results in:

$$h_1^* + \frac{1}{2h_1^{*2}(1+P/h_1)^2} = \frac{4}{3h_{1c}^{1/4}}\left(\frac{27B^2}{8\sigma}\right)^{1/4} \tag{13}$$

Eq. (13) can be rewritten as:

$$h_1^* + \frac{1}{2h_1^{*2}(1+P/h_1)^2} = \frac{4}{3}\times\left(\frac{27}{8}\right)^{1/4}\left(\frac{B^2}{\sigma h_1}\right)^{1/4} h_1^{*1/4} \tag{14}$$

Taking into account the definition of σ, Eq. (14) is reduced to:

$$h_1^* + \frac{1}{2h_1^{*2}(1+P/h_1)^2} = \frac{4}{3}\times\left(\frac{27}{8}\right)^{1/4}\frac{1}{\psi^{1/2}} h_1^{*1/4} \tag{15}$$

where ψ is expressed as:

$$\psi = \frac{T_m}{B}\sqrt{\frac{h_1}{y_m}} \tag{16}$$

Note that the ratio T_m/B represents a lateral contraction coefficient while h_1/y_m can be defined as the filling rate. After some arrangements, Eq. (15) can be written as:

$$h_1^{*3} - \left(\frac{32}{3}\right)^{1/4}\frac{1}{\psi^{1/2}} h_1^{*9/4} + \frac{1}{2(1+P/h_1)^2} = 0 \tag{17}$$

It should be noted that the flow in the section 1-1 is subcritical, meaning that $h_1 > h_{1c}$ or $h_1^* > 1$.

Eq. (1) allows writing that:

$$Q = \sqrt{g} B h_{1c}^{3/2} \tag{18}$$

Taking into account Eq. (12), Eq. (18) becomes:

$$Q = \sqrt{g} B \frac{h_1^{3/2}}{h_1^{*3/2}} \tag{19}$$

Eq. (19) can be rewritten as:

$$Q = \mu \sqrt{2g} B h_1^{3/2} \tag{20}$$

Eq. (20) is the theoretical depth-discharge relationship for the studied device, where μ is the discharge coefficient expressed as:

$$\mu = \frac{1}{\sqrt{2}\, h_1^{*3/2}} \tag{21}$$

The upstream depth h_1 of the flow is measured by a simple point gauge reading in a section located upstream of the weir.

On the other hand, the discharge Q for the parabolic weir is expressed as:

$$Q = C_d \frac{\pi}{8} \frac{T_m}{\sqrt{y_m}} \sqrt{2gh_1^4} \tag{22}$$

Eq. (19) can be rewritten as:

$$Q = \frac{1}{\sqrt{2}} \frac{B}{h_1^{*3/2} h_1^{1/2}} \sqrt{2gh_1^4} \tag{23}$$

Eliminating the discharge Q between Eqs. (22) and (23) results in:

$$C_d \frac{\pi}{8} \frac{T_m}{\sqrt{y_m}} \sqrt{2gh_1^4} = \frac{1}{\sqrt{2}} \frac{B}{h_1^{*3/2} h_1^{1/2}} \sqrt{2gh_1^4} \tag{24}$$

Hence:

$$C_{d,Th} = \frac{8/(\pi\sqrt{2})}{h_{1,Th}^{*3/2}\frac{T_m}{B}\sqrt{\frac{h_1}{y_m}}} \tag{25}$$

That is:

$$C_{d,Th} = \frac{8/(\pi\sqrt{2})}{\psi h_{1,Th}^{*3/2}} \tag{26}$$

The subscript "*Th*" denotes "Theoretical". Eq. (26) expresses the theoretical discharge coefficient of the parabolic weir as a function of P/h_1, T_m/B, and h_1/y_m. The last two parameters are involved in the variable ψ according to Eq. (16). The dimensionless parameter h_1^* is given by the fundamental theoretical relationship (17).

On the other hand, Eq. (22) gives:

$$C_{d,Exp} = \frac{8}{\pi}\left(\sqrt{y_m}/T_m\right)\frac{Q}{\sqrt{2gh_1^4}} \tag{22a}$$

The subscript "*Exp*" denotes "Experimental".

For the rest of the study, it is important to note that $C_{d,Exp}$ can be also determined by Eq. (26) by substituting $h_{1,Th}^*$ by $h_{1,Exp}^*$. Whence:

$$C_{d,Exp} = \frac{8/(\pi\sqrt{2})}{\psi h_{1,Exp}^{*3/2}} \tag{26a}$$

EXPERIMENTAL ANALYSIS

The experimental data were taken from the literature (Vatankhah and khamisabadi, 2019). The theoretical parameters indicated by the subscript "*Th*" were calculated according to the appropriate relationships described above. During the tests, 161 measurement points of flow rate Q and depth h_1 were considered in a rectangular channel of width B = 0.25 m. The depth h_1 (cm) has been varied in the range [3.2; 19.7] while the discharge Q (l/s) was in the range [0.38; 14.49]. Three height values of y_m of the considered parabolic weirs were taken, namely 0.10 m, 0.15 m and 0.20 m. The maximum top width T_m was taken constant such that T_m=0.15*m* implying T_m/B =0.60. Eight weir crest heights P were considered corresponding to P (cm): 5.4; 5.6; 5.8; 10.2; 10.3; 10.4; 15.3; and 15.4.

Consequently, the relative weir crest height P/h_1 has been varied in the range [0.313; 4.581], while the ratio h_1/y_m was in the range [0.16; 1].

Knowing experimentally Q, B and h_1, Eq. (1) gives $h_{1,c}$ and therefore the dimensionless parameter $h^*_{1,Exp}$ is easily worked out since $h^*_{1,Exp} = h_1 / h_{1,c}$. The dimensionless theoretical parameter $h^*_{1,Th}$ was calculated by applying the fundamental equation (17) using the "TI-84 Plus" handheld calculator solver, knowing both ψ and P / h_1 values. One can also use the MS Excel solver. The experimental measurements allowed calculating ψ values between 0.240 and 0.6045 according to Eq. (16).

The processing of the experimental data given by the study of Vatankhah and khamisabadi (2019) allowed graphically representing the variation between the dimensionless parameters $h^*_{1,Exp}$ and $h^*_{1,Th}$ as shown in the Fig. 3.

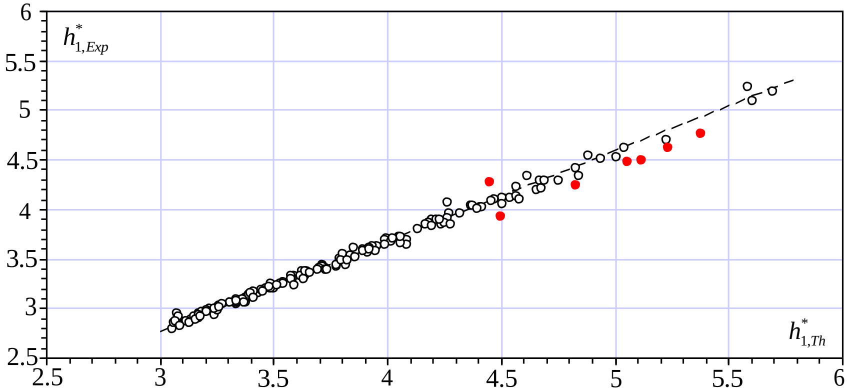

Figure 3: Variation of h^*_1 experimental versus h^*_1 theoretical

Due to the simplifying assumptions adopted during the study, $h^*_{1,Th}$ is different from $h^*_{1,Exp}$. One can observe in Fig. 3 that $h^*_{1,Th}$ is greater than $h^*_{1,Exp}$ meaning that $C_{d,Th}$ is less than $C_{d,Exp}$ according to both Eqs. (26) and (26a). Fig. 3 also highlights an almost linear trend for the variation of $h^*_{1,Exp}$ versus $h^*_{1,Th}$ represented by the broken line, with the exception of seven measurement points which appear to deviate from this trend. These points are represented in red color and appear to be marred by a measurement error. Therefore, they are excluded from the calculation. In total, a sample of (161-7) = 154 experimental points seem to be significant and which will serve as the basis for the analysis. The calculations showed that the linear trend line is governed by the following equation:

$$h^*_{1,Exp} = 0.915 h^*_{1,Th} \tag{27}$$

with a coefficient of determination $R^2 = 0.9936$.

Inserting Eq. (27) into Eq. (26a) results in:

$$C_{d,Exp} = \frac{2.05728}{\psi \, h^{*3/2}_{1,Th}} \tag{28}$$

It is useful to remember that $C_{d,Exp}$ corresponds to the actual or observed discharge coefficient. Eq. (28), along with Eq. (17), allows computing the discharge coefficient for a parabolic weir provided ψ and P/h_1 are given. The relative deviation between Eq. (22a) and (28) varies in the range [0.00045 %; 6 %] with an average relative error less than 1.3%. It is worth noting that among the 154 calculated values of the relative deviation on the computation of the discharge coefficient, 92.85% are less than 3% meaning that 7.15% only are greater than 3%, while 82.5% are less than 2%.

EXAMPLE

One of the tests carried out by Vatankhah and khamisabadi (2019) on a parabolic weir installed in a rectangular approach channel of width *B* is characterized by:

$Q = 14.49\, l/s$ measured using a flowmeter; $h_1 = 19.7\, cm$; $T_m = 15\, cm$; $y_m = 20\, cm$; $P = 15.4\, cm$; $B = 25\, cm$.

Check the flow rate *Q* measured experimentally with the flow rate calculated by applying the relationships developed previously.

SOLUTION

The given data allow computing the following parameters:

1. ψ according to Eq. (16) as:

$$\psi = \frac{T_m}{B}\sqrt{\frac{h_1}{y_m}} = \frac{15}{25} \times \sqrt{\frac{19.7}{20}} = 0.595483$$

2. The ratio P/h_1 such that:

$$P/h_1 = 15.4/19.7 = 0.78172589$$

3. Inserting the previous results into the fundamental relationship (17) gives:

$$h_1^{*3} - 2.34192239 h_1^{*9/4} + 0.15750278 = 0$$

The solution of this equation having a physical meaning, i.e. $h_1^* > 1$, is:

$$h_1^* = 3.08797836$$

This was obtained using the "TI-84 Plus" handheld calculator solver.

4. According to Eq. (27), the discharge coefficient is such as:

$$C_{d,Exp} = \frac{2.05728}{0.595483 \times 3.08797836^{3/2}} = 0.63666758$$

5. The discharge Q is given by Eq. (22) as:

$$Q = C_d \frac{\pi}{8} \frac{T_m}{\sqrt{y_m}} \sqrt{2gh_1^4} = 0.63666758 \times \frac{\pi}{8} \times \frac{0.15}{\sqrt{0.20}} \times \sqrt{2 \times 9.81 \times 0.197^4}$$

That is:

$$Q = 0.01441554\, m^3/s \approx 14.415\, l/s$$

One can therefore conclude that the flow rate thus calculated corresponds almost to the measured flow rate given in the problem statement as $Q = 14.49\, l/s$. The deviation between the two flow rates is 0.516%.

CONCLUSIONS

The parabolic weir was the subject of a theoretical investigation in order to determine the discharge coefficient relationship. For this, the energy equation was applied between two carefully chosen sections. The first section is located in the approach channel, upstream of the weir, while the second section is located at the weir whose crest is supposed to be crossed under critical flow conditions. By considering some simplifying assumptions, an implicit polynomial equation $\wp(h_1^*) = 0$ was established [Eq. (17)] where h_1^* is a dimensionless parameter closely related to the discharge coefficient of the studied weir [Eq. (26)]. Due to the simplifying assumptions, the theoretical discharge coefficient is different from the experimental discharge coefficient. This was observed by analyzing the experimental data available in the literature. Based on these data comprising 157 significant measurement points, the theoretical equation of the discharge coefficient has been corrected to be in conformity with the experimental values [Eq. (28)]. Calculations have shown that this equation causes an average relative error of less than 1.3% on the calculation of the discharge coefficient with a maximum deviation of 6%. A maximum error of 6% is not excessive in the field of discharge flow measurement using weirs. A statistical examination carried out on the 157 values of the deviation on the calculation of the discharge coefficient showed that almost 93% of the values were located below 3%, meaning that only 7% are greater than 3%. It was also observed that 82.5% of the values were below 2% deviation.

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