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TECHNICAL NOTE

THEORETICAL APPROACH TO STAGE-DISCHARGE RELATIONSHIP FOR A CIRCULAR SHARP-CRESTED WEIR

APPROCHE THEORIQUE DE LA RELATION HAUTEUR-DEBIT POUR UN DEVERSOIR A ECHANCRURE CIRCULAIRE A CRETE MINCE

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ABSTRACT

A novel theoretical approach for the flow discharge relationship in a circular sharp crested weir is developed in the present paper. Based on the application of the energy equation applied between a section taken in the approach channel and the weir under critical flow conditions, a general stage-discharge relationship is obtained taking into account width contraction effect and kinetic energy in the approaching channel. A correction function is introduced considering the simplifying assumptions assumed in the theory which neglected both the effects of viscosity, the curvature of the streamlines due to the contraction of the weir and surface tension.

Derivation of the discharge coefficient expression for classical stage-discharge equation is also given. In order to check the proposed model, verification with laboratory measurement data available in literature are made. Results show a very good agreement between predicted and actual discharge.

Keywords: Circular weir, Stage-discharge relationship, Discharge coefficient, Approach velocity.

RESUME

Une nouvelle approche théorique pour la relation de débit d'écoulement dans un déversoir circulaire à crête mince est développée dans le présent article. Sur la base de l'application de l'équation de l’énergie appliquée entre une section prise dans le canal d'approche et le déversoir sous des conditions d'écoulement critiques, une relation générale hauteur-débit est obtenue en tenant compte de l'effet de contraction de la largeur et de l'énergie cinétique dans le canal d'approche. Une fonction de correction est introduite qui tient compte des hypothèses simplificatrices émises dans la théorie qui a négligé à la fois les effets de la viscosité, de la courbure des lignes de courant due à la contraction du déversoir et enfin de la tension superficielle. La dérivation de l'expression du coefficient de débit pour l'équation classique du débit est également donnée. Afin de vérifier le modèle proposé, une vérification avec les données de mesures de laboratoire disponibles dans la littérature est effectuée. Les résultats montrent une très bonne concordance entre les débits calculés et les débits réels.

Mots-clés : Déversoir circulaire, Relation hauteur-débit, Coefficient de débit, Vitesse d’approche.

INTRODUCTION

One of the most important issues in open channel problems is flow measurement. To do so, a very common method of open-channel flow involves the use of a hydraulic structure such as a weir or flume (Henderson, 1966; French, 1985). The discharge of a weir is given by an equation called the stage-discharge relationship, that is to say, a relation that links the depth of the flow and the discharge as well as the geometric characteristics of the device. The depth of the water is measured at a specific location upstream of the weir. The stage discharge relationship can be determined theoretically or experimentally. The theoretical relationship is generally deduced using the weir theory along with Buckingham’s theorem of dimensional analysis (Vatankhah and Khamisabadi, 2019). The relationship must then be calibrated by experimental tests. The two relationships are unlikely to give the same result because the theoretical relationship does not take into account a number of influencing factors and is subject to simplifying assumptions. As a general rule, it is accepted that the head loss is neglected, the pressure distribution is considered to be hydrostatic, the effect of viscosity and surface tension is neglected, and that the effect of the flow streamlines curvature above the device are also overlooked. The ratio of the actual discharge, or the observed discharge, to the theoretical discharge is called the discharge coefficient also known as coefficient of discharge or efflux coefficient. A fact in practice is that the discharge coefficient, denoted C_d , cannot be greater than one.

The most widespread and commonly used weirs are the sharp-crested weirs with a rectangular, triangular, trapezoidal, parabolic or even circular notch which is of interest to this study. For the flow measurement of closed reservoirs, circular weirs or circular orifices inserted in one of the walls of the reservoir are used (Swamee and Swamee, 2010). For closed circular pipes, the semicircular weir is sometimes used for flow measurement when placed at the end of the pipe. Weirs with semicircular openings can be also used as flow-measuring devices in open channels. Experimentation has shown that the discharge coefficient of circular weirs is greater than that of the above mentioned opening shapes (Greve, 1932; Balachandar et al., 1991; Chunrong et al.; 2002).

This study focuses on the sharp-crested circular weir in order to define a novel approach to the theoretical stage-discharge relationship. To do this, the energy equation is applied between two well-chosen sections. The first section is located in the approach channel upstream of the weir, while the second is located at the location of the weir which is assumed to be crossed in a critical flow regime.

Under realistic simplifying assumptions, the theoretical stage-discharge relationship is obtained encompassing all the influencing parameters. This relationship is contrasted to experimental data available in the literature and interesting conclusions are then drawn.

THEORETICAL STAGE-DISCHARGE RELATIONSHIP

Let consider a free surface flow issuing from a sharp-crested circular weir of diameter D under a flow depth h_1 measured far from the weir crest where streamline curvature effects are negligible (Fig.1). The weir crest height is denoted P and the width of the rectangular approach channel is B. For the generalization of the theoretical approach, no restriction on the ratio D/B is considered which is less than or equal to unity. Similarly, the ratio P/D is considered free of limiting values.

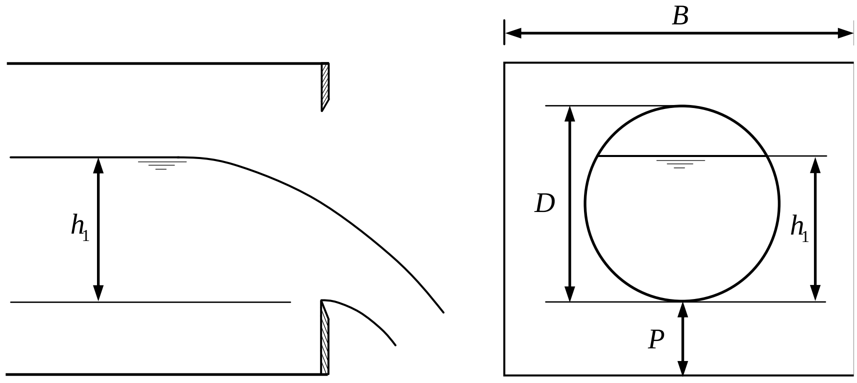

Figure 1: Definition sketch of the flow over a circular sharp-crested weir

Neglecting viscous, superficial tension and streamline curvature effects, one can establish a stage-discharge relationship of the considered weir from the critical flow theory together with the energy equation. From classical consideration, the critical depth in the rectangular cross-section (Fig. 1) is expressed as follows:

$$h_{1c} = \left(\frac{Q^2}{gB^2} \right)^{1/3} \tag{1}$$

where the subscript « c » denotes the critical conditions.

On the other hand, the critical depth in the circular cross-section (Fig. 1) can be approximated by the following simple expression (Hager, 2010):

$$h_{2c} = \left(\frac{Q}{\sqrt{gD}} \right)^{1/2} \tag{2}$$

where D is the circular weir diameter. Deviations of h_{2c} in Eq. (2) from its exact values are less than 4% in the following wide range $0.2 < h_{2c}/D < 0.91$.

Eliminating the discharge Q between Eq. (1) and Eq. (2) results in:

$$h_{2c} = \left(\frac{B}{\sqrt{D}} \right)^{1/2} h_{1c}^{3/4} \tag{3}$$

On the other hand the relation between the critical flow depth h_{2c} and the critical energy head H_2 at the weir location is (Hager, 2010):

$$H_2 = \left(\frac{3}{2} \right)^{6/5} \frac{h_{2c}^{6/5}}{D^{1/5}} \tag{4}$$

Assume that there is no head loss between section 1-1 taken in the approach channel where h_1 is measured (Fig. 1) and section 2-2 at the location of the weir. Equating total heads between sections 1-1 and 2-2 and considering Eq. (4) results in:

$$H_1 = H_2 = \left(\frac{3}{2} \right)^{6/5} \frac{h_{2c}^{6/5}}{D^{1/5}} \tag{5}$$

Combining Eqs. (3) and (5) yields:

$$H_1 = \left(\frac{3}{2} \right)^{6/5} \frac{B^{3/5}}{D^{1/2}} h_{1c}^{9/10} \tag{6}$$

The total head H_1 can be written as:

$$H_1 = h_1 + \frac{Q^2}{2gB^2 (h_1 + P)^2} \tag{7}$$

where h_1 is the upstream depth above the weir crest and P is the weir crest height (Fig. 2).

Eq. (7) can be rewritten as:

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{Q^2}{2gB^2 \left(h_1 + P\right)^2 h_{1c}} \tag{8}$$

Extracting h_1 from the quantity in parentheses, Eq. (8) can be then rewritten as:

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{Q^2}{2gB^2 h_1^2 \left(1 + P / h_1\right)^2 h_{1c}} \tag{9}$$

Eq. (1) allows writing that:

$$\frac{Q^2}{gB^2} = h_{1c}^3 \tag{10}$$

Combining Eqs. (9) and (10) yields:

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{1}{2(h_1 / h_{1c})^2 (1 + P / h_1)^2} \tag{11}$$

Eqs. (13) and (16) give what follows:

$$\frac{h_1}{h_{1c}} + \frac{1}{2\left(h_1 / h_{1c}\right)^2 \left(1 + P / h_1\right)^2} = \left(\frac{3}{2}\right)^{6/5} \frac{B^{3/5}}{D^{1/2} h_{1c}^{1/10}} \tag{12}$$

Introducing h_1 in the right-hand side of Eq. (12) results in:

$$\frac{h_1}{h_{1c}} + \frac{1}{2\left(h_1 / h_{1c}\right)^2 \left(1 + P / h_1\right)^2} = \left(\frac{3}{2}\right)^{6/5} \left(\frac{B^{3/5}}{D^{1/2} h_1^{1/10}}\right) \left(\frac{h_1}{h_{1c}}\right)^{1/10} \tag{13}$$

Considering the following non-dimensional parameters:

$$h_1^* = h_1 / h_{1c} \tag{14}$$

$$\psi = \frac{D}{B}\left(\frac{h_1}{D}\right)^{1/6} \tag{15}$$

$$P^* = P / h_1 \tag{16}$$

Equation (13) reads then as follows:

$$h_1^* + \frac{1}{2h_1^{*2}\left(1+P^*\right)^2} = \left(\frac{3}{2}\right)^{6/5} \psi^{-3/5} h_1^{*1/10} \tag{17}$$

After some rearrangements, Eq. (17) reduces to:

$$h_1^{*3} - \left(\frac{3}{2}\right)^{6/5} \psi^{-3/5} h_1^{*2.1} + \frac{1}{2\left(1+P^*\right)^2} = 0 \tag{18}$$

The relative weir crest height P^* expressed by Eq. (16) can be written as $P^* = \eta^{-1} P / D$ where $\eta = h_1 / D$ is the filling ratio. Introducing η in Eq. (15), the dimensionless parameter ψ can be written as $\psi = \eta^{1/6} D / B$. In practice, the known parameters are ψ and P^* which will be used to deduce h_1^* by solving the implicit Eq. (18). This can be achieved easily using any software package or a pocket calculator. Note that the flow in the section 1-1 is subcritical, meaning that $h_1 > h_{1c}$ i.e. $h_1^* > 1$. During one of their recent studies, the authors demonstrated that the dimensionless parameter h_1^* is closely related to the Froude number F_1 in the section of the approach channel where the depth h_1 is measured (Achour and Amara, 2021).

Eq. (18) is shown graphically in Fig.2 for a value of *D/P*=2 and different ratios *D/B*. It can be seen that for a given value of *D/B*, h_1^* decreases with the increase of h_1/D. On the other hand, solutions of h_1^* show similar trend for different values of *D/B***.**

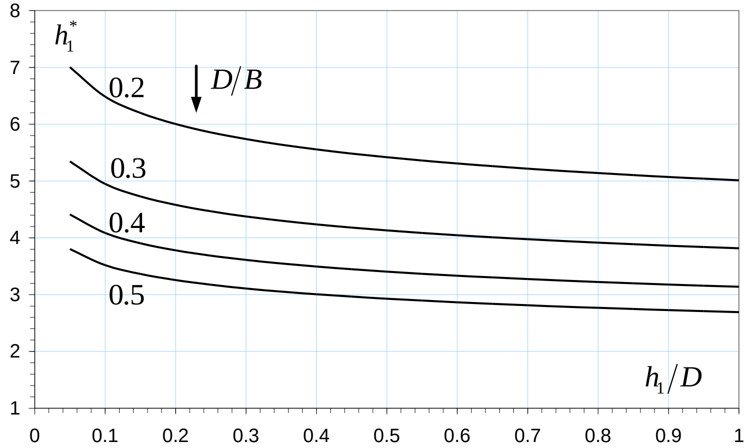

Figure 2: Variation of h_1^* with h_1/D for some values of D/B and $D/P=2$ according to Eq. (18).

Eq. (1) gives the discharge Q through the approach rectangular channel as:

$$Q = \sqrt{g} B h_{1c}^{3/2} \tag{19}$$

which can be rewritten as:

$$Q = \sqrt{g} B \frac{h_1^{3/2}}{h_1^{*3/2}} \tag{20}$$

or as:

$$Q = \frac{B\sqrt{g}}{h_1^{*3/2} h_1} h_1^{5/2} \tag{21}$$

It is obvious that Eq. (21) can be written as function of $h_1^{3/2}$ when dropping h_1 in the denominator, however expressing it under the form on $h_1^{5/2}$ is consistent to the dimensional analysis of the problem. Eq. (21) can finally be written as follows:

$$Q = \omega h_1^{5/2} \tag{22}$$

in which:

$$\omega = \frac{B\sqrt{g}}{h_1^{*3/2} h_1} \tag{23}$$

Because of viscous effects, head loss occurring over the crest and unequal velocity distribution, theoretical discharge is higher than effective one. Consequently, Eq. (22) must be corrected by a factor *k* which takes into account real flow effects. That is, the actual discharge is computed as:

$$Q = k\omega h_1^{5/2} \tag{24}$$

To determine the correction factor *k*, a comparison with experiments reported in literature was done. Principally, experiments carried out by Greve (1932) are used where several circular weirs from 0.076 m to 0.76 m in diameter were considered resulting in a total of 148 runs. Vantakhah (2010) performed a correlation of all Greve's experimental data and obtained an equation for the discharge coefficient for which 94% of the data have percentage error values less than 2.5%. This model was used as a reference to perform the correction of equation (22). A regression analysis of the data shows that the best fitting model is obtained for the *Hoerl* function with a coefficient of determination $R^2 = 0.999$, resulting in the following *k* relationship:

$$k = 1.49\left(0.66\right)^{\eta} \eta^{0.31} \tag{25}$$

Using expressions (25) and (23), the discharge relationship (22), after combining h_1 variable for simplicity, reads then:

$$Q = 1.49\left(0.66\right)^{\eta} \eta^{0.31} \frac{B\sqrt{g}}{h_1^{*3/2}} h_1^{3/2} \tag{26}$$

Comparison of Eq. (26) with Vantakhah's expression (Vantakhah, 2010) for the discharge estimation in the practical range of $0.1 \le \eta \le 0.95$ shows a maximum deviation of 0.8 % only.

Note that the correction factor *k* was deduced for the same conditions of deriving Stevens' equation and Vatankhah's equation i.e. the general installation conditions ($D/B \le 0.5$ and $D/P \le 2$ etc.). Under different conditions, another correction must be applied. It should be mentioned that, unlike classical formulas, Eq. (24) does not need correction for accounting the approach velocity effect, i.e. the kinetic energy in the approach channel, but only for viscous and velocity distribution effects. Figure (3) presents the variation of $k\omega$ term as function of the filling ratio for the aforementioned installation conditions.

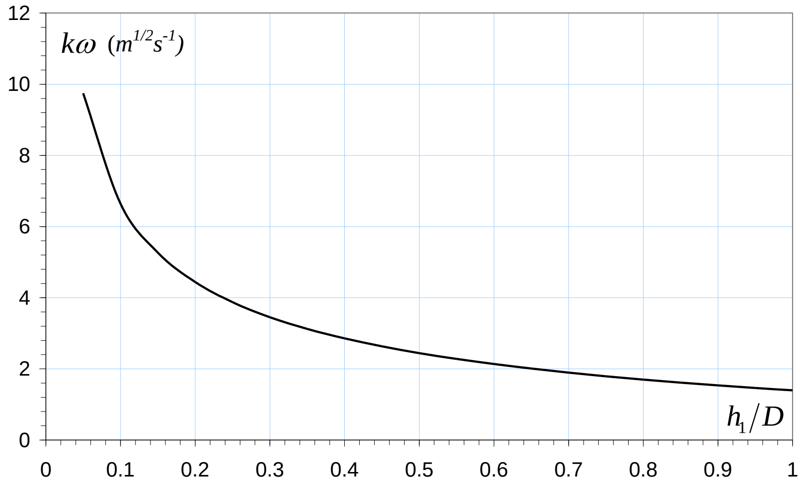

Figure 3: Variation of the parameter $k\omega$ **as function of** h_1/D **for** D/B =0.5 **and** $D/P=2$

DISCHARGE COEFFICIENT

The general flow discharge relationship Eq. (24) can be handled to be under the form of classical circular weir expression, i.e. as function of a discharge coefficient C_d. To do so, Eq. (23) is rewritten as:

$$\omega = \frac{\sqrt{g}}{h_1^{*3/2}\eta\frac{D}{B}} \tag{27}$$

Note that the ω function takes into account all parameters influencing the discharge through a circular sharp-crested weir such that $\omega = f\left(D/B, P/h_1, h_1/D\right)$, including in particular the contraction ratio D/B and the relative depth h_1/P , or the relative weir crest height P/h_1 accounting for the effect of approach velocity which is so far not considered in available formulas in the literature.

Replacing Eq. (27) together with the expression of *k* in Eq. (25) into Eq. (24) leads after some manipulations to:

$$Q = 1.49(0.66)^{\eta}\,\eta^{1.81}\sqrt{g}\left(\frac{D}{B}\right)^{-1} h_1^{*-3/2}\,D^{5/2} \tag{28}$$

which can be written in the following simple form:

$$Q = C_d \sqrt{2g}\, D^{5/2} \tag{29}$$

where the discharge coefficient C_d is expressed as:

$$C_d = \frac{1.49}{\sqrt{2}} (0.66)^{\eta} \eta^{1.81} \left(\frac{D}{B}\right)^{-1} h_1^{*-3/2} \tag{30}$$

It can be seen that actually the discharge coefficient is such that $C_d = f\left(\eta,\, D/B,\, P/h_1\right)$ or as $C_d = f\left(\eta,\, D/B,\, P/D\right)$. Comparing flow discharge expressions Eq. (29) and Eq. (24), the ω parameter can be considered as a weir coefficient having a dimension in contrast with the discharge coefficient C_d which is dimensionless.

RESULTS AND DISCUSSION

For the validation of the developed theoretical approach a comparison with experimental data was realized. Results concern the determination of flow discharge as function of the circular weir parameters and flow depth over its crest.

Figure (4) shows comparison between measured discharges by Stevens (1957) and the present theoretical model. Three different diameters *D* of the sharp crested semicircular weir are used (0.2 m, 0.5 m and 0.8 m). The crest height of these weirs is equal to the radius of the control section of the semicircular weir, i.e. $D/P=2$. It can be seen from this figure a very good agreement between the results obtained from the proposed model and experimental data. The maximum deviation for the three test cases is less than 5.95 %. Note however a slightly higher discrepancy for very small filling ratios i.e. h_1/D=0.5 due to the restriction on the validity of the correction factor *k* obtained for $\eta \geq 0.1$.

In the second validation test, comparison against experimental data of Greve (1932) was performed. Fig. (5) shows the comparison of the proposed approach and the experimental observation for three diameters D (0.203 m, 0.305 m and 0.457 m). As for the precedent test case, there is excellent match between predicted and measured discharges. The maximum deviations are respectively for the aforesaid diameters 1.4 %, 2 % and 12.4 %. The later error was noticed for the smallest filling ratios. It is worth to note that slightly larger maximum deviation is noticed when using Vantakhah's equation.

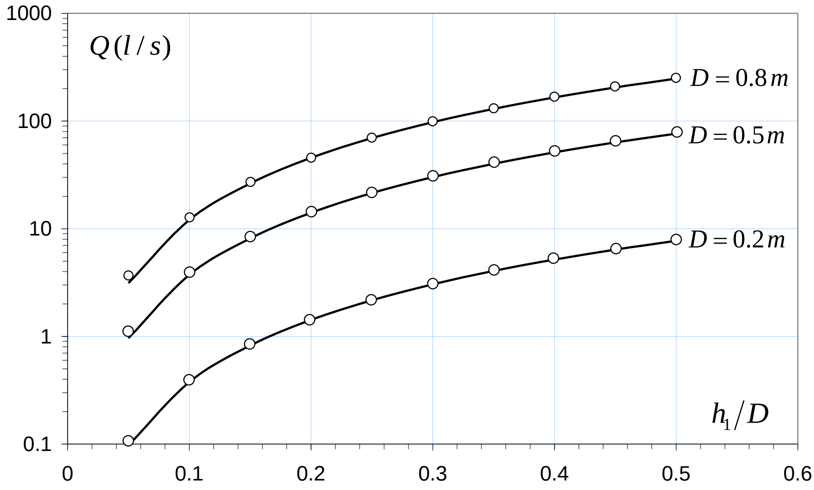

Figure 4: Comparison between: (—) computed discharge using Eq. (26) and (○) measured discharge by Stevens (1957)

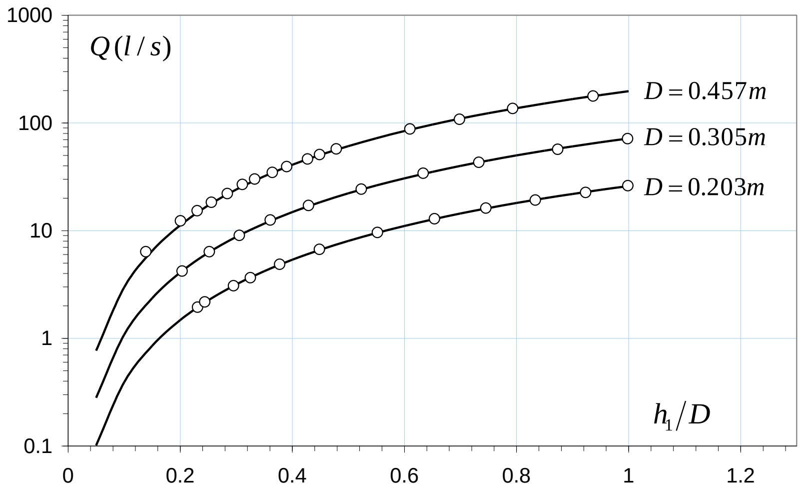

Figure 5: Comparison between: (—) computed discharge using Eq. (26) and (○) measured discharge by Greve (1932)

CONCLUSIONS

Using the critical flow theory, a novel approach to stage-discharge relationship was developed for circular sharp-crested weirs in open channel flow. The presented theory takes account of kinetic energy of approaching flow which is neglected in classical theories. Based on experimental data a correction function has been considered regarding the simplifying hypothesis underlying the present approach. For validation purpose, the comparison was done between the results obtained from the proposed model and experimental data of earlier researches; this comparison shows a very good agreement and confirms the accuracy of the theoretical relationship in a wide range of the filling ratio such as $0.1 \leq \eta \leq 0.95$.

Finally, although the present theoretical model is general and accounts of the velocity approach effect, the correction function needs however a slight modification when the ratios *D/B* and *D/P* are different from that for which the actual correction function was established. Note that an extensive experimental study is needed in such case to consider accurately the influence of the aforementioned ratios on the stage-discharge relationship and obtain a more general correction function.

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