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THEORETICAL DISCHARGE COEFFICIENT RELATIONSHIP FOR A CONTRACTED TRIANGULAR NOTCH WEIR. EXPERIMENTAL ANALYSIS FOR THE SPECIAL CASE OF THE 90-DEGREE V-NOTCH

RELATION THEORIQUE DU COEFFICIENT DE DÉBIT DES DEVERSOIRS CONTRACTES TRIANGULAIRES ANALYSE EXPÉRIMENTALE POUR LE CAS PARTICULIER D'UN ANGLE D'OUVERTURE DE 90°

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ABSTRACT

The study proposes a theoretical development in order to determine the relationship which governs the discharge coefficient of a contracted triangular weir. The theoretical approach takes into account the effect of the approach flow velocity which can be considered as a novelty in comparison with previous studies. The theoretical development consists in applying the energy equation between two well chosen sections with certain simplifying assumptions, namely: i) the head loss and the effect of both viscosity and surface tension are neglected, ii) the hydrostatic distribution of the pressure is considered hydrostatic, iii) the effect of flow streamlines curvature over the weir is neglected. The first section chosen is located in the supply channel at a certain distance upstream of the weir, while the second section corresponds to the location of the weir supposed to be crossed in a critical flow regime. Based on the data available in the literature on the 90° opening angle triangular weir, the experimental and theoretical discharge coefficients are compared and relative errors varying between 4% and 10% are observed. The theoretical discharge coefficient relationship is thus adjusted to be in conformity with the experimental data

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relative to this apex angle. The corrected theoretical relationship causes a maximum deviation of 1.5% on the calculation of the discharge coefficient, but this error is observed only for the ratio $P/B = 1$. For the other values of P/B, the maximum error varies between 0.35% and 1.3%, where P is the weir crest height and B is the width of the rectangular channel of approach.

Keywords: 90° V-notch, weir, discharge coefficient, weir crest height, approach velocity.

RESUME

L'étude propose un développement théorique afin de déterminer la relation qui régit le coefficient de débit d'un déversoir triangulaire contracté. L'approche théorique prend en compte l'effet de la vitesse d'approche de l'écoulement, ce qui peut être considéré comme une nouveauté par rapport aux études précédentes. Le développement théorique consiste à appliquer l'équation de l'énergie entre deux sections bien choisies avec certaines hypothèses simplificatrices, à savoir : i) la perte de charge et l'effet à la fois de la viscosité et de la tension superficielle sont négligés, ii) la distribution hydrostatique de la pression est considérée comme hydrostatique, iii) l'effet de la courbure des filets liquides de l'écoulement au-dessus du déversoir est négligé. La première section choisie se situe dans le canal d'amenée à une certaine distance à l'amont du déversoir, tandis que la deuxième section correspond à l'endroit du déversoir supposé être franchi en régime d'écoulement critique. Sur la base des données disponibles dans la littérature sur le déversoir triangulaire à angle d'ouverture de 90 °, les coefficients de débit expérimental et théorique sont comparés et des erreurs relatives variant entre 4% et 10% sont observées. La relation du coefficient de débit théorique est alors ajustée pour être conforme aux données expérimentales relatives à cet angle au sommet. La relation théorique corrigée entraîne un écart maximal de 1,5% sur le calcul du coefficient de débit, mais cette erreur n'est observée que pour le rapport $P/B = 1$. Pour les autres valeurs de P/B, l'erreur maximale varie entre 0,35% et 1,3%, où P est la hauteur de pelle du déversoir et B est la largeur du canal rectangulaire d'approche.

Mots clés : Déversoir triangulaire, échancrure 90°, coefficient de débit, hauteur de pelle, vitesse d'approche.

INTRODUCTION

Weirs are designed to measure the volumetric rate of water flow (Achour et al., 2003; Bos, 1976; 1989). The V-notch sharp-crested weir is one of the most precise discharge measuring devices suitable for a wide range of flow, used in laboratories and industry. In international literature, the 90° V-notch sharp-crested-weir is frequently referred to as the 'Thomson weir' (Thomson, 1858; 1861). Thomson recommended a constant value of the discharge coefficient C_d such that $C_d = 0.593$. However, according to the literature (USBR, 1997), the discharge coefficient C_d of a triangular weir is expressed by the following functional relationship $C_d = f(h_1 / B, h_1 / P, \alpha, h_1)$, where h_1 is the water depth measured above the vertex of the weir (Fig. 2), B is the width of the rectangular channel of approach, P is the weir crest height which represents the vertical distance between the vertex of the triangular notch and the bottom of the channel of approach, α *is* the apex angle, i.e. the opening angle of the notch.

The h_1/B ratio expressed by the previous functional relationship, in combination with the angle α, is a dimensionless parameter characterizing the width contraction which approximates the meaning of the b/B ratio for rectangular-notch weirs (Kindsvater and Carter, 1959). In order to play the same role of contraction effect, the P/B ratio is more suitable because it is a constant for a given weir installation. This ratio is obtained by dividing h_1/B by h_1/P, which is one of the three influencing dimensionless parameters included in the relationship. The new formulation of the discharge coefficient is then $C_d = f(h_1 / P, P / B, \alpha)$. The research workers Kulin and Compton (1975) proposed an experimental graph of the discharge coefficient $C_d = f(h_1 / P, P / B)$ only valid for a notch angle of 90° (Fig. 4). This graph will be the basis of the verification of the relationships resulting from the theoretical development, i.e. the results given by the theoretical discharge coefficient relationship will be contrasted to those given by the graph. Interesting conclusions will then be drawn concerning the accuracy of the proposed relationships.

In the present study, the theoretical approach is based on the energy equation (Henderson, 1966; Bos, 1989; Achour, 1989) applied between two well chosen sections. The first section is located upstream of the device, while the second section is at the location of the weir assumed to be crossed in a critical flow regime. Installing a weir in an open channel causes critical depth to form over the weir. It should be noted that, unlike previous studies, the approach velocity of the flow is taken into account.

DESCRIPTION OF THE DEVICE

Fig. 1 shows an overview of the rectangular approach channel of width B and the triangular weir of apex angle α. It is a symmetrical V-shaped notch in a vertical thin plate. The weir is characterized by a weir crest height P, i.e. the height of the notch vertex with respect to the floor of the approach channel. The side wall of the device is inclined at an angle θ with respect to the horizontal. The depth h_1 corresponds in fact to the upstream water depth measured above the vertex of the notch. The depth measuring section 1-1 (Fig. 2) is located a sufficient distance upstream of the device in order to avoid the surface drawdown zone. On the other hand, the depth measuring section must be close enough to the device so that the energy loss between sections 1-1 and 2-2 can be neglected. The discharge flowing through the approach rectangular channel is Q. One may define m as the side slope of the notch such that m = cotg (θ), i.e. m horizontal to 1 vertical.

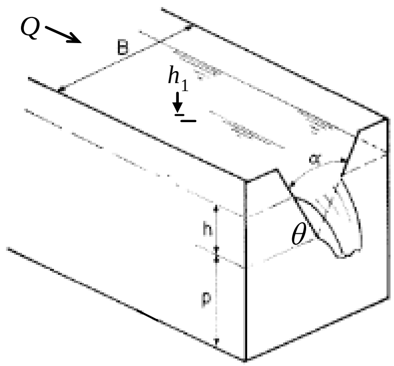

Figure 1: Definition sketch of the studied contracted weir

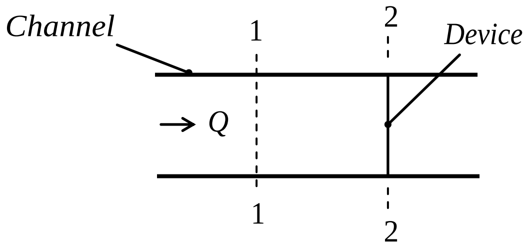

Figure 2: Plan view of the channel and the device

THEORETICAL DISCHARGE COEFFICIENT RELATIONSHIP

The critical depth in the rectangular cross-section 1-1 (Fig. 2) is written as:

$$h_{1c} = \left(\frac{Q^2}{g B^2} \right)^{1/3} \tag{1}$$

where the subscript « *c* » denotes the critical conditions.

On the other hand, the critical depth in the triangular cross-section 2-2 (Fig. 2) is as:

$$h_{2c} = \left(\frac{2Q^2}{g m^2} \right)^{1/5} \tag{2}$$

where $m = \text{cotg}\,\theta$ is the side slope of the notch: *m* horizontal to 1 vertical.

Eliminating the quantity Q^2 between Eqs. (1) and (2) results in:

$$h_{2c} = \left(\frac{2B^2}{m^2} \right)^{1/5} h_{1c}^{3/5} \tag{3}$$

Assume that there is no head loss between sections 1-1 and 2-2. Equal total heads between sections 1-1 and 2-2 translates into:

$$H_1 = H_2 = \frac{5}{4} h_{2c} \tag{4}$$

Combining Eqs. (2) and (4) results in:

$$H_1 = \frac{5}{4} \left(\frac{2B^2}{m^2} \right)^{1/5} h_{1c}^{3/5} \tag{5}$$

Taking into account the approach flow velocity, total head H_1 can be written as:

$$H_1 = h_1 + \frac{Q^2}{2gB^2 \left(h_1 + P \right)^2} \tag{6}$$

where h_1 is the upstream depth above the apex of the weir, and P is the crest weir height (Fig. 2). Eq. (6) can be rewritten as:

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{Q^2}{2gB^2 \left(h_1 + P \right)^2 h_{1c}} \tag{7}$$

Eq. (7) can be written as:

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{Q^2}{2gB^2 h_1^2 \left(1 + P / h_1 \right)^2 h_{1c}} \tag{7a}$$

Eq. (1) allows writing that:

$$\frac{Q^2}{gB^2} = h_{1c}^3 \tag{1a}$$

Combining Eqs. (7a) and (1a) yields:

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{1}{2(h_1 / h_{1c})^2 (1 + P / h_1)^2} \tag{8}$$

Eqs. (5) and (8) give what follows:

$$\frac{h_1}{h_{1c}}+\frac{1}{2(h_1/h_{1c})^2(1+P/h_1)^2}=\frac{5}{4}\left(\frac{2B^2}{m^2}\right)^{1/5}\frac{1}{h_{1c}^{2/5}} \tag{9}$$

Eq. (9) can be rewritten as:

$$\frac{h_1}{h_{1c}}+\frac{1}{2(h_1/h_{1c})^2(1+P/h_1)^2}=\frac{5}{4}\left(\frac{2B^2}{m^2h_1^2}\right)^{1/5}\frac{h_1^{2/5}}{h_{1c}^{2/5}} \tag{9a}$$

Let us adopt the following non-dimensional parameters:

$$h_1/h_{1c}=h_1^* \tag{10}$$

$$M_1=\frac{mh_1}{B} \tag{11}$$

$$P^*=P/h_1 \tag{12}$$

Inserting Eqs. (10), (11) and (12) into Eq. (9a) results in:

$$h_1^*+\frac{1}{2h_1^{*2}(1+P^*)^2}=\frac{5}{4}\left(\frac{\sqrt{2}}{M_1}\right)^{2/5}h_1^{*2/5} \tag{13}$$

After some rearrangements Eq. (13) reduces to:

$$h_1^{*3}-\frac{5}{4}\left(\frac{\sqrt{2}}{M_1}\right)^{2/5}h_1^{*12/5}+\frac{1}{2(1+P^*)^2}=0 \tag{14}$$

In practice, the known parameters are M_1 and P^* which will be used to deduce h_1^* by solving the implicit Eq. (14). Note that the flow in the section 1-1 is subcritical, meaning that $h_1>h_{1c}$ or $h_1^*>1$.

Eq. (1) gives the discharge Q through the rectangular channel of approach as:

$$Q=\sqrt{g}Bh_{1c}^{3/2} \tag{1b}$$

which can be rewritten as:

$$Q = \sqrt{g}\, B \frac{h_1^{3/2}}{h_1^{*\,3/2}} \tag{1c}$$

or as:

$$Q = \sqrt{g}\, \frac{B}{h_1} \frac{h_1^{5/2}}{h_1^{*\,3/2}} \tag{1d}$$

On the other hand, the discharge Q flowing through the V-notch (Fig. 1) is given by Shen's formula (1981). Neglecting the effect of viscosity and surface tension, this formula can be written as:

$$Q = C_d \frac{8}{15} \sqrt{2g}\, m\, h_1^{5/2} \tag{15}$$

Eliminating the discharge Q between Eqs. (1d) and (15) results in the following theoretical discharge coefficient relationship:

$$C_{d,Th} = \frac{15}{8\sqrt{2}} \frac{1}{M_1 h_{1,Th}^{*\ 3/2}} \tag{16}$$

where the subscript "*Th*" denotes "Theoretical". This is the general theoretical discharge relationship valid for all angles of the V-notch.

Fig. 4 shows that the experimental discharge coefficient for partially contracted 90-degree V-notch depends on both P/B and h_1/P. In order to use these parameters, the fundamental Eq. (14) can be rewritten as:

$$h_1^{*3} - \frac{5 \times 2^{1/5}}{4} \left(m \frac{h_1}{P} \times \frac{P}{B} \right)^{-2/5} h_1^{*12/5} + \frac{1}{2} \left(\frac{h_1/P}{1 + h_1/P} \right)^2 = 0 \tag{14a}$$

Eq. (16) can also be rewritten in the following form:

$$C_{d,Th} = \frac{15}{8\sqrt{2}} \frac{1}{\left(m \frac{h_1}{P} \times \frac{P}{B} \right) h_{1,Th}^{*\ 3/2}} = f(h_1/P; P/B; m) \tag{16a}$$

It is obvious that the theoretical development shows that the discharge coefficient depends on P/B, h_1/P, and m as predicted by the experiment. Eq. (16a) is theoretically applicable to triangular notch weirs of any apex angle. However, its experimental verification will be based on the data available on the apex angle of 90 °.

SPECIAL CASE OF PARTIALLY CONTRACTED 90-DEGREE V-NOTCHES

For this type of weir, the apex angle of the notch is 90 °, i.e. m = 1. The experimental effective discharge coefficient $C_{de,Exp}$ can be rated using Fig. 3.

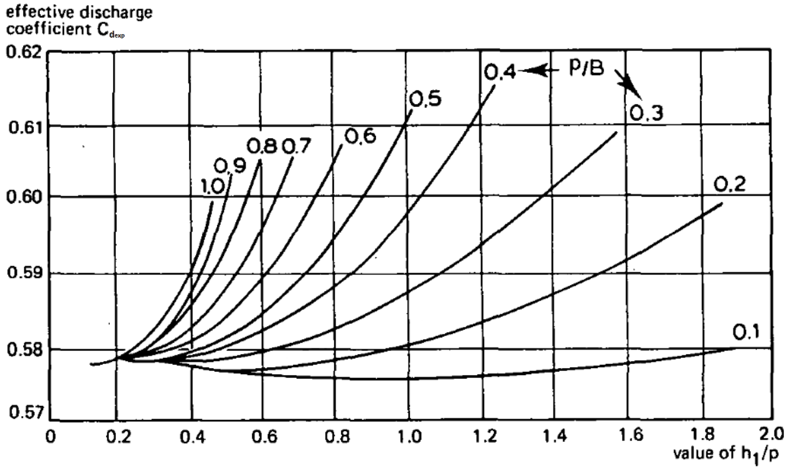

Figure 3: Experimental discharge coefficient for partially contracted 90-degree V-notches (Kulin and Compton, 1975).

If we were to take into account the effects of surface tension and viscosity, the measured depth h_1 should be adjusted by a factor k_h. Thus, Eq. (15) must be written as:

$$Q = C_d \frac{8}{15}\sqrt{2g}\ \left(h_1 + k_h\right)^{5/2} \tag{17}$$

The factor k_h is given graphically in the literature for 90-degree V-notch and it seems that it is approximately equal to 0.003 feet corresponding to roughly 0.001 m. Therefore, Eq. (17) is such that:

$$Q = C_d \frac{8}{15}\sqrt{2g}\ \left(h_1 + 0.001\right)^{5/2} \tag{17a}$$

Eq. (17a) can be rewritten in the following traditional form:

$$Q = C_d \left(1 + \frac{0.001}{h_1}\right)^{5/2} \frac{8}{15}\sqrt{2g}\ h_1^{5/2} \tag{17b}$$

One may then define a new discharge coefficient C_{d0} such as:

$$C_{d0} = C_d \left(1 + \frac{0.001}{h_1}\right)^{5/2} \tag{18}$$

For *m* = 1, Eq. (14a) becomes:

$$h_1^{*3} - \frac{5 \times 2^{1/5}}{4} \left(\frac{h_1}{P} \times \frac{P}{B}\right)^{-2/5} h_1^{*12/5} + \frac{1}{2} \left(\frac{h_1 / P}{1 + h_1 / P}\right)^2 = 0 \tag{14b}$$

Eq. (16a) is reduced to:

$$C_{d,Th} = \frac{15}{8\sqrt{2}} \frac{1}{\left(\frac{h_1}{P} \times \frac{P}{B}\right) h_{1,Th}^{*\ 3/2}} \tag{16b}$$

Eq. (14b) is shown graphically in Fig.4. One may observe that for a given value of *P/B*, h_1^* decreases with the increase of h_1 / P. On the other hand, for a given value of h_1 / P, h_1^* decreases with the increase of *P/B*.

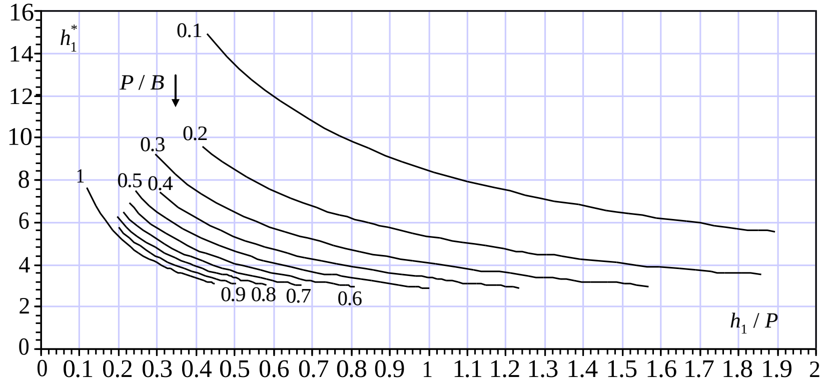

Figure 4: Variation of h_1^* with h_1 / P for some values of *P/B* according to Eq. (14b)

The deviation between theoretical C_d given by Eq. (16b) and experimental C_d given by Fig. 3 varies in the range [4%; 10%] depending on the value of *P/B* and h_1 / P. It should be noted that the theoretical discharge coefficient relationship was obtained by considering some simplifying assumptions with regard to head loss, pressure distribution of the flow passing over the weir, and both effects of viscosity and surface tension.

Calculations show that the relative error can be reduced to less than 2% if the experimental discharge coefficient is calculated by the following simple formula:

$$C_{d,Exp} = 4.1889 C_{d,Th} - 1.6668 \quad (19)$$

The advantage of this relation lies in the fact that it applies to all the values of *P/B* which have not been the subject of experimental investigations and such as $0.1 \le P/B \le 1$. Furthermore, the relationship applies to 90° V-notches weirs regardless of whether they are partially contracted or fully contracted, i.e. independently of the value of h_1/B.

If a smaller deviation is sought, the following relationship, a little more elaborate, is more appropriate giving a maximum deviation of 1.5% observed only for *P/B* = 1 and for $0.1163 \le h_1/P \le 0.1853$:

$$C_{d,Exp} = \left(4.1889 C_{d,Th} - 1.6668\right)\left(0.024\frac{P}{B} + 0.985\right) \quad (20)$$

Table 1 groups the maximum deviation caused by the relationship (20), along with Eqs. (14a) and (16b), according to the values of *P/B* indicated in Fig.3. The dimensionless theoretical parameter h_1^* is derived from the implicit Eq. (14a), knowing P/B and h_1/P values. The equation can be easily solved using a handheld calculator's solver. In this study, the h_1^* calculations were performed using the TI-84 Plus handheld calculator's solver.

Table 1: Maximum deviation caused by Eq. (20), along with Eqs. (14a) and (16b), for some values of *P/B*

P/B	**Max. Deviation %**
1	1.518
0.9	1.243
0.8	1.304
0.7	1.072
0.6	0.712
0.5	0.421
0.4	0.353
0.3	1.143
0.2	0.588
0.1	0.526

CONCLUSIONS

The study was devoted to the triangular-notch thin-plate Weir. The main objective was to define the discharge coefficient relationship by a theoretical approach. By choosing two sections, one upstream of the device and the other at the location of the weir crossed in a critical flow regime, the energy equation was applied, assuming some simplifying assumptions, in particular the head loss and the effect of both viscosity and surface tension have been neglected, the pressure distribution was assumed to be hydrostatic. It should be noted that the approach flow velocity is taken into account in the theoretical development which led to the establishment of Eq. (14a), applicable to any triangular weir. The relationship contains the dimensionless parameters h_1 / P and *P/B* as predicted by laboratory experimentation. The equality of discharges in the approach channel and crossing the weir allowed identifying the discharge coefficient governed by Eq. (16a). Applied to the case of the triangular weir with an opening angle of 90 °, Eq. (16a) was compared to available experimental data. The results of the calculation made it possible to establish a linear relationship between the theoretical and experimental discharge coefficients [Eq. (19)]. This causes a maximum deviation of less than 2%. If the user wants a smaller deviation, Eq. (20) is the most appropriate. It causes a maximum error of less than 1.5% for *P/B* = 1 and for $0.1163 \le h_1 / P \le 0.1853$. For other *P/B* values, the maximum error varies between 0.35% and 1.3% only, as shown in table 1.

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